Eötvös Loránd University

Integrable boundary sigma models

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THESIS STATEMENTS

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Introduction

The (1+1)-dimensional $O(N)$ sigma models are asymptotically free in perturbation theory and their classical conformal invariance is broken by a dynamically generated mass scale therefore these models are useful toy examples of quantum chromodynamics. Some special sigma models are integrable quantum field theories which implies that some physical quantities are exactly calculable. These models include the principal chiral models and the $O(N)$ sigma models. These are also relevant for AdS/CFT.

In this thesis, the principle chiral models and the $O(N)$ sigma models with boundaries are investigated. Classically, the integrability of field theories can be defined in different ways. One option is that the integrable theories are those that have zero curvature descriptions, i.e., their equation of motion can be written as the flatness condition of the so-called Lax connection. This representation can be used to construct infinitely many conserved charges. The zero curvature representation can be generalized to theories with boundaries which implies that the boundary condition can be written as the boundary flatness condition which contains the so-called classical reflection matrix.

At quantum level, the theory is integrable if there is no particle creation/annihilation at scattering processes and the n-particle scatterings factorize into 2-paricle scatterings uniquely which implies that the 2-

particle S-matrices satisfy the famous Yang-Baxter equation. Integrability can be generalized to theories with boundaries: a theory on the half line is integrable if there is no particle creation/annihilation at boundary scattering processes and the n-particle reflection factorizes into 2-particle bulk scatterings and 1-particle reflections uniquely which implies that the 1-paricle reflection matrices satisfy the boundary Yang-Baxter equation.

Integrability allows to calculate physical quantities (for example the energy spectrum) at finite volume. At asymptoticaly large volume we may regard the system as a free gas with point like interactions. The energy of the system is the sum of the one-particle energies therefore we can calculate the energy spectrum if we know the possible momenta of the particles. The quantization conditions of the momentas are the Bethe-Yang equations. The solution of these can be reduced to the diagonalization of the quantum transfer matrix. There are several methods for determining eigenvalues, e.g. coordinate-, algebraic-, analytical Bethe Ansatz. Among them, the coordinates- and the algebraic Bethe Ansatz can be used to obtain the eigenvectors, too.

In this thesis I investigated classical boundary conditions of principal chiral model and $O(N)$ sigma model, I classified the rational solutions of the boundary Yang-Baxter equation and I applied the algebraic Bethe Ansatz for the $SO(2N)$ sigma model on the finite strip.

Thesis statements

1. Spectral parameter dependent classical reflection matrices

In principal chiral models, I found new classical reflection matrices which depend non-trivially on the spectral parameter. I showed that these are consistent solutions of the boundary zero curvature condition. Using the local isometry $SO(4) \cong SU(2) \times SU(2)$, the new reflection matrix of the $SU(2)$ principal chiral model implies a new reflection matrix for $O(4)$ sigma model which can be generalized to $O(2N)$ sigma models. Furthermore, I have shown that these new solutions satisfy the classical boundary Yang-Baxter equation [3].

2. Classification of the classical boundary Yang-Baxter equation

I developed the Poisson algebra of the double row monodromy matrices including general field dependent κ-matrices. I showed that the existence of infinitely many conserved charges in involution implies that the κ-matrix satisfies the boundary Yang-Baxter equation. I systematically looked for solutions of this equation. The result of it suggested that a κ-matrix is a solutions of the boundary Yang-Baxter equation if and only if it is a consistent solution of the boundary flatness equation, too. In addition, I have determined a necessary condition for κ-matrices which are solutions of the boundary Yang-Baxter equation. Comparing this necessary condition and the explicitly constructed solutions, we can see that we have found all the solutions [3, 4].

3. Classification of the residual symmetry of the rational K-matrices

I proved a theorem which says that if the boundary breaks the bulk symmetry G to H then G/H has to be a symmetric space. This theorem allows us a systematic search of K-matrices, since the possible subgroups H are significantly narrowed down, which fixes the structure of the K-matrix.

4. Application of the algebraic Bethe Ansatz to $O(2N)$ sigma models with boundaries

I applied the algebraic Bethe Ansatz (ABA) to the $SO(2N)$ sigma model on the finite strip in two different ways. In the first method [1], the recursion is

$$SO(2N) \to SU(N) \to SU(N-1) \to \ldots \to SU(2)$$

Unfortunately, this method cannot be applied to $SO(2M+1) \times SO(2N-2M-1)$ symmetric boundaries. That is why I implemented another method [3], where the recursion is

$$SO(2N) \to SO(2N-2) \to SO(2N-4) \to \ldots \to SO(4)$$

I obtained that the $SO(2M+1) \times SO(2N-2M-1)$ symmetric boundaries lead to non-standard Bethe Ansatz equation. The reason was that the boundary decreased the rank of the symmetry algebra therefore there were less Bethe roots than in the periodic case.

References

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