Eötvös Loránd Tudományegyetem  
Bölcsészettudományi Kar  

Doktori Disszertáció  

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Deontic Logic and Formalizing Rights  

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Elnök: E. Szabó László, DSc, CSc, Habil.  
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Budapest, 2017
ADATLAP
a doktori értekezés nyilvánosságára hozatalához

I. A doktori értekezés adatai
A szerző neve: Markovich Réka
MTMT-azonosító: 10048348
DOI-azonosító: 10.15476/ELTE.2017.179
A doktori iskola neve: Filozófiatudományi Doktori Iskola
A doktori iskolán belüli doktori program neve: Logika és Tudományfilozófia
A témavezető neve és tudományos fokozata: Zvolenszky Zsófia, PhD, Habilit.
A témavezető munkahelye: egyetemi docens, Logika Tanszék, Filozófia Intézet

II. Nyilatkozatok
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a doktori értekezés szerzőjének aláírása
Deontic Logic and Formalizing Rights

Actions, Agents and Relations in the Hohfeldian Theory and Its Formalization

Réka Markovich
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1
Chapter 1

Motivation and Preface

My mother has a story about my childhood: I was five when I told her that I was going to be a lawyer when I grow up. Albeit I had a solid interest in natural sciences and mathematics—because of which I went into a science specialization in high school—, I was sticking to my early decision and enrolled in law school. One of my first courses there was logic, where I said: "That’s why I am here". For which utterance of mine a friend advisably whispered to me: "Well, this is not law". I didn’t believe him. Since back then I thought that everything I had known about argumentation, inferences and their beauty culminated in law (and given I had no idea then that there is such a thing as a logician profession), I was quite satisfied with my choice, and completed my law degree five years later. Working in one of the most interesting areas of law—advertising law—I still felt later, though, as something was missing from the way I used my brain in everyday work. So I started to attend logic classes at ELTE’s Science Faculty, and was delighted to apply to the brand new Logic and Theory of Science master’s program launched in 2010 at the Faculty of Humanities. The decision two years later to quit my job as a lawyer and become a researcher instead had many reasons, but the main one is that I believed—and still believe now—that logic should play a far greater role in law, that law really could be an area in which reason and rationality culminate, and I still have the vision that we can utilize powerful mathematical and philosophical tools to make progress in this direction.

This thesis extends beyond logical considerations and theories, touching upon issues in the philosophy of law and philosophy of language—reflecting nicely to the disciplinary complexity the topic discussed here involves.

As the reader already knows from my life-story above, first I was a lawyer. This is partly why I am confining my inquiry in the legal interpretation of the various conceptions and sentences I will discuss in this dissertation, even
though, in many cases, they have alternative normative interpretations also, e.g. in ethics.

Law as a system of norms is a crucial subsystem of the society, the primary medium of human institutions. By its nature it belongs to a given society, a given state. Since I have been raised in Hungary, was taught in a Hungarian university of law, and worked as a lawyer with Hungarian legal texts, the Hungarian—and therefore a civil-law-based—legal culture and system inescapably determine my legal thinking. This does not just influence the examples and references I use, but—even when viewed with utmost care—might affect my way of thinking about law in general, therefore also my conclusions. I have no intention to overplay it, but want to call the reader’s attention to the fact—and that I am aware of it.

Acknowledgements

There are a lot of people who played a smaller or a greater role in completing this dissertation. First of all, of course, I thank to my supervisor, Zsófia Zvolenszky, whose professionalism, dedication and benignity make her a role model and whom having as an academic ancestor makes me at least as proud as having Franz Brentano, Arthur Prior or Kit Fine. But I am also grateful to the other colleagues of the Department of Logic for having me as a student and being so great professors and mates—especially to András Máté for launching and persistently managing the master’s program without which I would never have been a PhD student; and Attila Molnár, from whom I learnt everything that I know about modal logic. I owe many thanks to Kinga Pázmándi, the then head of the Department of Business Law for letting me do my research and showing trust, steadiness and affection on a level that makes her another role model of mine. I am grateful to István Szakadát and Gábor Hamp, since they were the first ones I had long discussions with and shared enthusiasm about Hohfeld, and the ones with whom I experienced what a great thing it is to have research colleagues. I thank the suggestions and comments of Mátyás Bódig, Ferenc Csaba, Valentin Goreenko, Guido Governatori, Zalán Gyenis, John F. Hory, Joris Hulstijn, David Makinson, Paul McNamara, Péter Mekis, Xavier Parent, Kinga Pétervári, Corrado Roversi and Giovanni Sartor; and particularly for the detailed comments and help of Miklós Szabó and Antonino Rotolo. I am especially grateful to the two professors I had the longest, liveliest and deepest discussions with about the topic of this dissertation, and from whom I learnt a lot, wishing continuously to have the chance to learn more: Burkhard Schafer and Marek Sergot. And last but far from least: I am thankful to Dániel Kárpáti for his love and amazing support during writing, because of whom becoming a logician is only the second best thing that happened to me at the Department of Logic.
Chapter 2

Introduction

Hohfeld’s analysis (Fundamental Legal Conceptions as Applied in Judicial Reasoning, 1913, 1917) on the different types of rights and duties is highly influential in analytical legal theory. Yet a century later, the formalization of his theory remains, in various ways, unresolved. In this dissertation—after presenting Hohfeld’s original ideas, the classical formalizations of them by Kanger and Lindahl, and the reception and shortcomings of those—I provide my own amended version of the Hohfeldian conceptions’ formal representation.

2.1 Logic and Law

Providing a formal representation for a legal theory immediately presents logicians with a dilemma. Should they engage in a broader discussion about the relationship, history, problems and developments of law and logic in general, or, take the validity of that relationship granted. Even if such a topic seems to provide an excellent opportunity to argue for a strong relationship, the obvious applicability and need for logic in law, also to introduce the prospering research developments, call for resisting the temptation if one wants to keep the focus on the very result they write about. This is how I am going to proceed, thus, after a fast cursory survey of the broader context, the introduction will be restricted only to the most essential and core historical and contextual background.

The topic of logic and law is immensely rich with a long history, and a broad range of exciting, innovative (current) results—most likely because of the recent explosion in progress and interest in AI, including AI’s connections to Law. In Bench-Capon and Prakken [2008] we find a concise survey on how this connection began, expanded and varied—partly in order to provide approaches increasingly relevant to AI. The original use of logic in law, the ground work for explicating this connection has been as representation: "rep-
representation of law in a clear and unambiguous manner." Investigation can concern revealing *syntactic ambiguities* in legislation\(^1\); defining the meaning of *terms* often used in legal norms like ‘may’, ‘must’ and ‘shall’; analyzing the *normative positions and relation* created by these legal norms—which involves the investigation of *actions* regulated by these norms creating the normative positions. As the need for considering the complexity of these norms has been realized, factors like *time* and *change* have become part of investigations. These considerations have required (or contributed to the birth of) different types of logics, starting from *deontic logic*, *action logic*, *dynamic logic*, *legal ontologies* with *description logics*; and then, recognizing the relevance of exemptions and conditions under which the legislation is (or is not) applicable, led to the use of non-monoton logics. This turn in the area of logic, law and AI directed attention to how reasoning actually carried out in law, and resulted in abandoning the classical models of logic. Bench-Capon and Prakken [2008] mention *case-based reasoning*, *practical and teleological reasoning*, *theory formation*, *reasoning about evidence*, *argumentation frameworks*, and *argument schemes* as some of the most prominent research areas in this field that pushed the investigation toward informal logic; meanwhile, research on (legal) interaction put the emphasis of *computational modelling* of legal procedure and *multi-agent systems*.

The topic of this dissertation mostly stays within the representation of legal conceptions and systems as it analyzes a legal theory whose formal representation has remained so far, in various ways, unresolved. A foundational consideration like this can serve, however, as a basis for progressive modelling solutions as the formalized theory is of crucial importance in multi-agent systems. This investigation includes primarily the conceptual formal analysis of normative positions and relations—using deontic logic, but, inevitably, involves the discussion of how actions and their formal representation influence what we can tell about (legal) norms and situations. Therefore, the brief historical survey I start with concerns only these aspects and their immediate context.

### 2.2 Logical Inferences in Law—and Where Inquiries about them Led

The fact that in law we use arguments and inferences seems self-evident. Straightforward and fraught with problems. Such worries have been instrumental in why the whole idea of involving logic in analyzing and handling law is often refused. The story goes further back than Joergen Joergensen phrased his famous dilemma; still, it is practical to make this 1937 dilemma

\(^1\)from the author’s work, see for instance Markovich et al. [2015]
our starting point. It is apparently the case that inferences—which are obviously entities (and operations) featured in a system or theory of logic—are pretty ordinary in law; what is more, using inferences is actually how the application of law occurs: in legal norms we have rules phrased more or less as conditionals like "any person who causes damage to another person wrongfully has to pay for it", which can be translated to "if someone causes damage to another person wrongfully, then he has to pay for it". What happens in the very action of operation of law is that the judge decides whether things that happened can be considered as causing damage to someone else wrongfully. If so, then she applies the legal consequences (which is in itself a thing whose name obviously invokes logic); we can formulate this process as an inference:

Premise 1: If $x$ causes damage wrongfully to $y$, then $x$ has to pay for it (to $y$).
Premise 2: $a$ caused damage wrongfully to $b$.
Conclusion: $a$ has to pay for it (to $b$).

This seems like a nice and classical type of valid inference called *modus ponens*—and this is how law has been working since the beginnings. The problem is, says Joergensen, that the consequence of the conditional in Premise 1, and the conclusion are not that type of sentences that can be either true or false, meanwhile the scope of logic—and therefore of classically valid inferences—extends to true and false sentences only: we define a valid deductive inference as the conclusion must be true if the premises are true. Then how can it be the case that using inferences in law, containing *normative* sentences can work so well, on analogy with valid inferences. Our choice is to develop an extended notion of logical validity, or to forego trusting our intuitions about validity in the above case. Both options are problematic. This then is Joergensen’s dilemma.

One of the attempts to reply—or to solve—the dilemma (as Solt [1996] describes it) is Ota Weinberger’s, according to whom there are three alternative responses:

a) rules of inferences should be extended to be applicable to norms
b) norms should be translated or reduced to propositions, descriptive sentences
c) we should accept that there is no notion of a valid inference that could be applied to norms.

Even Weinberger himself did not consider c) as a real solution, said about b) that it is not possible, therefore, the only solution is a), that is, to extend the rules of logical inferences. Weinberger said we simply need to apply an
analogy between true/false sentences and legally valid/invalid norms. The problem is that this step would only solve the inferences containing exclusively norms. But—as life and law practice show—the paradigmatic cases to cover are mixed inferences, like the one above that we have just found problematic. Another logician sought a solution analogue to b) and founded deontic logic: he is Georg von Wright.

2.3 A Brief Historical Survey of (von Wright’s) Deontic Logic

The starting point of the classic paper Von Wright [1951] is that norms’ way of existence is their validity. Therefore, we can assign to a valid norm a true deontic sentence (‘deontos’ coming from Greek meaning ‘should’ or ‘ought’) and a false one to each invalid norm. Even if von Wright himself exceeds this model later, we still call the system of deontic logic von Wright practically drew in this paper ‘standard’. He introduces the modalities (and modal operators) ‘obligatory’ (\(O\)), ‘permitted’ (\(P\)), ‘forbidden’ (\(F\)) and ‘indifferent’ (\(I\)) which categories affect on actions (\(A\)), and the performance-value (modeled on truth-value)—as the sentence ‘\(A\)’ means ‘the action \(A\) is performed’. On sentences about acts being performed he uses logical constants, where negation is understood as refraining from performing the given act (‘\(\neg A\)’ is to be understood as ‘the action \(A\) is not performed’). von Wright takes the ‘permitted’ as an undefined, basic category in terms of which he defines the others:

\[
\begin{align*}
OA & \iff \neg P\neg A \\
FA & \iff \neg PA \\
IA & \iff PA \land P\neg A
\end{align*}
\]

von Wright does not provide his system in an axiomatic way but describes important theorems on how logical constants influence the truth value of deontic sentences. He, of course, calls attention to the crucial difference between alethic modal logic (basic logic for necessity and possibility) and de-

\[2\] Validity here refers to legal validity and has no relation to logical validity. In civil law countries legal validity means—in brief and simplifying somewhat—the requirement that the given statute was created by those who have the power to and it was created in the way it has to be created. On whether it is enough to consider this feature of law, or we have to involve the feature of being in force too, see Markovich [2014] (in Hungarian)

\[3\] If we take seriously that validity is the existential way of norms, an "invalid norm" does not strictly make sense—but we need to presuppose it does in order to fill in the details of the parallel.

\[4\] Later von Wright’s opinion changes about whether it is really obvious how the modalities can be defined in terms of one another, but in this paper he considers these equivalences obvious.
ontic modal logic. In alethic modal logic

\[ p \rightarrow \lozenge p \]

is a theorem (where \( p \) is a propositional letter), meanwhile the parallel schema

\[ A \rightarrow PA \]

obviously cannot be a theorem of deontic logic (since it would mean that everything that is done is permitted, that is, unlawful actions are never performed). von Wright characterized another schema of deontic logic as defining the connection between obligation and permission:

\[ OA \rightarrow PA \]

To this day this remains the feature of deontic logic distinguishing it from other modal logics. According to this: what is obligatory cannot be forbidden—which is not a strong requirement of a modal system, it means a system cannot to be contradictory (and has to be serial from a semantic point of view, but this comes later).

It is well to highlight the relativity of rules, a point von Wright emphasized at the end of his paper: even if the rules of deontic logic he drew seem to be absolute, they are to be understood as being relative to a given system of norms (which of course can be not just a legal code but a system of moral or religious norms as well).

Deontic Logic is the fundamental paper on deontic logic, but it is by no means the first such paper. von Wright himself mentions that St. Thomas Aquinas, for example, was dealing with the rule according to which an action which commits us to perform a forbidden one is forbidden itself. We also know that Leibniz, who was not only a mathematician but also a lawyer, differentiated the following modalities in Elementa juris naturalis in 1672:

debitum (obligatory) — licitum (permitted) — illicitum (prohibited) — indifferentum (indifferent)

which obviously correspond to the modalities von Wright introduces. It is also usual to consider Jeremy Bentham’s The Principles of Morals and Legislations from 1789 as a precursor, just as Ernst Mally’s book Grundgesetze de Sollens. Elemente der Logik des Willens (The Basic Laws of Ought. Element of the Logic of Willing) from 1926. And if we consider philosophy of law, according to Szabó [1995], Hans Kelsen’s doubts on the logic of norms were among the main inspirations for von Wright.
Whether or not this paper of von Wright can be credited with founding a new discipline—deontic logic—, he certainly did not take Deontic Logic to be his greatest contributions to the file. What he regarded as his masterpiece is Norm and Action from 1963. In this book, beside enhancing his earlier (as he calls here: old) system presented in Von Wright [1951], he also—sometimes radically—criticizes it. One of the main enhancements is working out a complex logic of action, which was not addressed in 1951. In Wright [1963] we can find a structure of action logic built step by step. The starting point is the logic of change. This logic handles propositions \( p, q \), etc. representing states of affairs in the world, and the symbol \( T \) which stands for transition or transfer, having two sides to fill with propositions between which transition is represented with \( T \)—which is therefore an event, or, in some occasions, as we will see, its absence. If proposition \( p \) stands for the window being open, then the proposition \( \neg pTp \) means that the window opens, \( pT\neg p \) that it closes, while for instance \( pTp \) represents the window staying open. When we have two—logically independent—propositions \( p \) and \( q \) the possibilities get an accession since:

\[
pTq \leftrightarrow ((p \land q) \lor (p \land \neg q))T((p \land q) \lor (\neg p \land q))
\]

In this logic we already can have contradictories like

\[
(pTp) \land (\neg pTp)
\]

and tautologies like

\[
(pTp) \lor (pT\neg p) \lor (\neg pTp) \lor (\neg pT\neg p)
\]

but we do not have actions yet: I intentionally used the phrases above as "the window opens" or "stays open"—there is no intervention yet, only events happening without reference to agents bringing about actions constituting those events.

For actions von Wright introduces two operators: \( d \) for doing and \( f \) for forbearance, and adds rules resulting in the DF-calculus. With one operator we immediately have four types of actions: \( d(pTp) \), \( d(pT\neg p) \), \( d(\neg pT\neg p) \), and \( d(\neg pTp) \) meaning keeping the window open, closing the window, keeping the window closed, and opening the window, respectively. It is visible that there are two conditions that have to be fulfilled in order for an action to be performed: one is the state of affairs on the left side of \( T \)—no one can open a window which is already open; the other one is about how the things would happen without human intervention: if the window would open anyway (because of some automatism for instance), then there is no possible action to
open it; it norder to open the window, it has to be the case that otherwise the closed window would stay closed. The forbearance operator gets a similar interpretation: \( f(\neg pTp) \) results in \( \neg pT\neg p \), that is the window stays closed (because it has not been opened). The DF-calculus is intensional that we can show with the following example starting from:

\[
(df1) \quad d(pTp)
\]

In propositional logic:

\[
(df2) \quad p \leftrightarrow (p \land q) \lor (p \land \neg q)
\]

which we can substitute into \((df1)\):

\[
(df3) \quad d(( (p \land q) \lor (p \land \neg q))T((p \land q) \lor (p \land \neg q)))
\]

According to the rules of the logic of change:

\[
(df4) \quad ((p \land q) \lor (p \land \neg q))T((p \land q) \lor (p \land \neg q)) \leftrightarrow (pTp) \land ((qTq) \lor (qT\neg q) \lor (\neg qTq) \lor (\neg qT\neg q))
\]

Substituting \((df4)\) we get:

\[
(df5) \quad d((pTp) \land ( (qTq) \lor (qT\neg q) \lor (\neg qTq) \lor (\neg qT\neg q)))
\]

which—according to the logic of actions von Wright creates—is equivalent with the following d-sentence:

\[
(df6) \quad d(pTp) \land d((qTq) \lor (qT\neg q) \lor (\neg qTq) \lor (\neg qT\neg q))
\]

which is—also because of the rules of von Wright’s action logic—is equivalent with this:

\[
(df7) \quad d(pTp) \land d(qTq) \lor d(qT\neg q) \lor d(\neg qTq) \lor d(\neg qT\neg q)
\]

We can see that, meanwhile equivalence is obviously transitive in this system too, \((df1)\) and \((df7)\) are not equivalent. Since it is obvious that we can describe with 4 formulas all the possibilities that can happen with a window, but it is not obvious that the change that happens is produced by someone’s action. Therefore, it is plausible that DF-calculus is intensional.

A fascinating system of action logic emerges from von Wright’s work. Putting the deontic operators into this system we get a rather elaborate deontic system (for example instead of having two options \(OA\) and \(O\neg A\) we had in his
old system, here there are eight atomic O-norms). This elaboration is not the only development von Wright makes in *Norm and Action*; for instance, he rethinks the operators too, but we set this aside, since the chapters to follow won’t rely on it. Meanwhile his contribution about action within deontic logic summarized here will provide a relevant backdrop for subsequent discussion.

### 2.4 Normative Matter vs. Normative Relations

The deontic modalities we have been talking about (*obligation*, *permission*, and *prohibition*) Sileno [2016] calls ‘normative matter’—being the matter of normative specification. Beside works investigating this normative matter, Sileno identifies another direction of modern analytic literature handling legal conceptions with logical tools: the study of *normative relations*. This branch of investigation started with Wesley Newcomb Hohfeld’s paper on fundamental legal concepts—who did not use formal tools, but continued with many others, who did, and this is the line I would like to adjust. Normative relations are held between two parties, and often referred to as—the two expressions that will arise the most in this dissertation—directed obligations (duties) and rights. While Hohfeld’s essay is considerably earlier than von Wright’s discipline-founding paper, the formal representation of normative relations often uses conceptions of deontic logic. This exposes strong and widely drawn relation between the two approaches in formalization. Deontic logic is not the only formal background used in this approach, however: Allen and Saxon [1995] use relevance logic, Makinson and van der Torre [2000] build upon input/output logic, Sileno [2016] creates computationally applicable semi-formal explanatory and requirement models, while van Eijck and Ju [2016] use dynamic logic. This dissertation presents a formal representation of normative (legal) relations and Hohfeldian conceptions building mainly on deontic logic and a simple stit action logic.
Chapter 3

Hohfeld and the Fundamental Legal Conceptions—The Article

We don’t know a lot about Wesley Newcomb Hohfeld’s life. He was born in 1879, before turning to law he studied chemistry. He obtained his law degree at the University of California, Berkeley, and Harvard Law School, after which he worked as a law professor at Stanford Law School. His first essay on fundamental legal conceptions in Yale Law Journal made such an impression that he was recruited to Yale Law Faculty, where he taught until his premature death in 1918. Hohfeld held courses in Equity and Conflict of Laws, where he required that his students master and use the basic concepts of his analytical legal theory evoking both refusal and admiration, mainly depending on the students’ abilities. The amount of Hohfeld’s publications is exiguous, while their significance is the exact opposite. His analysis on rights, whose formalization is the topic of this dissertation, is the masterpiece of his regrettably short life, as Nigel Simmonds [2001] phrases: “few works have made such a lasting contribution to analytical jurisprudence”. Before his death the material behind this analysis was published in Yale Law Journal in 1913 (Some Fundamental Legal Conceptions as Applied in Judicial Reasoning I) and in 1917 (Fundamental Legal Conceptions as Applied in Judicial Reasoning II); but after his death they had been revised and edited to be published together in 1919, and then in 1923 in a book accompanied by seven other essays by the Yale University Press. The primary focus of this dissertation is on the first essay (the quotations refer to the edition from 1923), more precisely on its last (and longest) chapter on Fundamental Jural Relations Contrasted with One Another; while the second essay will be used to provide a clearer picture about how Hohfeld

\[\text{Simmonds [2001] notes that it has been suggested (by Twining) that Hohfeld’s analysis resembles chemical analysis in that it breaks law down into constituent elements.}\]
intended his conceptions in the first one. When we will use the terms about Hohfeldian fundamental legal conceptions, we will refer to the relations and positions described and analyzed in the last chapter of the first essay.

3.1 Hohfeld’s Motivation and How to Read It

About the feelings and findings one might face reading the famous paper of Hohfeld, Pierre Schlag [2015] writes the following: "Wesley Newcomb Hohfeld’s 1913 *Fundamental Legal Conceptions as Applied in Judicial Reasoning* is a brilliant article. A thrilling read it is not—more like chewing on sawdust. The arguments are dense, the examples unwieldy, and the prose turgid." Well, it is true enough that there is no clear structure in presenting the examples—meanwhile these examples represent the great majority of the text: Hohfeld lists the examples from judicial decisions and arguments, one after another, but rarely expresses clearly why he does or does not agree with the cited arguments. He analyzes them scrupulously, though—as Simmonds [2001] says: "throughout the book, technical legal doctrines or institutions (...) are subjected to meticulous analysis in a manner calculated to mystify the student and irritate the practitioner". But before he gives the considerable amount of examples where the conceptions he considers are mentioned or disputed, Hohfeld starts with—and this is absolutely not negligible—a short preliminary declaration how these conceptions are related to each other. As he briefly puts:

<table>
<thead>
<tr>
<th>Jural opposites</th>
<th>right</th>
<th>privilege</th>
<th>power</th>
<th>immunity</th>
</tr>
</thead>
<tbody>
<tr>
<td>no-right</td>
<td>duty</td>
<td>disability</td>
<td>liability</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Jural correlatives</th>
<th>right</th>
<th>privilege</th>
<th>power</th>
<th>immunity</th>
</tr>
</thead>
<tbody>
<tr>
<td>duty</td>
<td>no-right</td>
<td>liability</td>
<td>disability</td>
<td></td>
</tr>
</tbody>
</table>

The well-known reason Hohfeld differentiates these types of rights (the upper lines) and duties (lower lines) is that he finds the word ‘right’ is overused and "even if the difficulty related merely to inadequacy and ambiguity of terminology, its seriousness would nevertheless be worthy of definite recognition and persistent effort toward improvement". Out of this scheme, we, indeed, for a better solution, are practically forced to lean on the examples. Unless we provide a clear formalization which helps us understand the theory.³ But

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²This is literal citation of Schlag’s phrasing, I did not change the cited title, since it was *Some Fundamental Legal Conceptions as Applied in Judicial Reasoning* in 1913, but it is common to refer to it in this way.

³As Ruzsa [1988] refers to the role of formal languages: they have a function in a more accurate description of a given scientific discipline’s conceptions and rules.
we also have to keep in mind what Hohfeld says about the definability of his conceptions: "The strictly fundamental legal relations are, after all, *sui generis*; and thus it is that attempts at formal definition are always unsatisfactory, if not altogether useless." Instead of taking this as disheartening as it sounds, we only need to be confined to the authorial intentions which at least meant not to try to reduce these conceptions to anything else (like will or interest). We have no reason to think that by ‘formal definition’ Hohfeld meant formal logic as a tool to express what these conceptions mean, we rather presume that he refused the definition of these legal conceptions in terms of *something else*. This still presents a nice challenge from a formal logical point of view since the usual expectation toward explicit definitions is that they have to be identity statements with definiendum on one side and definiens on the other—which must not contain the definiendum. This practically means that if we take seriously Hohfeld’s consideration of these conceptions being *sui generis* and his refusal to give a formal definition, we surely cannot provide an explicit one—with or without formal logic. The question arises of course: what can we provide then with our formal tools that, at least, goes some distance toward defining these conceptions? "Accordingly, the most promising line of procedure seems to consist in exhibiting all of the various relations in a scheme of *opposites* and *correlatives*"—this is what Hohfeld sketched with the scheme above, and this will be what we will also do, only more formally, and—hopefully—in a more comprehensive fashion.

As a first step—and this is how far we go in this section—we can draw a figure on the way Hohfeld differentiated the 8 legal positions and 2 types of relation between them. The conceptions and relations indicated above can be depicted together in the following way:

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4 The will theory and the interest theory are two well-known approaches on how we should describe rights according to their function, by providing candidates we can reduce them to. As Wenar [2015] writes, "a will theorist asserts that the function of a right is to give its holder control over another’s duty", while "interest theorists maintain that the function of a right is to further the right-holder’s interests". Introducing these contesting approaches would take us far from the focus, so I do not go into details (shortly will refer to them, when relevant, though).

5 See for instance Ruzza [2000].

6 In the reception one can find references to this system as containing "8 legal relations" (see for instance Hansson [2001]). I think this way of phrasing is misleading since there are 4 relations (the relations or pairs into which the correlativity orders the positions) involving 8 legal positions. This will be the terminology we are going to use.
By drawing this figure, it becomes clear that there are two separate groups of rights and duties, which we will often call first (the one on the left) and second group (the one on the right). Before delving into Hohfeld’s examples let’s see a complex—and complete—one from the reception since it is an excellent tool to represent the difference between the types of right and the necessity of its exploitation. We find it in Szabó [2009]: the sentence "Peter has a right to be in this house" can refer any of these types above depending on where we say it and on who Peter is: if this house is a building rented by Peter then his right is a claim-right toward the owner to ensure his occupancy (which means that the owner has the duty to do so); if this building is a public one then Peter’s right is a privilege to be inside as being a citizen (which means other people have no claim toward him not to be there); if Peter is a detective dashing in a house with an official search warrant in his hand then he has power (which means that the house’s owner has a liability to this action of Peter’s); but if Peter chains himself to the radiator stressing that he is exempt from the execution of an eviction, then what we refer to with the word ‘right’ is his immunity (which also involves the other person’s disability to conduct the eviction). After briefly outlining how multilayer the meaning of the word ‘right’ is, we can discover the legal theoretical layers described by Hohfeld. Following his presentation, we take them as vertical, that is, correlative pairs, and we try to characterized what we can extract from the examples and explanations provided by the author.

3.2 Claim-Right and Duty

Meanwhile Hohfeld calls this basic type of right also ‘Right’ "in the narrow sense", the reception often refers to it as ‘Claim-right’ simply to avoid mixing them up. (Hohfeld himself mentions ‘claim’ as the alternative to ‘right’). Hereafter we also will refer to this type of right as ‘Claim-right’\(^7\) (still leaving the literal citations of Hohfeld in their original form).

\(^7\)To make reading easier I will write the name of the rights and duties with upper case when I talk about the conception and not a particular right or duty.
After describing the judiciary discourse as using inconsistently the words above—sometimes as synonyms, sometimes differently, sometimes referring to all of them with the only word ‘right’—, Hohfeld writes what he probably judges the most important feature of this relation: "A duty or a legal obligation is that which one ought or ought not to do. ‘Duty’ and ‘right’ are correlative terms. When a right is invaded, a duty is violated. In other words, if X has a right against Y that he shall stay off the former’s land, the correlative (and equivalent) is that Y is under a duty toward X to stay off the place." Very important conclusions we have got already: correlativity means equivalency, which statement is confirmed a bit later, where Hohfeld says that a correlative is logically implied. Equivalence, from a formal point of view, is a quite straightforward feature. We can already note that the action involved in this relation is an action of Y, that is, the duty-bearer, not the one who has the claim-right. Also, Hohfeld seems to identify his notion of Duty with legal obligation, which can be important too from the viewpoint of formalization when we would like to use standard deontic logic as a background—but we come back to this point later.

### 3.3 Privilege and No-claim

Adopting the terminology above where we called Right in a narrow sense ‘Claim-right’, we rather use ‘No-claim’ to the notion Hohfeld called ‘No-right’ (although preserving his term when citing him). These conceptions—being a vertical pair in the drawing above—are correlative, but they also can be defined in the terms of the previous pair with the relation type ‘opposite’. As it is visible in the name of No-claim, it is intended to be the negation of a (claim-)right, and we can also call the Hohfeldian Privilege as ‘No-duty’. And this is very important: the Hohfeldian notion of Privilege (or freedom, or liberty—as he later considers these as more or less adequate synonyms) is merely the negation, the lack of (a given) duty. Let’s see what he writes regarding this relation: "In the example last put, whereas X has a right or claim that Y, the other man, should stay off the land, he himself has the privilege of entering on the land; or, in equivalent words, X does not have a duty to stay off. The privilege of entering is the negation of a duty to stay off. As indicated by this case, some caution is necessary at this point; for, always, when it is said that a given privilege is the mere negation of a duty, what is meant, of course, is a duty having a content or tenor precisely the opposite of the content of the privilege in question. Thus, if, for some special reason, X has contracted with Y to go on the former’s own land, it is obvious that X has, as regards Y, both the privilege of entering and the duty of entering. The privilege is perfectly consistent with this sort of duty—for the latter is of the same content or tenor as the privilege;—but
it still holds good that, as regards Y, X's privilege of entering is the precise negation of a duty to stay off. Similarly, if A has not contracted with B to perform certain work for the latter, A's privilege of not doing so is the very negation of a duty of doing so. Here again the duty contrasted is of a content or tenor exactly opposite to that of the privilege. Hohfeld sees the difference between Claim-right and Privilege provable with emphasizing that their correlatives are different: "the correlative of X's right that Y shall not enter on the land is Y's duty not to enter; but the correlative of X's privilege of entering himself is manifestly Y's "no-right" that X shall not enter."

But again: Privilege is only the lack of a given duty, and not a protected one, not a privilege in the sense this word is often used in everyday life referring to e.g. a kind of civil liberty. Hohfeld stresses this difference with refusing Professor Grey’s example, which says: "The eating of shrimp salad is an interest of mine, and, if I can pay for it, the law will protect that interest, and it is therefore a right of mine to eat shrimp salad which I have paid for, although I know that shrimp salad always gives me the colic." According to Hohfeld, this kind of protection by law is not contained in the notion of Privilege, since that is only the lack of duty of refraining from eating the shrimp salad. The protection that is missing here would mean a claim-right against the other people not to interfere—and, he says, that is "perfectly distinct: the privileges could, in a given case, exist even though the rights mentioned did not. A, B, C and D, being the owners of the salad, might say to X: »Eat the salad, if you can; you have our license to do so, but we don't agree not to interfere with you.« In such a case the privileges exist, so that if X succeeds in eating the salad, he has violated no rights of any of the parties. But it is equally clear that if A had succeeded in holding so fast to the dish that X couldn’t eat the contents, no right of X would have been violated."

One could raise the question of how a lack of something could bear the property of being correlative, since if I do not have a duty to refrain from something, that privilege of mine sounds pretty general and not correlative. The refinement Hohfeld offers for this latter example helps us understand: "Suppose that X, being already the legal owner of the salad, contracts with Y that he (X) will never eat this particular food. With A, B, C, D and others no such contract has been made. One of the relations now existing between X and Y is, as a consequence, fundamentally different from the relation between X and A. As regards Y, X has no privilege of eating the salad; but as regards either A or any of the others, X has such a privilege. It is to be observed incidentally that X’s right that Y should not eat the food persists even though X’s own privilege of doing so has been extinguished." The expression ‘as regards a given agent’ is crucial in understanding the notion of correlativity, and thus will be crucial in formalizing these correlative notions. Hohfeld simply says: "A right is one’s affirmative claim against another, and
a privilege is one’s freedom from the right or claim of another." The general-
ity that we mentioned above comes with the rights that Hohfeld calls general
political liberties, and that he considers perfectly distinct from the privilege
he defines—these will be discussed later.

### 3.4 Power and Liability

According to the diagram above, Power, Liability, Immunity and Disabil-
ity create a separate group from the group of Claim-right, Duty, Privilege
and No-claim. The first correlative pair of this group is Power and Liability.

Hohfeld, after giving Power’s "position" in terms of the relations (correla-
tive pair of Liability, and opposite of Disability), inquires whether it would
be possible to know about its intrinsic nature. But, just like in the case of
rights and duties discussed previously, he dismisses metaphysical considera-
tions and confines himself to "an approximate explanation, sufficient for all
practical purposes". And pretty soon we get to the essential point of Power:
a person, whose volitional action has a result in a change in legal relation has
the power to effect the particular change in the given legal relation. Hohfeld
considers the possible synonyms: he finds ‘ability’ acceptable, but refutes
‘capacity’ (meanwhile, as we will see soon, the reception assigns the word
‘capacity’ to the modality type borne by the rights and duties in the group
of power). Then he mentions a host of examples from judicial reasoning from
the area of the law of escrow, and of contracts. Hohfeld emphasizes the dif-
ference between legal power and physical power, and also stresses that Power
bears that correlative nature that all the other legal positions, and so is pos-
sible to exist together or separately of a privilege regarding the very same
action: "if X, a landowner, has contracted with Y that the former will not
alienate to Z, the acts of X necessary to exercise the power of alienating to
Z are privileged as between X and every party other than Y; but, obviously,
as between X and Y, the former has no privilege of doing the necessary acts;
or conversely, he is under a duty to Y not to do what is necessary to exercise
the power." This feature and agent-dependency is pivotal in understanding
correlativity, also if we would like to map normative contradictions: as we
saw it can be the case that I have a power to do something but I do not have
a privilege to do so, which may sound as a contradiction at first glance, but
Hohfeld shows it depends on the agents involved.

In the light of all that were told on Power, Hohfeld does not feel neces-
sary to tell a lot on Liability, as it is the correlative duty-type of Power as
a right: whenever X has a power to do something regarding Y, Y is liable
to this action of X. He, citing the case Booth vs. Commonwealth, mentions
one of the best examples of (Power and) Liability from a Virginia statute:
"all free white male persons who are twenty-one years of age and not over sixty, shall be liable to serve as jurors, except as hereinafter provided" and, agreeing the terminology used in the case, Hohfeld confirms that "it is plain that this enactment imposed only a liability and not a duty". And we get the most important sentence about being liable: "it is a liability to have a duty created". Actually the translation of Miklós Szabó (Hohfeld [2000]) expresses in more depth what is said here: "Beavatkozásnak kitettség az, ha kötelezettséget róhatnak ránk". If we were to "retranslate" strictly literally the Hungarian version to English in order to show why it is much more expressive, it would be something like this: Liability is the situation when it can happen that someone creates a duty to us. What is missing—or at least not emphasized enough—in the original Hohfeldian sentence’s literal level is that a liability is a liability even before the duty is created: we are not just liable when it happens, we are liable if it is the case that a duty can arise. It is true and important that liability (and obviously power) becomes "visible" when the action on which the given agent has the power is exercised. But the conditional nature is essential: if someone practices his power, then a duty arises. Of course the sentence above does not mean that only a duty can be the result of exercising one’s power: as it has been written, any kind of legal relation (and therefore position) can arise. Having a duty created is simply the situation that is most easy to grasp, since people feel liable typically when they are "threatened" by a duty, and less if they are "threatened" by a right; presumably that is why Hohfeld phrased things in this way.

3.5 Immunity and Disability

Immunity is the correlative pair of Disability and the opposite, the negation of Liability. Also, its relation to Power can be described in the light of what we saw in the case of the group of Claim-right: "A right is one’s affirmative claim against another, and a privilege is one’s freedom from the right or claim of another. Similarly, a power is one’s affirmative "control" over a given legal relation as against another; whereas an immunity is one’s freedom from the legal power or "control" of another as regards some legal relation." And the reader gets some examples too: "X, a landowner, has, as we have seen, power to alienate to Y or to any other ordinary party. On the other hand, X has also various immunities as against Y, and all other ordinary parties. For Y is under a disability (i.e., has no power) so far as shifting the legal interest either to himself or to a third party is concerned; and what is true of Y applies similarly to every one else who has not by virtue of special operative facts acquired a power to alienate X’s property. If, indeed, a sheriff has been duly empowered by a writ of execution to sell X’s interest, that is a very
different matter: correlative to such sheriff’s power would be the liability of X,—the very opposite of immunity (or exemption). It is elementary, too, that as against the sheriff, X might be immune or exempt in relation to certain parcels of property, and be liable as to others. Similarly, if an agent has been duly appointed by X to sell a given piece of property, then, as to the latter, X has, in relation to such agent, a liability rather than an immunity.

3.6 Hohfeld’s intentions

After the analysis each conception by their pairs, Hohfeld summarizes his intentions and what he thinks are the most important features of these conceptions. He stresses their atomic nature by using the metaphor that these conceptions are the "lowest common denominators of the law". As it works in the case of fractions: they, superficially, can seem so different, difficult to compare. But once we change to their lowest common denominators, their structural, fundamental similarity becomes apparent, and comparison becomes easy. As Hohfeld says, "the same thing is of course true as regards the lowest generic conceptions to which any and all »legal quantities« may be reduced". This confirms our claim at the beginning of this chapter: these conceptions are the fundamental ones, the further reduction is useless. About his intentions Hohfeld says: "eight conceptions of the law have been analyzed and compared in some detail, the purpose having been to exhibit not only their intrinsic meaning and scope, but also their relations to one another and the methods by which they are applied, in judicial reasoning, to the solution of concrete problems of litigation." It sounds as this analysis was clearly _descriptive_ about how these notions are used in judicial reasoning, but during the analysis the reader often feels that it is rather _normative_, since Hohfeld’s starting point was precisely that there is an "inadequacy and ambiguity of terminology" which needs improvement: "the above mentioned inadequacy and ambiguity of terms unfortunately reflect, all too often, corresponding paucity and confusion as regards actual legal conceptions. That this is so may appear in some measure from the discussion to follow." It is hard not to understand these introductory sentences as giving a description of use and its incoherency with providing a _sound_ analysis on the conceptions and its suggested terminology. Which means that this analysis, which became decisive in the last century, is _normative_. We reflect on this duality of the nature of Hohfeld’s analysis in the later chapters.
Chapter 4

Hohfeld and the Fundamental Legal Conceptions—The Reception

The significance of Hohfeld’s work is shown by the sheer volume of reflections his papers got and the fact that the literature engaging his work—after a hundred years—continues to expand. As Simmonds [2001] says: "the analysis of rights that Hohfeld offers is still regularly cited and relied upon by both lawyers and philosophers, and it is treated as a source of insights into the nature of moral rights as well as the legal rights that were Hohfeld’s own focus of concern." This importance—at least partly, for sure—is given by the fact that Hohfeld’s work is considered the first complete and structured description of a system of rights; and also one, which, in many ways, has remained unsurpassed since then. Simmonds [2001] reflects on this uniqueness as "although some of his analytical distinction were anticipated by earlier jurists, their insights were fragmentary and imperfect by comparison."

About this point, it is worth taking a look at the opinion of Kocourek [1920], who was Hohfeld’s contemporary, and who, while agreeing with what is written above, leaves the reader with no doubt that he blames Hohfeld for not knowing these previous works: "It was the first attempt at a complete systematic arrangement of jural relations. A half-dozen or more Germans had already treated in a thorough way the active (power) side of jural relations. The most complete of these attempts was that of Bierling, but no writer in any country, prior to Hohfeld, had sought to give a systematic account, with suitable terminology, of the passive side of jural relations. Partial efforts to state the correlatives (the active and passive sides of jural relations) had been made by Terry and Salmond." Then in his conclusion Kocourek says: "[Hohfeld’s] literary apparatus shows an intimate acquaintance with everything on the subject printed in English, but it shows no acquaintance at any point with an important literature, especially in German, which has explored juristic ideas in various directions which have not yet been made familiar to us in our own language. This isolation must be regarded as a great hindrance to any investigator in jurisprudence, but in spite of it, or rather because of it, Hohfeld succeeded in building up a structure which has the unquestioned merit of originality and ingenuity."

It is well to add that not only German thinkers preceded Hohfeld in making attempts to
systematic and apparently exhaustive (yet concise) treatment is generally regarded as unsurpassed." In this survey we only review those works from the rich reception that are relevant from the viewpoint of the formalization, especially the ones relevant for the way of formalizing the Hohfeldian theory that will be presented in this dissertation.

4.1 Kocourek’s Critique on Hohfeld

Hohfeld’s work was discussed, praised and critized by his contemporaries. Albert Kocourek, who published a voluminous book on *Jural relations* in 1928, and knew Hohfeld personally, published an analysis of his system not long after Hohfeld’s death. Kocourek [1920]—next to admitting Hohfeld’s genius—criticizes the *Fundamental Legal Conceptions* on several points. One of these is using the word ‘opposite’ for describing the logical relation between Claim-right and No-claim, Duty and Privilege, Power and Disability, and between Liability and Immunity. As Kocourek says: "In logic, opposites as distinguished from contradictories are the extreme terms of quantity. Thus +a is the opposite of -a. In the case of legal relations to have a claim to payment of $100 would be the opposite of a duty in the same person to pay $100. Yet we find in Professor Hohfeld’s table that the ‘opposite’ of ‘right’ is not ‘duty,’ but ‘no-right.’ Now it is clear that ‘right’ and ‘no-right’ are not ‘opposites’—at least not in the sense of logic—but are rather ‘contradictories’ (negatives)." The later reception seems steady in handling the Hohfeldian ‘opposition’ indeed as negation (this will be the way we are going to treat it too), what is more, the Hohfeldian phrasing also tells about this intention independently from the word chosen to label this relation.

Another series of criticism by Kocurek—from a legal theoretical position—concerns the notion of privilege. One problem he raises is that—as being the negation, that is, the absence of the given duty—Privilege is not a legal notion at all, and as such it has no juristic value: "No one has a claim against A that he shall not smoke the cigar. What is the possible juristic significance of the act? Does the law in any way undertake for the advantage of others to say that A shall, or shall not, smoke the cigar? Not at all. Then where is the juristic significance?" Another problem with the Hohfeldian Privilege, according to Kocourek [1920] is that it—as all the other conceptions of Hohfeld—is described as relational, meanwhile "there is no more of ‘relation’ in ‘privilege’ than may be found in a windmill or a table." Kocourek shows that these two features connect: "it is clear that ‘no-duty’

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systematize rights, but Hungarians too: Gusztáv Szászy-Schwarz published a paper with a system of rights—which system-description still has an influence on the Hungarian private law—in 1907. A thorough comparison of Hohfeld’s and Szászy-Schwarz’s system can be read in Hungarian in Blutman [2012].
and ‘no-right’ are both mere negations and that as such they cannot be in relation in any logical sense”; also: "non-existence is the most absolute thing in the world, and incidentally it is perhaps one of the few logical absolutes". Well, the way Kocourek refers to A having privilege as "no one has a claim against A that he shall not smoke" does not ease to see what Hohfeld meant, indeed. But the mistake is not Hohfeld’s, but Kocourek’s in this case: A having a privilege to smoke means that a specific another person, say B, has no claim against A that he shall not smoke, not that no one has. We will discuss this in detail later.

4.2 Active and Passive Rights

Active rights are those where the right-owner (the agent who has the given claim-right/privilege/power/immunity) and the actor (the agent whose action is concerned in the given right) is the same: I have privilege on my own action—it does not even make sense to say that I have privilege that someone else do something. Same with Power: I have the power to do something—not that someone else do something. While passive rights involve different agents in the places of right-holder and actor: I can only have a claim-right on someone else’s action, also I can have immunity against someone else’s action. It is the same with duties: where the duty-bearer and the actor are the same, we talk about an active duty; while about a passive duty if they differ.

The differentiation between active and passive rights is associated with the paper of David Lyons et al. [1970] entitled The Correlativity of Rights and Duties (see for instance Wenar [2015] and Syi [2014]). Actually, this distinction is not clearly explained in the paper of Lyons et al.—maybe because the distinction is not its point. The aim of this paper is to wonder about correlativity (as its title shows) and to contest its doctrine. Lyons et al. first put forth significant considerations on how to understand the generality of correlativity: they stress that specificity and determination involved.

"Suppose that Bernard owes Alvin ten dollars: we then have equal reason to ascribe a right to Alvin and a corresponding obligation to Bernard. Bernard’s obligation is to pay Alvin ten dollars; but his obligation is also to Alvin—or, as we say, it is "owed" to Alvin in particular. Alvin has a corresponding right, to be paid ten dollars by Bernard, which is held "against" him specifically. Alvin’s right and Bernard’s obligation do not merely coexist: their coexistence is necessary, not contingent. Neither the right nor the obligation could arise without the other, and if one is discharged, waived, cancelled, voided, forfeited or otherwise extinguished the other must be extinguished as well. For the
"ground" of the obligation—the undischarged debt—is the "ti-
tle" of the right. This right and obligation entail one another. A
statement ascribing one warrants fully an inference to the other,
without appeal to contingent facts or substantive principles. It
is not that facts or principles have no bearing on the case: asser-
tions of the right or obligation may presuppose principles deriving
them from certain kinds of fact. But, if we are given either the
right or the obligation we can infer the existence of the other.
Moreover, such implications are, as we might say, specific and
the correlations determinate. A full statement of the right or the
obligation implies a full specification of the other. It is not that
Alvin’s right implies merely that there is some coexisting obli-
gation, but that Alvin’s having this particular right implies that
Bernard is under an obligation, to Alvin, to pay him ten dollars
(and vice versa)."

And this nature of correlativity is something Lyons et al. do not find in
the cases like the right to free speech. More precisely, they claim that this
conceptual level of correlativity is restricted to the cases of passive rights
and active duties. Meanwhile in the case of free speech, which the authors
consider a privilege (even if they do not use this word), it is not clear how
we should consider the expectation of all other people to not interfere. Well,
Hohfeld declared that he did not intend to consider the so-called political
liberties among privileges. Although, it is natural that these types of rights
to do something come to one’s mind when thinking about privileges. We will
discuss this overlap in detail when formalizing, and show why these cases of
freedom do not influence correlativity. Anyway, even if not in its main goal,
Lyons et al.‘s paper did contribute to Hohfeld’s reception by highlighting the
essential difference between active and passive rights. The categorization is
the following:

- active rights (right-owner and actor is the same): privilege, power
- passive rights (right-owner and actor are different): claim-right, immu-
nity

Naturally, an active right’s correlative pair is a passive duty, while a passive
right involves an active duty. Therefore:

- passive duties: no-right (no-claim), liability
- active duties: duty, disability

It may sound strange to call Disability active in any sense when it is all about
lacking something. The point is that within the negated proposition we talk
about the legal position of the actor. But this relation will be articulated
more precisely later.
4.3 Negative and Positive Rights

It’s well to present also the distinction between negative and positive rights. The introduction of this distinction, suggested by Wenar [2015] popular among normative theorists, starts with the following definitions: "The holder of a negative right is entitled to non-interference, while the holder of a positive right is entitled to provision of some good or service." Probably the reader has already realized that this distinction makes sense only within the category of passive rights, therefore Privilege and Power are neither negative nor positive types of rights. At least, if we insist (which we do) on maintaining the Hohfeldian notion of Privilege: the free speech type of freedom is—as this distinction shows too—not a Hohfeldian privilege (again, we will discuss this later). There is a quite interesting note later in this paragraph of Wenar [2015] citing Holmes and Sunstein [1999] according to whom when it comes to the state enforcement, the difference disappears: "in the context of citizens’ rights to state enforcement, all rights are positive". This is a point that will be also reflected upon during the formalization.

There are other interpretations according to which we can talk about positive and negative freedoms, too, see for example Syi [2014] citing Isaiah Berlin and Jan Naverson in his book on action theory, but these do not really cover the Hohfeldian privilege, and are not even legal, but are instead political or action theoretical considerations that do not really add to our current investigation.

4.4 Fitch and the Capacitative Modalities

Frederic Fitch’s, American logician’s article A Revision on Hohfeld’s Theory of Legal Concepts contains crucial considerations from philosophical and logical viewpoints. On the one hand, Fitch [1967] calls attention to the difference between the action in Privilege’s scope and the action in the corresponding Duty’s scope: while the word ‘opposite’ could be understood so the only task is to negate the proposition in order to get one form from the other, we also have to put a negation in front of the action. With Fitch’s words: "Y has a right R against X that X pay him a dollar if and only if X has the duty D to pay Y a dollar. Notice, however, that if D is the duty of paying a dollar, then the corresponding privilege P is the privilege of not paying a dollar, rather than the privilege of paying a dollar." Actually, Hohfeld himself calls attention to this specificity in Duty’s and Privilege’s opposition writing: "when it is said that a given privilege is the mere negation of a duty, what is meant, of course, is a duty having a content or tenor precisely opposite to that of the privilege in question". What Fitch adding here is this: he analyzes further
this specificity with calling the latter (privilege of paying a dollar) \( P' \), and shows that \( P' \) is not contradictory to \( D \), but is implied by it. This means, privilege works as permission works with obligation in standard deontic logic. By naming further positions Fitch shows how traditional logic's A-, I-, E-, O-propositions can be drawn:

<table>
<thead>
<tr>
<th>Contraries</th>
<th>Subcontraries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duty (A)</td>
<td>Privilege (I)</td>
</tr>
<tr>
<td>Prohibition (E)</td>
<td>Exemption (O)</td>
</tr>
</tbody>
</table>

where privilege is understood as \( P' \) above, and exemption is understood as the Hohfeldian privilege, which should be 'privilege-not' according to Fitch; while prohibition is simply (obviously) understood as 'duty-not'. With the same method the following square can also be drawn:

<table>
<thead>
<tr>
<th>Contraries</th>
<th>Subcontraries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Right (A)</td>
<td>No-right-not (I)</td>
</tr>
<tr>
<td>Right-not (E)</td>
<td>No-right (O)</td>
</tr>
</tbody>
</table>

Fitch—tacitly—makes another interesting observation. After the investigation above, he introduces a formalization in which he tacitly reduces power to capacity. He introduces undefined symbols ‘C’ for causation and ‘S’ for "strives to realize":

\[
f(a) \quad \text{is} \quad a \text{ is a member of (class) } f
\]

\[
[pCq] \quad \text{is} \quad p \text{ is a (partial) cause of } q
\]

\[
[xSp] \quad \text{is} \quad x \text{ strives to make } p \text{ true}
\]

and defines ability (capacity) and power in the following way:

\[
\text{Def: } \text{cando}(x, p) = (\exists q)[[xS[p \land q]]Cp]
\]

\[
\text{Def: } \text{power}(x, f, y) = \text{cando}(x, f(y))
\]

\[
\text{Def: } \text{liability}(y, f, x) = \text{cando}(x, f(y))
\]

which means that Fitch actually identifies Power with ability or capacity. Since he does not say anything about the intended meaning regarding the nature of this capacity or ability, we can understand it as legal capacity, but this legal feature has no sign either. The only thing we know from other formulas of his is that having a right is a kind of class membership, so the definition above covers the capacity of changing someone’s legal situation regarding her rights and duties—and this is indeed the intrinsic nature of the Hohfeldian Power.
With this separate way of handling Power and its group (Liability, Immunity and Disability) compared to (Claim-)right and its group can be seen as differentiating the types of their modality: Fitch showed that he finds Claim-right, Duty and Privilege reducible to classical modalities, but his apparent reductions of Power to capacity is, actually, a characterization of Power as a capacitative modality.

4.5 Makinson’s Critique of the Hohfeldian Theory

David Makinson also calls attention to some structural differences between the two groups of the Hohfeldian positions or modalities in his article *On the Formal Representation of Rights Relations* in 1986.

Makinson refers to the deontic and capacitative modalities as two distinct dimensions. First he affirms that there are differences that Hohfeld already realized and clarified. One of these is separating permission and Power. Hohfeld himself did not use the expression ‘permission’ but as we saw above in Fitch’s work, the first order rights can be assigned to the standard deontic modalities—in case of the Hohfeldian Privilege the assignable modality is the so-called weak permission (which is equivalent to the negation of the obligation to do the given thing’s opposite $P \leftrightarrow \neg O \neg A$). Makinson cites Hohfeld’s example where Power and Privilege’s possible interaction is mentioned: "As regards all the »legal powers« thus far considered, possibly some caution is necessary. If, for example, we consider the ordinary property owner’s power of alienation, it is necessary to distinguish carefully between (i) the legal power, (ii) the physical power to do the things necessary for the »exercise« of the legal power, and, finally, (iii) the privilege of doing these things—that is, if such privilege does really exist. It may or may not. Thus, if X, a landowner, has contracted with Y that the former will not alienate to Z, the acts of X necessary to exercise the power of alienating to Z are privileged as between X and every party other than Y; but, obviously, as between X and Y, the former has no privilege of doing the necessary acts; or conversely, he is under a duty to Y not to do what is necessary to exercise the power." But Makinson adds an important example of his own about the difference between (i) and (iii): "Consider the case of a priest of a certain religion who does not have permission, according to instructions issued by the ecclesiastical authorities, to marry two people, only one of whom is of that religion, unless they both promise to bring up the children in that religion. He may nevertheless have the power to marry the couple even in the absence of such a promise, in the sense that if he goes ahead and performs the ceremony, it still counts as a valid act of marriage under the rules of the same church even though the priest may be subject to reprimand or more
severe penalty for having performed it."

Makinson also indicates that—in accordance with his time’s jurisprudential theory standards—Hohfeld realized that Power has a much more restricted domain than privilege (or permission): while any voluntary human action can be permitted (can be a subject to a privilege), a power is understood as something with which legal relations, positions can be changed. With Makinson’s words: "power, in the sense that we are discussing, is always a power to »perform a legal act« or »create a legal fact«—in other words, to alter, by means of a performative utterance or inscription or some other conventionally recognized gesture or procedure, the legal or other normative relations (themselves either deontic or in turn capacitative) between parties, within a framework of general laws that, except for some limiting cases, remains unaltered. For example, one may have or lack permission to go naked on a beach, but one neither has nor lacks a power to do so. A municipal officer may, however, have a power to determine whether people will be permitted to go naked on a certain stretch of sand." These are very important statements.

First, the different scope of deontic and—invoking Fitch’s distinction—capacitative modalities has been clarified: both types of modal operators have actions in their argument, but quite different ones, we can say, different sets of actions. But this does not mean disjoint sets: as Makinson’s own example shows we can have or lack permission for an action on which we have or lack a power. But the other way around does not sound plausible as the power-required actions are specific—which specificity will be explicated in details during the formalization. So far it seems, thus, the actions that can be the object of a power form a proper subset of the set of actions that we can have permission for.

Second, even if, as Makinson stresses, with showing the intrinsic nature of Power(-related rights and duties) Hohfeld circumscribed the actions which require power to do, Hohfeld never used ‘perform a legal act’ or ‘create a legal fact’ to characterize what happens in these cases. These characterization added by Makinson are crucial from a legal theoretical viewpoint, also from a viewpoint of getting a deeper understanding of the Hohfeldian ideas. Furthermore, these characterization relate to some theories of philosophy of language, especially by the clause mentioning performative utterances—we will discuss these in detail in subsequent chapters.

4.5.1 Structural Differences

After acknowledging Hohfeld’s clarifications on some crucial issues, Makinson introduces some structural differences that have been completely passed over—we might think, even unrealized—by the original author, at least, as
Makinson puts it: "Hohfeld writes as if in all other respects the logic of the class of all possible types of capacitative relation between two parties is a mirror image of that of the rights relation, and as if the classification of the types in one dimension corresponds entirely to that of the other." Makinson quotes parts of the Hohfeldian text where the stylistic trouble to present the two dimensions in parallel fashion—according to Makinson—can be detected. Such a paragraph of Hohfeld’s is the following:

"Perhaps it will also be plain, from the preliminary outline and from the discussion down to this point, that a power bears the same general contrast to an immunity that a right does to a privilege. A right is one’s affirmative claim against another, and a privilege is one’s freedom from the right or claim of another. Similarly, a power is one’s affirmative »control« over a given legal relation as against another; whereas an immunity is one’s freedom from the legal power or »control« of another as regards some legal relation."

Let’s consider again the diagram showing the relations under scrutiny:

The first thing Hohfeld claims here is "the logical relation between a claim-right (to have done) and a privilege (not to do) is the same as the relation between a power (to do) and an immunity (from having done)". This can be shown with formal tools (Kanger did show, this to be examined later in detail).

But there is another point "Hohfeld appears to be making": the deontic relation of a right of x (to have done by y) itself corresponds to the given capacitative relation power to x (to do) in a way that a privilege of x (not to do) corresponds to the given immunity of x (from having done by y). Hohfeld’s basis to talk about this parallel of the correspondences is that, as Makinson rephrases him, "the first two both confer a degree of ‘control’ on the part of x, and so correspond in that respect, whilst the latter two deny such a control on the part of y."
What Makinson is ready to admit is that there is a similarity if we understand the notion of (Claim-)right of x as involving Power on the part of x to take legal action in the case of non-fulfillment. But in this case we just say that both Claim-right and Power itself imply attribution of power to x—whose expression formally is not yet solved. (At this point Makinson indicates that he is going to give a proposal informally, regarding which, though, even he has some doubts. This proposal will be one of our starting points later in this dissertation.) The connotation of the word ‘control’ indeed suggests some direct connection to Power. The intuition invoked by Makinson is plausible: ‘control’ can have a clear procedural understanding: in Hungarian there is a general term referring to the party in any kind of procedure who has power to decide about it, ‘az ügy ura’ (literal translation: ‘the lord of the case’, or better: ‘the master of the case’). Let’s see what Hohfeld’s description of this parallel can be rendered with $P$ and $P'$ standing for permission and Power respectively:

the relation in the deontic dimension between:
$$O(y \text{ do } F) = \neg P \neg (y \text{ do } F) \text{ and } P \neg (x \text{ do } F)$$

the relation in the capacitative dimension between:
$$P'(x \text{ do } F) \text{ and } \neg P'(y \text{ do } F)$$

True enough, formally the parallel Hohfeld referred to is not visible.

Makinson’s opinion—which is not new, as we saw above—is that out of the deontic notions, rather permission (which is a view of privilege) is with which it would be worth to compare Power in order to observe a parallel. If we consider permission as x’s privilege to do $F$ and y’s prohibition to do $F$ as x’s (claim-)right towards y to not $F$, we see an elegant correspondence with x’s power and x’s immunity, respectively. With formulas:

$P'(x \text{ do } F)$ corresponds to $P(x \text{ do } F)$, rather than a right to have done. Likewisely, $\neg P'(x \text{ do } F)$ corresponds to $\neg P(x \text{ do } F)$, rather than a permission not to do.

The correspondence is clear indeed, but this does not mean that the parallel is perfect. There are two structural differences between the dimensions Makinson calls attention to: (i) an (ii).

(i) Difference in consequences

The first difference is about acting without the given position: doing something that is not permitted comes (well, at least may or should come) with
the consequence of punishment. While doing something without power comes with the consequence that the given thing is actually not done: "if a person tries, say, to celebrate a marriage or issue a passport without having the power to do so, then we say that he has not in fact celebrated a marriage or issued a passport (for emphasis: has not issued a valid passport) but has only gone through the motions or given the appearance of doing so." Formally:

$$\neg P'(x \text{ do } F) \rightarrow \neg(x \text{ do } F)$$

the contraposition of which is:

$$x \text{ do } F \rightarrow P'(x \text{ do } F)$$

(with the restriction of F’s domain to "legal facts", as Makinson calls them)

This property of Power is obviously not a property of permission (otherwise the axiom T would be an axiom of SDL—we will discuss this later).

(ii) Tripodono

The other structural difference Makinson raises is, as he says, a "more subtle and less noticed" one. While it makes perfect sense in the deontic dimension that someone has a permission to perform a given action, also that she has a permission not to do that, in the case of the capacitative dimension it is different: it of course makes sense to assert that someone has power to perform a given action, but it is rather senseless to say that she has power not to do so. Makinson’s conclusion: "It would seem that statements asserting that x has a legal power not to see to it that F are either meaningless or, more accurately, always trivially true." For mnemonic purposes Makinson calls this thesis of his tripodono principle (for ‘trivial truth of powers not to see to it’).

In his defense of the tripodono principle Makinson considers some potential counterexamples. The first is a logical one: since F can be a tautology, or any thesis of the underlying logic (Makinson—indicating this property—calls it F0), then if the given logic is closed under the rule that if F is a thesis then so is x do F, and also closed under the thesis (that Makinson calls standard rule for any kind of capacitative operator) that when F is a thesis then so is $$\neg P' \neg F$$, we can derive that $$\neg P' \neg (x \text{ do } F0)$$. Which means in this case x does indeed lack power not to see to it that F0. One obvious way to counter this problem is to deny that from F being thesis we could derive x do F as a thesis (this is what Kanger and Lindahl did, as we will see later). Insisting on the difference between the physical power and legal power can serve a good basis to deny the latter rules: just because I am physically unable to create a situation that is always true, I can have a legal power to do so. But Makinson opts instead for accepting these rules in our
underlying logic and admitting that the triponodo principle has exceptions in these very special limiting cases where \( F_0 \) is a thesis.

There are other, "more meaty in content" counterexamples, however. Makinson mentions one specific case: "suppose that under certain rules of procedure, the accused is required either to make ‘guilty’ plea or to make a ‘not guilty’. He is required to make one plea or the other—required, not in the sense (or at least, not only in the sense) that this is obligatory, but also in the sense that he just does not have the legal power not to make some plea or other. If he refuses to speak when asked, the court will, we suppose for our example, consider him as having made a plea of ‘not guilty’.

A possible answer is that, in this case, what \( x \) lacks is the power to see to it that no plea is entered, but does have a power not to see to it that some plea is entered—and that when he exercises this power, it is the court and not him that sees that the specific plea is entered. But we could also say that when \( x \) fails to response to the question, the court considers him as having entered a plea of ‘not guilty’. "It may be granted that the court is seeing to it that \( x \) is seeing to it that that is the plea recorded, but it is nevertheless \( x \) that is seeing to it that that is the plea, and not (or: not only) the court. Such a way of speaking, and such a notion of agency, may indeed differ from those familiar in common sense, physical, or philosophical discourse but is a feasible (and perhaps practiced) legal way of attributing agency. According to this response, then, the plea example does indeed provide a counter-example of a substantive nature to the triponodo principle."

As Makinson says these are two—equally coherent and legitimate—ways of conceptualizing agency. But even if we want to commit ourselves the second way of speaking, when we have to admit this example as a real counterexample to the triponodo principle, we see that this kind of counterexample is quite infrequent. That is, the principle still survives as a very general, although not universal, rule: "when a legal state of affairs \( F \) is not a truth of logic, then it is almost always trivially true that a party \( x \) has a legal power not to see to it that \( F \)."

Makinson points the following out as a possible reason behind Hohfeld’s reasoning: while Hohfeld continuously speaks about deontic and capacitative dimensions as parallels, never gives any example of the formal counterpart—under the \( F \) to \( F' \) map—of an obligation (duty) to see to it that something is the case. According to Makinson, the reason is that it would be a permission not to see to it that \( F \) is the case, which in the capacitative counterpart would be a power not to see to it that \( F \)—which, as we saw, there are not many examples for, actually "rarely (if ever) holds in a consistent system of norms". But the situation is different with the parallel Hohfeld emphasized:
the capacitative counterpart of a prohibition on seeing to it that $F$ is the denial of a power to see to it that $F$—and this situation frequently holds.

4.5.2 Interference

As it occurred above, interference—actually the lack thereof—is something, intuitively, rather connected to the common notion of liberty or privilege. What Bentham [1843] calls ‘vested liberty’ to see to it that $F$—and what Wright [1963] calls simply ‘right’—means not only a permission to see to it that $F$, but also that no one is permitted to interfere with his so doing. But as we saw, Hohfeld puts great emphasis on differentiating his ‘Privilege’ from this common notion of liberty (see e.g. the salad example). Makinson rather considers Hohfeld’s emphasis as him being "indeed at pains to distinguish" Privilege from its stronger conjunction with a prohibition on interference. It is difficult not to interpret Makinson’s comments on this issue as he thinks interference is of a great importance when considering rights, and as he finds Hohfeld’s reclusion a bit unsatisfactory. Kanger and Lindahl will deal with this notion somehow (not in the way Makinson would be satisfied with), therefore, we come back to Makinson’s further considerations after investigation the classical formalizations.
Chapter 5

The Classical Formalizations—The Theory of Normative Positions

Makinson [1986] talks about the Kanger-Lindahl theory of normative positions as the best known response to the challenge offered by the Hohfeld’s discussion’s abstract and systematizing character, disciplined and regimented language which is still missing the assistance of formal logic’s tools. As Marek Sergot [2013] introduces it, the Kanger-Lindahl theory of normative positions is an attempt to apply the tools of modal logic to the formalization of the ‘fundamental legal conceptions’. The word ‘attempt’ and the phrasing not saying directly that "it is the formalization of the Hohfeldian theory" tells a lot: even if the reception considers this theory as the classical formalization, the same reception admits that these formalizations (if we consider them separately) do not really, or at least not precisely represent (formally) the original Hohfeldian theory. Nevertheless, it is common to conceive the results and merits of these solutions by referring to the "original" theory of fundamental legal conceptions created by Hohfeld; as Sergot sums it up (by which we already get a fade light on the relation between the ‘normative positions’ and the notions of Hohfeld):

"The Kanger-Lindahl theory is generally regarded as the most comprehensive and best developed attempt to formalize distinctions such as Hohfeld’s. For example, Hohfeld identified four distinct legal/normative relations that could hold between any two agents with respect to some given act type. (...) Kanger’s systematic, formal analysis yielded 26 distinct ‘atomic types of rights relations’ or ‘normative positions’ as a refinement of Hohfeld’s four. Lindahl’s subsequent analysis produced 35 of the same basic kind as Kanger’s and 127 if a more precise set of possible relationships is considered instead. (...) There are more still than
are accounted for in Lindahl’s version: employing the same logics, 255 distinct relationships can be generated refining Kanger’s 26 and Lindahl’s 127, and many more if we include more complex act types and more agents than two.

The point of the Kanger-Lindahl theory is to map the space of logically possible legal relations between two given agents. We will investigate in details these results. We will do it following Sergot [2013]’s conversion, though, using the notation of Makinson [1986], to make the theory’s versions made by Kanger and Lindahl comparable. We do it so especially because this way of presenting these theories was the first guidance for me to amend an own version of the formalization of Hohfeld’s original theory on the fundamental legal conceptions, and hopefully this presentation also helps the reader relate my solution with the classical ones—even if the goals are different, but we will talk about this later in detail.

The Kanger-Lindahl theory is built on two logics: a deontic logical component, and an action logical component; and it has a method taken from set algebra to generate the space of all logically possible positions. The language Sergot uses is not perfectly the same as the original ones, since they used Shall, May as deontic operators and Do as an action operator, while Sergot uses propositional logic augmented with modal operators O for ‘obligation’ and its dual P for ‘permission’; and the action operator is an agent-relativised modal operator $E_a, E_b$ to refer to actions as it has been seen to it that a given state of affairs $F$ by the agent $a$ and $b$, respectively.

Let’s see the deontic logical component first. In the argument of the modal operators there are propositions: an expression $OA$ is referring to ‘it is obligatory that $A$’, that is, ‘it ought to be the case that $A$’. Defining $P$ happens as usual: $PA \overset{\text{def}}{=} \neg O \neg A$. As Sergot says: "the deontic logic employed by Kanger and Lindahl is—for all intents and purposes—the system usually referred to as Standard Deontic Logic (SDL). Specifically, the deontic logic employed is the smallest system containing propositional logic (PL) and the following axiom schemas and rules":

\[
\begin{align*}
\text{O.RE} & \quad A \leftrightarrow B \\
& \quad OA \leftrightarrow OB \\
\text{O.M} & \quad OA \land B \rightarrow (OA \rightarrow OB) \\
\text{O.C} & \quad (OA \land OB) \rightarrow OA \rightarrow OB \\
\text{O.P} & \quad \neg OA \rightarrow \bot \\
\end{align*}
\]

These names of the axiom schemas used by Sergot here are the ones Chellas [1980] used, this logic of $O$ is a classical modal type of the one Chellas called...
EMPC. For those who are familiar with Standard Deontic Logic it might quickly rise to view that the derivation rule of SDL

\[ \text{O.RN} \]

\[
\begin{array}{c}
A \\
\hline
\text{O}A \\
\end{array}
\]

(which can also be written as the schema \( O\top \)) is missing here. The reason is that the absence or presence of this rule does not affect the generation of normative positions (that’s why Sergot writes that the Kanger-Lindahl theory’s system is "for all intents and purposes" SDL). It also might seem strange at first glance that the characteristic axiom of deontic logic

\[ \text{O.D} \quad OP \rightarrow PA \]

is not in the list above: the reason is it follows from O.C and O.P.

Sergot touches upon a crucial issue: it is a well-known fact of SDL that it has a lot of "limitations" and "inadequacies", but, as he writes, "in combination with the logic of action, and in the restricted ways it is employed in generation of normative positions, these inadequacies are relatively benign."

The action logic part employed is the one called ET by Chellas [1980]. This is a very simple logic, which is important to emphasize since calling the \( E_x \) operator 'sees to it that' easily makes the readers associate it to STIT logics like S4 or S5. But in our case the relativized modal operator \( E_x \) has ET behind, that is, the smallest system containing PL, closed under

\[ \text{E.RE} \]

\[
\begin{array}{c}
A \leftrightarrow B \\
\hline
E_xA \leftrightarrow E_xB \\
\end{array}
\]

and containing the following axiom scheme (of reflexivity):

\[ \text{E.T} \quad E_xA \rightarrow A \]

in order to have successful actions.\(^1\) But this is the only requirement or feature we would like to count on regarding actions, nothing else—for instance intentionality or whether x saw to it knowingly or unknowingly—matters.

\(^1\)This restriction raises the question of attempt, which is a given actualization stage of an inchoate crime, which can be punishable. If we work with a system of action logic having the axiom E.T (and most of them contain it) we won’t be able to cover those cases. Therefore, in order to cover the punishable attempts we need to formalize them as separate actions (which solution is not so far from what happens in law: somehow it is needed to be phrased which stages count as attempts...).
I think the generality of an action operator is a good choice, but we will discuss it at my own proposal for rights’ formal representation.

5.1 Kanger’s Theory on Normative Positions

First of all we have to lay down that we do not speak about a single article, and actually we do not speak about a single Kanger: meanwhile Stig Kanger is the person to whom we primarily ascribe the famous formalization—with, of course, good reason—, his wife, Helle Kanger also worked in this topic; they have a famous paper together (Kanger and Kanger [1966]), and she wrote separately about it too: Kanger [1981]

The theory of Stig Kanger has been published in different places in different forms. Next to their common paper, Kanger [1971], Kanger [1972], and Kanger [1985] are the sources from which we can know the theory. (The way of the following presentation still follows Sergot [2013], though.)

Kanger’s starting point is considering simple types of rights relations between two given agents, \(a\) and \(b\), with respect to a given state of affairs, \(F\). The possible variations can be covered with a scheme suggested by Makinson [1986] and used by Sergot [2013]:

\[ \pm O \pm \left( \frac{E_a}{E_b} \right) \pm F \]

Here \(\pm\) stands for the two possibilities of affirmation and negation, while the choice scheme \(\left( \frac{E_a}{E_b} \right)\), which is a shorthand for a set of expressions (and therefore is mixed by Makinson/Sergot freely with standard set notations) is for indicating the two alternatives of the ‘sees to it that’ operator indexed by each of the two possible agents. The 16 expressions covered by the scheme range from \(OE_aF\) to \(\neg OE_b\neg F\). Kanger uses names for these simple types of rights (and here Sergot refers to the summery of Lindahl [1994]):

"(...) from the perspective of \(a\)’s rights versus \(b\), those in the scheme \(O \pm E_b \pm F\) are called Claim, Counter-claim, Immunity, Counter-immunity; those in the scheme \(\neg O \pm E_a \pm F\) (equivalently, \(P \pm E_a \pm F\)) are called Power, Counter-power, Freedom, Counter-freedom."

What do we see from these names already? The ‘Claim’ is the correspondent of the Hohfeldian ‘(Claim-)right’, its formula is: \(OE_bF\) if we talk about \(a\)’s right versus \(b\). The next formula covered by the scheme is \(O\neg E_b\neg F\), which is called ‘Counter-claim’ by Kanger, is not a separately considered and named version of \(a\)’s right versus \(b\) in the Hohfeldian theory: it is still a (claim-)right
of him that \( b \) refrain from doing something. The formula \( OE_b \neg F \) is still a (claim-)right in the Hohfeldian system (to see to it that something is not the case) and not the Hohfeldian ‘Immunity’. While \( O \neg E_b \neg F \) is a Hohfeldian ‘(claim-)right’ of \( a \) against \( b \) to refrain from seeing to it that something is not the case. This means that in spite of the names recalling the Hohfeldian theory at two points, so far we only got the Hohfeldian ‘(Claim-)right’ back in Kanger’s formal representation. Well, leastwise if we accept this indicated reading convention regarding the role of \( a \), since that agent is not in the formula: what we see on the very formal level is that Kanger considers the (Hohfeldian) duty of \( b \), even if he does not use the expression ‘duty’, and gives a reading convention to it as \( a \)’s right. Let’s see the other ones mentioned in the quote above. The formula \( \neg O E_a F \) is called ‘Power’ by Kanger but has nothing to do with the Hohfeldian Power. What it means actually is that it is permitted for \( a \) to refrain from seeing to it that \( F \), since \( \neg O E_a F \) is equivalent with \( P \neg E_a F \), which is actually a Hohfeldian privilege: with the negation of the obligation (Duty) Hohfeld defines Privilege—which in this case is held for refraining something. The formula \( \neg O \neg E_a F \) is the "basic" type of Privilege to do something in the Hohfeldian system (even if it is called ‘Counter-power’ by Kanger), since it is equivalent with \( PE_a F \). The formula \( \neg O \neg E_a \neg F \) is called ‘Freedom’ by Kanger, means a permission to refrain from seeing to it that \( F \) is the case, that is, it’s still a Hohfeldian privilege; while the \( \neg O \neg E_a \neg F \) is a permission to "do \( F \)", and as such, it would still be considered a version of Privilege in the Hohfeldian terms, having or not the name ‘Counter-freedom’ in Kanger.

Therefore, we can already lay down that the Kangerian terms for his simple right types do not correspond to the Hohfeldian ones—or, more precisely, they cover two of those, providing 4-4 refined versions including the possibilities of doing and refraining, and the "affirmative" (positive) and negated states of affairs.

But, we can say, Kanger, and his reception, was more interested in various compounds that can be created from the simple types—Kanger called these ‘atomic types of rights relation’. This label might sound strange regarding the Hohfeldian approach compared to which these constructions are rather molecular than atomic; but it is not strange at all regarding it from the viewpoint of Boolean algebra: these are the atoms of a Boolean algebra.

Makinson [1986] observed that these atomic types of Kanger can be covered by a set produced in the following way:

\[
\text{(KAT)}
\]

\[
\left[ \pm O \pm \left( \frac{E_a}{E_b} \right) \pm F \right]
\]
The brackets denote *maxi-conjunctions*: if \( \Phi \) is a choice-scheme (which is a set of sentences) \([\Phi]\) is the set of maxi-conjunction of \( \Phi \): the maximal consistent conjunctions of expressions belonging to \( \Phi \). This consistency refers to the underlying logic behind \( O \) and \( E_x \). 'Conjunction' is understood as without repetition\(^2\), and 'maximal consistent' means that any addition of any other conjunct from \( \Phi \) would make the conjunction inconsistent, that is: "a conjunction \( \Gamma \) is a maxi-conjunction of \( \Phi \) if and only if \( \Gamma \) is consistent, and every expression of \( \Phi \) either appears as a conjunct in \( \Gamma \) or is inconsistent with \( \Gamma \).

The atomic types of Kanger can be written in the following way, too:

\[
(KAT_x) \quad \left[ \left[ \pm O \pm \left( E_a \pm E_b \pm F \right) \right] = \left[ [\pm O \pm E_a \pm F] \cdot \left[ [\pm O \pm E_b \pm F] \right] \right] \right.
\]

How to understand the notation above? If we have two sets of expressions, like \( P \) and \( Q \), \( P \cdot Q \) stands for the set of all the consistent conjunctions that can be formed by conjoining an expression from set \( P \) with an expression from set \( Q \).

The maxi-conjunctions in

\[
(KAT_a) \quad [[\pm O \pm E_a \pm F]]
\]

are called in Jones and Sergot [1996] as 'Kanger’s normative one-agent actions'. According to the logic Kanger used,—removing the redundancies—there are six elements in it:\(^3\)

(K1) \( PE_a F \land PE_a \neg F \)
(K2) \( O \land E_a F \land O \land E_a \neg F \)
(K3) \( O E_a F \)
(K4) \( PE_a F \land P \land E_a F \land O \land E_a \neg F \)
(K5) \( O E_a \neg F \)
(K6) \( O \land E_a F \land P \land E_a \neg F \land P \land E_a \neg F \)

By the virtue of the way they have been constructed, these expressions are consistent, mutually exclusive and their disjunction is a tautology. "In any given situation precisely one of them must be true, according to the logical principles employed"—an explanation Sergot [2013] adds. The content of

\(^2\)No repetition does not ensure that there is no logical redundancies: it might be the case that one or more conjuncts may be implied by the others.

\(^3\)the numbering is from Lindahl [1977]
some formulas above is pretty straightforward: (K1) is about \( a \) is permitted
to do and to refrain from \( F \), while in the case described by (K3) \( a \) is obliged
to see to it that \( F \). (K2) might not be so talkative at first glance, but if we
write it in this way:

\[
(K2') \quad \neg PE_a F \land \neg PE_a \neg F
\]

it is more visible why Lindahl abbreviated it as referring to passivity: (as
Sergot says) it is "an obligation on \( a \) to remain passive with respect to \( F \)."

The 26 atomic types of rights relations that we mentioned in the introduction
come with using the (KATx), with which we would get 6x6=36, but 10 out of
them turned out to be inconsistent. Therefore, for two agents, with respect
to the bringing about of some given state of affairs, there are 26 atomic types
of rights relations in the Kangerian analysis. These are—by construction—
internally consistent, mutually exclusive and their disjunction is a tautology.
In any given situation only exactly one of them is true. Therefore, as Sergot
says: "it is in this sense that Kanger can be said to provide a complete and
exhaustive analysis of all the logically possible normative positions." But, I
feel the need to notice here again shortly that this analysis, while using some
notions of the Hohfeldian theory, is not the actual formalization of it.

5.2 Lindahl’s Contribution to the Theory of Normative Positions

What Lars Lindahl did is usually referred to as a refinement and further de-
velopment of the Kangerian theory. Working in the same spirit and adding
this development was enough to have called the theory of normative posi-
tions Kanger-Lindahl theory.

Lindahl also has a way of construction of the normative one-agent act posi-
tions, but it differs from (KATa), in the notation of Makinson [1986] it looks
like the following:

\[
(LATa) \quad [[\pm P ([\pm E_a \pm F] ])]
\]

where there is a maxi-conjunction expression within the scope of the \( P \)
operator. In words (LATa) "is the set of maxi-conjunction expression of the

4Sergot notes that this development is mainly of considering the change of normative
positions, but does not introduce this work of Lindahl, neither we'll do, since that is out
of the scope of our investigation.
form \( \pm PA \), where each \( A \) is itself a maxi-conjunctions of sentences of the form \( \pm E_a \pm F \)." In the set of

\[(LAPa) \quad \llbracket \pm E_a \pm F \rrbracket \]

there are three act positions:

- (LA1) \( E_a F \)
- (LA2) \( E_a \neg F \)
- (LA3) \( \neg E_a F \land \neg E_a \neg F \)

the third of which is about passivity of agent \( a \) with respect to the state of affairs \( F \), and which is abbreviated by Lindahl as \( Pass_a F \).

Removing the redundancies, there are 7 expressions in the set (LATa):

- (L1) \( PE_a F \land PE_a \neg F \land PPass_a F \)
- (L2) \( PE_a F \land O \neg E_a \neg F \land PPass_a F \)
- (L3) \( PE_a F \land PE_a \neg F \land \neg PPass_a F \)
- (L4) \( O \neg E_a F \land PE_a \neg F \land PPass_a F \)
- (L5) \( OE_a F \)
- (L6) \( OPass_a F \)
- (L7) \( OE_a \neg F \)

Five of the six expressions in (KATa) are equivalent to five of the seven in (LATa), while one Kangerian expression is equivalent to a disjunction of the "left-over" two of Lindahl:

- (K1) is logically equivalent to (L1 \lor L3)
- (K2) is logically equivalent to (L6)
- (K3) is logically equivalent to (L5)
- (K4) is logically equivalent to (L2)
- (K5) is logically equivalent to (L7)
- (K6) is logically equivalent to (L4)

Instead of the Kangerian two-agents types set (KATx), in Lindahl we have:

\[(LATx) \quad \llbracket \pm O \llbracket \pm E_a \pm F \rrbracket \rrbracket \cdot \llbracket \pm O \llbracket \pm E_b \pm F \rrbracket \rrbracket \]

which of course causes that instead of 26, here we have \( 7 \times 7 = 49 \) minus the inconsistent ones, that is, 35 normative two-agent normative positions.

Sergot discusses one more extension of the theory of Kanger made by Lindahl considering the two-agents atomic types: this is introducing collectivistic
two-agent types, "to cover the case, for instance, there is an obligation on two agents which does not imply to either of them individually":

\[
\text{(LCAT)} \quad O(E_a F \lor E_b F) \land \neg O E_a F \land \neg O E_b F
\]

which is actually about coordination of a’s and b’s actions. The reason that (LCAT) can be true is that in the logics employed here O does not distribute over disjunction (nor P over conjunctions). This is an important feature considering everyday cases, let us add an example of transport companies’ requests toward passengers: "Please use all doors to get on and off". It is obvious that any of the passengers has a specific obligation in advance, but seeing that (all the) others in front of them are waiting at the front door, one is required to go to the back door. (The italic is for emphasizing the coordination feature.) Behaving according to this, the requirement using all doors will be satisfied by the passengers together.

The following set operation shows the way of constructing Lindahl’s collectivistic two-agent normative positions:

\[
\text{(LCATx)} \quad \left[ \pm P \left[ \pm \left( \frac{E_a}{E_b} \right) \pm F \right] \right] = \left[ \pm P \left[ \left[ \pm E_a \pm F \right] \cdot \left[ \pm E_b \pm F \right] \right] \right]
\]

This is the way we get the 127 collectivistic normative two-agent act positions that were mentioned in the introduction. Even if these operations are very interesting from a computability point of view, it can be seen that, by now, we have gone quite far from the Hohfeldian theory: Hohfeld does not consider action coordination, what is more, not even the possible ways of combining the rights he describes. So instead of going into further computational details of these theories, we gather some impressions expressed by the reception of them, with special attention on their relation to the original Hohfeldian theory of fundamental legal concepts.

To put a summing up type short notice here we already can lay down that what Kanger and Lindahl did with introducing an action operator into the rights relations/positions is, next to being a computationally relevant enrichment (with which, as Makinson [1986] says—as it allows internal and external negation—the simple type relationship is exactly twice as numerous as without), is also a conceptual refinement.
5.3 Notes on, Supplements to, Reception and Shortcomings of the Classical Formalizations

Because of the reasons detailed above, we only briefly touch upon three papers first, since these all relate to the computability features of the rights relations; then I discuss two papers in detail since those two provide the basis of my proposal.

5.3.1 Jones and Sergot’s Refinement

Jones and Sergot [1996] provide substantive analysis primarily focusing on the computability features of normative positions’ theory (which analysis is worth to mention in spite of our leading goals). They call attention to that, since \( P \) is the dual of \( O \), \((\text{KATa})\) can be written in the following way:

\[
(\text{KATa'})
\]

\[
[[\pm P \pm E_a \pm F]]
\]

Sergot [2013] emphasizes the importance of this step as: "the expressions within the maxi-conjunction brackets may be seen in two ways: either as a scheme of four (not mutually exclusive) act positions \( \pm E_a \pm F \) prefixed by \( \pm P \), or as two (mutually exclusive) act positions \( E_a \pm F \) prefixed by \( \pm P \). What is obtained by combining the second view, \( \pm P \pm \), with the three mutually exclusive act positions \( [[\pm E_a \pm F]] \)?" For the logics used by Kanger and Lindahl, we can get the same seven expressions as Lindahl got from \((\text{LATa})\):

\[
(\text{JSATa})
\]

\[
[[\pm P \pm [[\pm E_a \pm F]]]] = [[\pm O \pm [[\pm E_a \pm F]]]]
\]

And this means, that in the case of logics EMCP (or stronger) for \( O \) and ET for \( E_x \), the following holds:

\[
(\text{JSATaT})
\]

\[
[[\pm P \pm [[\pm E_a \pm F]]]] = [[\pm P [[\pm E_a \pm F]]]]
\]

5.3.2 Alchourrón and Bulygin’s Normative Systems

It is well to also briefly touch upon the Kanger-Lindahl formalizations’—actually their Sergot-presentation’s—relation with the formal representation of normative systems developed in the famous book of Alchourrón and Bulygin [1971]. A normative system \( \mathcal{N} \) maps the universe of cases to solutions. We get the universe of cases taking the set of all possible fact combinations generated from a given set of propositional variables \( \text{Props} \) which in the
maxi-conjunction notation is \([\pm Props]\). To a given action \(F\) we can draw a consistent and complete normative system with the solutions in the following way:

\[
\mathcal{N} : \quad [\pm Props] \mapsto [\pm O \pm F]
\]

If the logic of \(O\) is type EMPC (or the Standard Deontic Logic, type KD) then (with removing the logical redundancies) the set of mutually exclusive normative ‘fact positions’ is \(\{OF, O \neg F, PF \land P \neg F\}\) (that is, ‘obligatory’, ‘forbidden’, ‘facultative’). Sergot [2013] mentions the way of generalization giving a consistent and complete normative system \(\mathcal{N}\) mapping the universe of cases to solutions for actions \(\{F_1, ..., F_n\}\):

\[
\mathcal{N} : \quad [\pm Props] \mapsto [\pm O \pm F_1] \cdot \ldots \cdot [\pm O \pm F_n]
\]

which is

\[
\mathcal{N} : \quad [\pm Props] \mapsto \begin{pmatrix}
OF_1 \\
O \neg F_1 \\
PF_1 \land P \neg F_1
\end{pmatrix} \cdot \ldots \cdot \begin{pmatrix}
OF_n \\
O \neg F_n \\
PF_n \land P \neg F_1
\end{pmatrix}
\]

5.3.3 A Graphic Representation by Syi

Before investigating the comments and critics of Makinson and Sergot—which will be the most important part of the reception regarding the inspiration of my proposal—, there is one more work it is nice have a look at. It is a graphic representation inspired by the same criticism on Hohfeld’s oversimplifying actions that led Kanger and Lindhal to their enhancement. Our reason of this short survey is using the graphic representation’s illuminating power on the variations raising and disappearing with combining right positions and actions.

In his book on action theory, Syi [2014] uses a graphic representation to show the structure of the Hohfeldian first group’s rights and duties and to prepare the Kangerian formalization’s presentation. In this graphic illustration Syi works with the following structure to build the logically possible right positions and then put them on the proper place of the drawings:

- type of the right: claim-right | duty | privilege | no-claim
- the owner of the right: Ego (E) | Alter (A)
- the action ("content" of the right): Do | \neg Do
- the actor of the action: Ego (E) | Alter (A)

Thus, we can have 32 combinations. If we arrange a given right-owner’s right positions to a given action into a plane, we get 8 groups of right positions. Syi arranges them into four quarters of space according to what kind
of action of whom we are talking about. On the following figure we can see in a) the doings of Alter; in b) the refrainings of Alter; in c) the doings of Ego; and in d) the refrainings of Ego:

![Diagram showing the relationships between claims, privileges, duties, and rights involving Alter and Ego.]

Not all of these logical possibilities make sense regarding the Hohfeldian intentions, though: a claim-right for instance only makes sense as having different right-owner and actor—being a passive right, while privilege only makes sense if it has the same agent in the position of the right-owner as the one in the position of the actor. If we delete these senseless positions from the diagram, and we add the correlativity as it has been indicated in Hohfeld we get the following figure:
Syi emphasizes that the correlativity lines going through quarters of space show that the action of a privilege and of a no-claim are different, a fact Hohfeld also mentions\(^5\), but maybe does not stress enough (the original Kangerian version of analysis differentiated inverse and converse correlativity to point the difference between the relation of Claim-right and Duty and the relation of Privilege and No-claim have).

Further clearing of the diagram can be done with slipping quarters into each other. Syi does it with quarters being ordered vertically (a)-c) and b)-d)), ending up in planes showing a given right-owner’s positions: the upper planes will show Alter’s rights relations, while the two underneath show Ego’s ones. In order to have a rather clear look on correlativity relations, Syi mirrors the planes on the right and then we get the following drawing:

---

\(^5\)As it is cited in this paper more times, Hohfeld says: "always, when it is said that a given privilege is the mere negation of a duty, what is meant, of course, is a duty having a content or tenor precisely opposite to that of the privilege in question."
A last making up on the drawing enables us to see the relations practically in the way Hohfeld draws them ('practically', since Hohfeld does not make the distinction between doing something and refraining from doing it). In this change Syi rotates the upper planes with 180 degrees around their horizontal axis, interchanges their privileges and no-claims, and he mirrors the right cube’s vertices to the vertical plane (being perpendicular to the planes). Then we still will have Alter’s right positions on the upper planes, and Ego’s ones on the lowers, but the correlative pairs get underneath each other:

What we see is that the structure divides into two separate cubes differing only in describing the "opposite doings" (the other of the doing-refraining pair).
5.3.4 Makinson's Critique of the Theory of Normative Positions

Interference

As it was indicated earlier, Makinson has a definitive opinion on interference and on the necessity of its implementation into the formal theory of rights. He admits that Kanger and Lindhal had some intention to deal with what role interference has in the notion Bentham called 'vested liberty', and Wright [1963] called simply 'right'—in which, next to a privilege of the agent, there is a prohibition of others to interfere; but as Makinson declares at the beginning of his discussion, what Kanger and Lindahl actually dealt with is not interference but—as he says—"the much more manageable notion of counter-performance". This notion and its handling is not immediately visible from the way Sergot—and therefore we—rephrased their theory, but it is enough to know for this investigation at the moment that Kanger and Lindahl—as Makinson describes—"instead of trying to express in formal terms the notion of \( y \) interfering with \( x \)'s doing \( F \), (...) work with the simpler notion of \( y \) seeing to it that not \( F \)." The problem is that neither notion implies the other. For displaying this, Makinson draws two situations: "In those situations in which \( x \) has no intention of bringing about \( F \), \( y \)'s seeing to it that not \( F \) would hardly seem to constitute interference with an action that \( x \) has not even envisaged, much less attempted. On the other hand, in those situations where \( x \) is already trying to bring it about \( F \), \( y \)'s seeing to it that not \( F \) is presumably a sufficient condition for interference. As regards the converse, however, there are many ways in which \( y \) can interfere with \( x \)'s seeing to it that \( F \) without actually bringing about the opposite; \( y \) may hinder, impede, harass, or create various difficulties or costs without succeeding in preventing \( F \) from coming about—indeed without even intending to prevent it, only make life more difficult." These thoughts of Makinson will be important for us when formalizing Privilege—and vested liberty (that we will call civil liberty).

Thus, the question how to formally represent interference and then vested liberty stayed open after the Kanger-Lindahl theory. Makinson is wondering about two options to solve this problem: "one approach is to treat interference in a formal manner, representing it by an additional component in general scheme for simple rights relationships. Another approach is to treat it as a matter of content only, whose place in a formal representation is only in possible instantiations of the predicate variable \( F \) of the scheme." Makinson indicates that Kanger and Lindahl were aware of the difference between interference and counterperformance, and they did not consider themselves representing the complete notion of vested liberty, only the simpler notion which is still closely related. Makinson consequently calls \( F \) as 'predicate variable' but I have to think he means what we mean by ‘propositional letter’.
son votes for the latter for two main reasons: the philosophical reason is that interference seem rather content laden (for which he takes the note that the 
\textit{sees to it that} operator is already quite content laden, more content laden than most logicians would like to see in such a role); the semantic reason is that interference has many variants that would require different structural properties and thus modes of symbolization if they were treated as part of the form. Makinson also adds that the representation should deal with intentionally, but since I do not deal with intentionality in my proposal, we do not go into details here.

Another interesting comment of his, though, that he calls attention to the difference between agent-indexed and agent-unindexed interference: "suppose that $x$ is permitted to see to it that $F$, and $y$ is forbidden to interfere. To interfere with what? With it coming to be the case that $F$, or specifically with $x$’s seeing to it that $F$? It can happen, for example, that $y$ interferes with $x$’s seeing to it that $F$ not because he has anything against the realization of $F$, but rather because he wants himself to be the one who brings it about, or has something against $x$’s bringing it about." We will try to reflect these considerations when formalizing.

Also, another differentiation makes sense, between modulated and non-modulated prohibitions on interference, according to Makinson: "suppose that $x$ is permitted to see to it that $F$, and $y$ is forbidden to interfere with $x$’s seeing to it that $F$." 8 Now notoriously, a permission to see to it that $F$ does not imply a permission to do so in any way whatsoever. For example, $x$ may be permitted to build a house, and yet consistently forbidden to do so in certain ways—indeed may be required to satisfy quite stringent conditions of safety, appearance and so on. So when $y$ is forbidden to interfere with $x$’s seeing to it that $F$, this may be understood either as a prohibition of interfering with any of $x$’s executions of $F$, or merely, as one on interfering with any of $x$’s permitted executions of $F$." These are examples of unmodulated and modulated kinds of prohibition (respectively).

I am not convinced that this last differentiation should happen directly when formalizing. I reckon the way $F$ is given solves—at least can solve—the problem: even if a specific rule contains only the description of a liberty with simply phrasing its content, other rules (usually in the very same paragraph only in other sections) provide the constraints. If we put the restricted content in the place of $F$ this problem will arise much fewer times.

---

8Note that at this point Makinson himself chose the version of what there is a prohibition to interfere with—about he was wondering a paragraph earlier—since he says "and $y$ is forbidden to interfere with $x$’s seeing to it that $F$". Agreeing with that, we will formalize vested liberty as having the prohibition on this kind of interfere instead of seeing to it that not $F$. 

50
Makinson closes the part of his paper discussing interference that its notion is both complex and ambiguous, which ambiguities he sees to be necessary to resolve priorly to formally representing it.

Formal Representation of the Bearer and Counterparty

Makinson [1986] has several observations on the classical formalizations. One is about relationality. As Makinson stresses while Hohfeld’s theory is resolutely relational, it needs a layered analysis to decide whether Kanger and Lindahl capture this feature in the formal representation. What we can see at first sight already is that they do not use explicit notational indexation for either bearer or counterparty: all the formulas falling under the scheme they use, looking like:

\[ \pm O \pm \left( \sum_y \right) \text{do} \pm F \]

containing a standard deontic operator to the obligation, whose argument is a proposition containing only one of the agents. Notwithstanding, since the do operator is to be read as having an agent who sees to it that something is the case, according to Makinson, it can be suggested that the identification of duty-bearers is implicitly in the formalism. But this still does not add anything to the notion of counterparty. In a relativized way it occurs in the atomic rights of Kanger, but talking about bearer and counterparty absolutely makes sense (what is more, in the Hohfeldian spirit, essential) in the case of simple rights alone—for which there is no solution of interpretation.

Makinson deems it necessary to introduce some explicit indexing of counterparties in the formal representation in order to properly capture the full relationality of rights relationships—even if it can be redundant in certain contexts. But if we do not want it to be necessary, but optional to indicate the bearer and the counterparty, then, according to Makinson, an explicit logic is needed to handle the relation between agent-indexed and not-indexed operators. About this logic he tells the following considerations:

- On the bearer’s side Makinson finds the most important to deal with

\[^9\text{Makinson refers to the parties in a rights relation as one of them is the bearer—the one who has the duty—, and the other one is the counterparty. I would see reasonable to refer the two parties as ‘counterparties’ seeing this relation symmetric in the sense of being the one or the other of the given relation, but I—at least in this part of the paper—will follow Makinson’s terminology.}\]

\[^10\text{in which we—following Sergot [2013]—use the notation of the action operator and the choice scheme together as } \left( \sum E \right).\]

\[^11\text{which means the complex ones in his case since he considers Boolean atoms.}\]

\[^12\text{The bearer will be indicated before (at the left side of) the } O \text{ operator, while the counterparty on its right (after the operator).}\]
the general ‘debearer’ implication, to express the plausible implication from someone bearing an obligation to the fact that there is an obligation:

\[ xOF \vdash OF \]

In spite of its plausibility, it has a seemingly strange or unwanted consequence: from this rule—if we substitute \( \neg F \) for \( F \), contrapose the implication and use permission the usual way being the dual of obligation, then— it also follows that

\[ PF \vdash xPF \]

But according to Makinson, it is not that odd as it firstly seems: "when we bear firmly in mind that \( F \) must be held unchanged on the left and right hand sides: there must be no surreptitious change of visible or hidden variable within it. Thus the principle does not tell us that it is permitted for \( x \) that \( x \) open the door if it is permitted that \( y \) open the door. Nor does it say that it is permitted for \( x \) that \( x \) open the door if it permitted that the door be opened by somebody. In each case the left-hand instantiation of \( F \) must be exactly the same as the right-hand one. It should also be remembered that ordinary language formulations of statements of permission tend to be be very vague and sloppy in their treatment of variables, with frequency ambiguity between, say, existential and universal readings (...). For this reason, any suspected counterexample to the above contraposed form of the ‘debearer’ implication needs to be formulated and examined very scupulously."

Makinson also suggests two "bearer monotony" rules to the logic containing agent-indexed rights relations:

\[ F \vdash F' \Rightarrow xOF(x \ do \ F) \vdash xOF(x \ do \ F') \]
\[ F \vdash F' \Rightarrow xO\neg(x \ do \ F) \vdash xO\neg(x \ do \ F') \]

or as an alternative, there is a "more general and less cautious" bearer monotony rule that covers the two above:

\[ F \vdash F' \Rightarrow xOF \vdash xOF' \]

or even more generally:

\[ F_1 \land ... \land F_n \vdash F' \Rightarrow xOF_1 \land ... \land xOF_n \vdash xOF' \]

or, what is more, we can accept "redundancy principles":

52
$$O(x \text{ do } F) \vdash xO(x \text{ do } F) \quad \text{and} \quad O\neg(x \text{ do } F) \vdash xO\neg(x \text{ do } F).$$

But according to Makinson, we should "resist the temptation to strengthen the debearer principle" which could take the form of

$$OF \vdash \exists w(wOF)$$

(where w is any variable with no free occurrences in F) since then the following situation could cause a problem: let’s suppose that $xO(x$ do $F)$ and $yO(y$ do $F)$ are both true. Then because of the debearer rule $O(x$ do $F)$ and $O(y$ do $F)$ would both hold, and then—because of Standard Deontic Logic—$O(x$ do $F \land y$ do $F)$ would also be true, meanwhile neither $x$ nor $y$ could be reasonably regarded as the bearer of this obligation, which means that from left to right direction in the formula above would fail.

- On the side of the counterparty Makinson also deems the consideration of a ‘decounter’ principle:

$$OyF \vdash OF$$

About the "counterparty monotony rules", he puts them in their "cautious" form:

$$F \vdash F' \Rightarrow Oy(x \text{ do } F) \vdash Oy(x \text{ do } F')$$
$$F \vdash F' \Rightarrow Oy\neg(x \text{ do } F) \vdash Oy\neg(x \text{ do } F')$$

and in the more general forms:

$$F \vdash F' \Rightarrow OyF \vdash OyF'$$
$$F_1 \land ... \land F_n \Rightarrow OyF_1 \land ... \land OyF_n \vdash OyF'$$

In contrast with the case of the bearer, in the case of counterparty we do not need to worry about "redundancy rules". It is still a question, though, whether the ‘decounter principle’ can be formed as an existential equivalence

$$OF \vdash \exists w(OWF)$$

(where w is any variable with no free occurrence in F). The possible situation we have to consider here is the following: $Oy(x$ do $F)$ and $Oz(x$ do $G)$ are both true. Then because of the decounter and Standard Deontic Logic $O(x$ do $F \land x$ do $G)$ is also true. But neither $y$ nor
z can be reasonably regarded as the counterparty of the conjunctive obligation. Thus, again, the left to right direction of the existential equivalence would fail.

- Considering the bearer and the counterparty together Makinson only makes a supplement with saying that we can add the other index to the principles above and then we get them in the forms:

\[
xOyF \vdash OyF \\
xOyF \vdash xOF
\]

After providing these preliminary sketches about plausible rules of the logic reflecting explicitly to the bearer and counterparty, Makinson feels necessary to explicate informally what motivations he has on the notions of these agents. But he, in advance, keeps aloof from developing a full formal system with model-theoretic valuation for indexed formulas, since, as he phrases: "whilst it is relatively easy to set out an informal explication that is sufficiently clear to assist in the appreciation of postulates, it is quite another matter to be able to render such an explication precise, extensional, or formal." Therefore, again: without aiming to provide a full suitable logic, Makinson lays down the informal definition-like description of a rights relation where the bearer and the counterparty are explicitly built on:

\[
x \text{ bears an obligation to } y \text{ that } F \text{ under the system } N \text{ of norms } \iff \\
in the case that } F \text{ is not true then } y \text{ has the power under the code } N \text{ to initiate legal action against } x \text{ for non-fulfillment of } F
\]

Makinson immediately calls attention to two features which make it difficult to convert this definition into an evaluation system, because, as he says: "both of these two »grains of intensionality« are essential to the account; both are difficult to treat model-theoretically":

- First, it is obvious that the informal implication ‘in the case that’ cannot be formally a material implication, because the simple propositional logic rules behind the material condition: whenever \( F \) in fact would be true, the whole conditional would be true, which, because of the ‘if and only if’ would ended up in a situation in which everybody would bear an obligation to everybody that \( F \) is the case, that is, whatever we see to it that that is the case would be obligatory for us to see to it that.

- Second: maybe it seems just as an addition, but ‘for non-fulfillment of \( F \)” is crucial to include, because, as Makinson explains: "it is needed to exclude the case that \( y \) happens to have a power to initiate legal action against \( x \) for quite different reasons that have nothing to do with \( F \)."
Makinson countervails the modesty of this proposal above as being informal with emphasizing its philosophical significance; he says: "in order to give an account of what it is to be a bearer or counterparty of an obligation, it makes essential to use of the notion of a power to initiate legal proceedings, and the counterparty of the obligation is taken as the bearer of the power. This seems to suggest the importance of the notion of a power, which has hitherto been rather neglected by logicians, for a full understanding of obligation, permission, and other concepts of the deontic dimension."

These consideration of Makinson will be the ones from we I proceed when developing my own proposal of rights formalization.

5.3.5 Sergot’s Critique of the Theory of Normative Positions

Several considerations of Marek Sergot has already been touched upon, since the introduction of the classical formalizations has happened following his introductory and evaluative article on normative positions. This is the point, though, where we need to investigate the evaluative comments and summary of him made on the theory of Kanger and Lindahl—and on the theory of Makinson.

Handling Counterparty

The first of the shortcomings or limitations of the Kanger-Lindahl theory, as Sergot [2013] refers to the points criticised by him, is the role (actually its lack) of the counterparty. As it already has come up a few times, relationality, the notion of having parties—bearer and counterparty—has been found fundamental in the Hohfeldian theory by its reception, therefore, it is not surprising that its lack is one of the main points Sergot considered as shortcomings of the most notable formalizations. He writes in his article presenting and discussing the work of Kanger and Lindahl: "the theory of normative positions, when viewed as a theory of duties and rights or as a formalisation of the Hohfeldian framework fails to deal with the notion of counterparty—the idea that when a party $x$ owes an obligation or duty to party $y$ that such–and–such, or when $y$ has a claim-right against $x$ that such–and–such, then the counterparty $y$ has a special relationship in the normative relation between $x$ and $y$ that is not shared by other agents". Sergot introduces two directions of conceptual handling the nature of the counterparty’s role. These two are treating counterparty as (i) claimant and as (ii) beneficiary. Arguing for one or the other as the proper interpretation of counterparty makes them seemingly competing account, meanwhile, according to Sergot, it is better to regard them as meaningful and distinct notions on their own right. At this point Sergot notes something very interesting: "In some cases claimant and beneficiary coincide, in other cases they do not". We do not get examples,
but the reference to Makinson Sergot has here shows a potential reason to say this—but we come back to this point later.

(i) Introducing the two—competing or not—approaches to the counterparty’s role starts with claimant, for which Sergot says: "counterparty as a claimant notion is associated with ‘power’. Thus a commonly expressed view of what it means to be a counterparty is in the terms of a conditional power". Sergot first cites Wellmann: "A relative duty in the law is owed to the party who has the legal power to initiate proceedings to enforce that duty", then cites Makinson giving a definition for obligation we already discussed above:

\[
x \text{ bears an obligation to } y \text{ that } F \text{ under the system } N \text{ of norms iff }
\]

\[
in the case that \ F \text{ is not true then } y \text{ has the power under the code } N \text{ to initiate legal action against } x \text{ for non-fulfillment of } F
\]

Sergot does not discuss the difficulties Makinson himself did in his paper on the possible formal correspondence of this approach, but discusses another, arising independently from any formal consideration being already within. The obvious difficulty with this intuitivesounding definition of Makinson is the right-left direction of the biconditional: \( y \) can initiate a legal action against anyone without having a claim-right. It’s just that he won’t win the case, or as Sergot phrases: "a party \( y \) has a power to initiate a legal action against \( x \) even when \( x \) has no obligation to \( y \), even when legal action is initiated on what will turn out to be completely unsubstantiated grounds, or perhaps even frivolously." It might help to involve some kind of expectation of success into the definition, a point Sergot raises. Not a guarantee, because legal actions by their nature are not that certain. "But an extra ingredient is essential to eliminate speculative, unsubstantiated or frivolous legal actions. It is very far from clear how one might approach a characterisation of that idea" Sergot says. We will come back to this and discuss it in detail when formalizing.

(ii) The other approach to describe the nature of the counterparty is speaking about it as the beneficiary. Here Sergot emphasizes the difficulties of formal representation. He introduces the solution of Herrestad and Krogh [1995], who—along with others—proposed adding a directed index to the obligation operator denoting beneficiary in a way

\[
O_{x \rightarrow y} F
\]

represents ‘there is a directed obligation that \( F \) on the bearer \( x \) that is for the benefit of the counterparty \( y \)’. Meanwhile, as Sergot says,
this notation enables us to express important distinctions, adding a
notation in itself does not add any insight into the notion we would
like to use it for. He also mentions Lindahl [1994]’s proposal, which he
evaluates showing much more promise, a variation of the Andersonian
reduction. At this moment it seems enough to tell about the Anderso-
nian reduction that is a proposal of reducing deontic logic to alethic
modal logic with using the constant referring to the common notion of
sanction with the definition

$$S \overset{\text{def}}{=} \neg d$$

with the meaning of ‘some (relevant) normative demands has been
violated’, and defining obligatoriness in a way of

$$Op \overset{\text{def}}{=} \Box (\neg p \to S).$$

In Lindahl’s version, let’s say that $x$ and $y$ are names of agents, and
we have a propositional constant $W$ in a way ‘$W(x, y)$’ expresses ‘$x$ is
wronged by $y$’. Then the $O_{x \to y} F$ that is, $x$ is the bearer of a directive
obligation (relative duty) to $y$ that $F$ is to be understand as ‘$y$ has
a right-proper versus $x$ to the effect that $F$. Then we can define this
directed obligation with $W(x, y)$ as follows:

$$O_{x \to y} F \overset{\text{def}}{=} \Box (\neg F \to W(y, x))$$

In words: $x$ owes an obligation to $y$ that $F$ ($y$ has a right-proper versus
$x$ that $F$) when, if it is not the case that $F$, then $y$ is wronged by $x$.
As in the version of Anderson, Lindahl handles $\Box$ as a normal alethic
modality of logic type that Chellas called KT.\footnote{Having the axioms D, T, B, 4 and 5, that is, the logic for which we also often refer as S5} We can introduce the
Andersonian propositional constant $S$ adding the axiom:

$$W(x, y) \to S$$

so with the Andersonian reduction

$$OF \leftrightarrow \Box (\neg F \to S) \quad \text{and}$$

$$\neg \Box S$$

we get a logic of $O_{x \to y} F$ being Standard Deontic Logic (a normal logic
of type KD).

\footnote{\textit{having the axioms D, T, B, 4 and 5, that is, the logic for which we also often refer as S5}}
Sergot mentions a possible way of enriching this idea with some simple
general properties of □ (which he does not list because of the space)
and introducing $E_x$ so it will be derivable that

$$O_{x\rightarrow y} E_x F \rightarrow O_{y\rightarrow x} E_y \neg F$$

that is, if $x$ owes a duty to $y$ to see to it that $F$ then $y$ has a duty
towards $x$ not to see to it that $\neg F$, which is quite plausible.

**Formalizing Power**

The other—already mentioned—shortcoming of the Kanger-Lindahl theory
of normative positions—if we consider it from a Hohfeldian viewpoint—is
that it absolutely misses to address the feature of the Hohfeldian Power. As
Sergot says (partly referring to what we saw in Fitch [1967]), "it has long
been understood that ‘power’ in the sense of (legal) capacity or ‘competence’
cannot be reduced to permission, and must also be distinguished from the
‘can’ of practical possibility. An agent can have ‘power’, to effect a mar-
riage say, without necessarily having the the permission nor the practical
possibility of exercising that power"; as Makinson [1986] explained it, too.
Sergot also refers to the paper Jones and Sergot [1996] in which they argue
that Hohfeldian Power is to be understood as falling within the more general
phenomenon of ‘counts as’: "in the context of a given normative system or
institution, designated kind of acts, when performed by designated agents
in specific circumstances, count as acts that create or modify specific kinds
of institutional relations and states of affairs. This switches attention from
the formalisation of permission to the formalisation of the count as relation
more generally." Grossi and Jones [2013] provides a summary and analysis
of which directions have been followed and what results have been showed
so far in the formal analysis on ‘counts as’ relations, actually ‘counts as’
conditionals.

As Grossi and Jones [2013] introduces their analysis, the constitutive norms
have been analyzed formally building on the somewhat canonical form of
them declared by John Searle:

$$X \text{ counts as } Y \text{ in the context } C.$$  

The pioneer and trigger of all the latter papers was indeed Jones and Sergot
[1996] as Sergot himself referred to it at the end of his article on normative
positions. In this paper Jones and Sergot introduced a special connective $\Rightarrow_c$
to represent the Searlian presentation above in a way: $\varphi_1 \Rightarrow_c \varphi_2$ with the
following principles:
expressing right logical equivalence, left logical equivalence, properties of conjunction of the consequent, properties of disjunction of the antecedent, and transitivity—respectively (we come back later to some of these principles when formalizing). In addition the authors link the logic of a modality called $D_c$, which logic is a modal system KD (that is, SDL, Standard Deontic Logic). The intuitive reading of $D_c\varphi$ is "it is a constraint of institution $c$ that $\varphi". This logic is linked to the principles with the following schemata:

(JS6) \((\varphi_1 \Rightarrow_c \varphi_2) \rightarrow D_c(\varphi_1 \rightarrow \varphi_2)\)

(JS7) \((\varphi_1 \Rightarrow_c \varphi_2) \rightarrow (\varphi_1 \rightarrow D_c \varphi_1)\)

These formulae are given back in words by Grossi and Jones [2013] as (JS6) "states, intuitively, that counts-as conditionals of a given institution $c$ are a subset of the constraints operative in institution $c"$, and (JS7) as saying that "if a state-of-affairs occurs as an antecedent in a counts-as conditional, then, if that state-of-affairs is the case it is also "institutionally" the case, that is, it is recognized by the institution concerned".

In this logic then such reasoning patterns that describe how an officer announce a couple married will be valid (and called ‘institutional detachment’), with $p$ referring to the state-of-affairs in which the officer pronounces the couple husband and wife, $c$ to the context of institution, and $m$ that the couple is married:

\[ p \Rightarrow_c m, p \vdash D_c m \]

Its proof is written in Grossi and Jones [2013] as: from $p \Rightarrow_c m$, formula (JS7) and modus ponens we can infer to $(p \rightarrow D_c)$, from which, by $p$, modal principles and modus ponens we can conclude that $D_c m$.

The other formalizations introduced in Grossi and Jones [2013] are refinements of this theory of Jones and Sergot [1996] (i) staying within modal logic like by Gelati et al. [2004], Grossi et al. [2006] or Lorini et al. [2009]—the two latter of which are rather based on semantic intuitions; or (ii) using alternative formalisms like Governatori and Rotolo 2008 do with defeasible logic, Boella and der Torre 2003 do using input/output logic, or Lindahl and Odelstad 2000 do with algebraic analysis. (We do not analyze these solutions since the point of these formalizations is not exactly the same as
Returning to the original paper of Sergot on normative positions (Sergot [2013]), his conclusion on the theory of normative positions developed by Kanger and Lindahl is that it is "an important but limited component" of the Hohfeldian rights relations’ formal treatment. Sergot remarks that it makes no real point to seek for an identifiable set of basic types in terms of which all logically possible relations between two agents could be articulated, since this representation can be taken to arbitrary levels with enriching the complexity. But, and this is how Sergot closes his paper on the normative positions in the *Handbook of Deontic Logic and Normative Systems*, "there are nevertheless grounds to believe that a more comprehensive formal account could be developed, together with the automated support tools necessary for its practical use."

This is what I attempt with the proposal detailed in the following chapters.
Chapter 6


6.1 Major Underlying Considerations and Arguments when Formalizing

Before discussing the specific formal proposal I provide, it is useful to share the considerations along which I have developed it. This concurrently provides the possibility to tell which of the previously introduced theoretical viewpoints from the last hundred years I agree with and have built upon, and which others I see necessary to include to provide a formal representation of rights I deem comprehensive and—at the same time—close enough to the Hohfeldian intentions.

6.1.1 Aims

My starting point is to use the formal tools that the theories I previously introduced do, but I pursue to satisfy the closing wish of Sergot [2013] by providing a more comprehensive conceptual formal account for the Hohfeldian conceptions in a uniform manner\(^1\), in order to reveal their nature and

\(^1\)On a much earlier version of my formalization, a reviewer wrote that "There is every reason to presume that the eight Hohfeldian categories and their inter-relations form a coherent whole, and therefore that their formalizations should exhibit a strong similarity and that their inter-relations should flow very directly from their formalizations." This is something I deeply agree with, and I have changed several things while developing the formal theory in order to tend to the manner required in the quote above. I hope now, at least, my formalization provides the promise of a level of work meeting this requirement. The review was a blind one so I have no chance to thank for the reviewer directly, but at this place I would like tell how useful it was.
the difference between them. This, at the same time, means that I do not
strive to assess computational features and possibilities of atomic or molecu-
lar right structures—this has been accomplished by the normative positions’
theory, I have introduced above, and I have no intention to compete with
it: on the contrary, starting from the same formal tools, I put the emphasis
completely on what can be settled about Hohfeldian legal conceptions, in
addition to what already has been settled about their computational prop-
erties. While I would like to provide substantial formulas, I insist on staying
within the Hohfeldian intentions and meaning by regarding the rights and
duties he differentiated as *sui generis*. Thus, I refuse to involve out-of-law
factors, like *will* or *interest* with which such an explanation could be pro-
vided on what rights mean that avoids any possibility of redundancy. Still,
I—obviously—aim to avoid redundancy or circularity, to the extent possible,
but I do so staying within *law*.

6.1.2 Level of Abstraction and Practical Use

Formalizing notions raises a lot of questions about the necessary level of
abstraction. On the one hand, we already have the notions we would like to
formalize, provided in a way that is already abstract: Hohfeld himself refers to
the fact that the conceptions he describes do not reflect on very specific cases;
he rather aims at grasping generalities, common base and differences behind
the specific cases, and that already involves abstraction. On the other hand, if
we would like to provide a formal description of the system of rights, it would
be nice to show in detail how and why they fit specific cases and norms of *legal
reality*. Going into details formally seems especially reasonable if we pursue
the practical use aiming at automated support tools usable in softwares. I
will proceed in a way which rather tends toward an abstract direction (which
sometimes involves simplification from a legal point of view) but show at
different points how the set of tools we generate is capable of expressing
more specific considerations. I do this assuming that grasping the essence
of conceptions and showing a system’s structure in a comprehensive way
requires a given level of generality and abstraction. Regarding the grounds
for automation, I do not provide meta-theorems, but rather stay on the
philosophical side of the discussion hoping that if that is worthy enough,
future considerations and developments can be built on it, with an eye to
specific computational applications.

6.1.3 State of Affairs in Logic—and in Law

States of affairs can be referred to with propositions. In the language we
are going to use, states of affairs will mostly be referred to by propositional
letters. The reason for this choice is that I would like to concentrate on
the rights and duties, and their interaction with actions. Since law aims to
grasp and refer to the real world, (descriptions of) states of affairs in law are extremely various. Even to define types of references would be very hard, but to picture what differences have to be imagined it is enough to think of differences—even within a given branch of law—between statutory references and states of affairs referred to in a contract: the first will be phrased in a general way in order to cover all the possible future situations that the legislator considers to be the same from the viewpoint of the given regulatory goal; while, in the case of a contract, parties will describe the state of affairs they consider and agree in a very specific way. For instance, according to the Civil Code, "any person who causes damage to another person wrongfully shall be liable for such damage", that is, it speaks about states in which someone caused damage to another. No specific person is named, no specific type of damage is referred to, since the Code is designed to cover all of them. What happens here is the creation of a type of situation by describing the relevant features of different states of affairs that are similar enough due to these common features. In the case of a contract, however, parties usually agree about seeing to it that a specific situation is indeed the case. They do not consider a whole type of states of affairs, but describe a specific one; for instance, agent A and agent B agree that A sells his car to B, which means that A has to see to it that B (not anyone) receives the car (not any car) in a given time (not any time), maybe with specifying also the conveyance’s place and other circumstances. What they describe is a token of a state of affairs.

If we consider criminal law, we see the same duality: the Criminal Code describes several types of state of affairs (well, mostly describes actions, but these actions have a result: a state of affairs), which the legislator actually does not consider desirable (that’s why seeing to it that any of these states of affairs is the case, that is, any committing any crime is ordered to be punished). If something happens, and a legal action is initiated, the judge has to decide whether what really happened (the token) is a token of the type of the specific state of affairs that is described in the Criminal Code. This difference is of great theoretical importance, which we will refer to when formalizing, but as a starting point, we are going to use propositional letters to refer to states of affairs. A propositional letter enables us to set aside (at least to pretend that we could set aside) this difference, since it means we do not look inside the proposition: we, therefore, do not bother ourselves by whether it is a general description of a type of state of affairs; or a specific description, that is, a reference to a token of states of affairs. This usage of propositional letters does not mean that we won’t reflect on the differences: I will either look for a formal solution to reflect on when needed, or I will mention whether it is a limitation of this simple formal solution.
6.1.4 Actions in Logic—and in Law

The formal study of actions started in philosophy, but bears great relevance in linguistics, computer science and in the study of artificial intelligence. Since legal rules are tools for regulating human behaviour, it is natural that the logic of (legal) norms or deontic logic involves some logic of actions. The formal representation of actions is far from obvious—not so surprisingly of course, if we consider that the philosophy or theory of actions itself has several different approaches to, and descriptions of what exactly we should consider an action. In the logic of actions, there are two main directions to describe an action: one is the so called stit saga—after the expression: one 'sees to it that' something is the case; the other is dynamic logic. In stit we describe an action by its result, which has been seen to, without referring to the process itself by which the result has been obtained; while in dynamic logic the focus is on the action, which is usually described as a transition from one state to another (coming with an ontology having actions and events). Both approaches can be considered branches of modal logic. Dynamic logic was practically "invented" by computer science, but as we saw, Wright [1963] draws a logic of actions exactly with this kind of approach. The other formalizations we saw, however, use stit operators as propositional operators added to the propositional language. This operator is usually agent-indexed referring to the agent who "sees to it that", and the argument of the operator contains the proposition which describes what comes after the ‘that’ above. Stit logics can be different according to the axioms they work with. The most usual approaches come with the axiomatic systems of S4 or S5 building on branching time models behind. The most well-known systems are the ones that were developed in Chellas [1969] and Belnap [2001].

Just like the philosophy of actions, the (stit) logic of actions can involve or reflect on a lot of different factors and features of actions. Sergot [2013], after listing a lot of papers applying different action logics in deontic logic or with formalization of rights, writes: "it is likely that a comprehensive theory of rights and/or organisations would require several different notions of action and agency", and, as an example, refers to two papers of Filipe Santos suggesting that distinguishing between direct and indirect action can be useful (in describing certain organisational structures). But immediately adds that nothing in the Kanger-Lindahl theory’s account depends on such choices. I have also been hoping to develop a formal system that is essence-focused and insightful enough to be expressive and comprehensive even without being lost in the details of action. What is more, I believe that using a simple action logic helps keep the nature of rights in the focus while keeping the problem involving details of action theories away. Along with these considerations, I have deemed it better to keep the formal representation of the actions quite simple, just as it happened with the papers introduced in the
previous chapters. Therefore we will use the same action logic as the presented theories used, the one Chellas [1980] called ET, that has only two axioms (one is a rule about the interchangeability of equivalent propositions in its scope, the other is axiom T) leaving a lot of the actions' potential features in the background. This generality enables us to occasionally handle seemingly quite different actions in the very same fashion, shedding light on what the only relevant point is in these actions from the viewpoint of the given legal relation in which they occur.

In my proposal actions, in addition to their role of being in the argument of rights and duties, will play crucial role in showing these latters' nature with the formulas reflecting on how given (ways of) actions and given rights or duties affect on each-other and result in other rights and duties.

Using a stit operator, of course, results in action descriptions that usually do not sound like those we read about in legal texts. In stit logics actions are not named, while in statutes we usually find rules naming actions—but not always: in some contexts the typical phrasing of a rule is referring to the ending state of an action, a result state regardless of what action exactly led to it. For instance, in advertising law there is a prohibition saying no advertisement should be misleading. When the authority (consumer protection or the competition authority) finds an advertising misleading, it also decides whose actions led to this state of affairs. But it is not really possible to list all the potential actions which could lead to such a situation. Therefore, focusing on the end state of an action is not something completely alien to law.

6.1.5 Agents in Logic—and in Law

Since from the viewpoint of deontic logic the point in the Hohfeldian theory is that it refers to agency, it is natural to build the formal representation "around" the agents. This involves agent-indexed operators, and formulas in which the agents’ role is crucial. The parties will be denoted by variables $x$, $y$, and a special one, the judge, by $j$, which is, at first, to be understood as a variable running over a proper subset of agents containing judges, then, later, as a constant standing for judiciary as a whole. The set of agents will be the basis of the semantics, we won’t quantify over them—we stay within zero-order (multi-)modal logic.

What we will see is that correlativity of rights and duties—which is present in all pairs of them—sometimes naturally arises from the formal rephrasing showing two involved agents, sometimes (in the case of Duty, Privilege, Power and Disability, that is, in the case of active rights and duties), however, we need to add special notations to indicate it.
Binding rights and duties to agents on both sides, that is, the theory according to which there is no undirected duty and right is an often discussed point of the Hohfeldian system. Criminal law is a specific area frequently mentioned as one handling rights without a specific duty bearer on the other side. I will formally specify what answer can be given to this criticism by showing how agents can be referred to in the case of rights like the right to life, the right to physical integrity, etc. This does not mean that I will overtly and comprehensively argue that the Hohfeldian theory can be applied to every area of law without exception, but I do argue that some of the counterarguments can be refuted by formal representations of what happens in these special areas of law.

It is important to discuss the issue of a special possible agent of rights relations: the state. The role of the state is a frequently discussed point in connection with rights, especially human rights. The correlative nature of rights and duties, the fact that we always speak about agents involved in a given rights relation is actually well embedded in legal and political thinking and human rights arguments, just like the fact that the state is also involved (somehow). But these two issues, intendedly or not, are often merged into one another: this usually achieved by putting the state placed in the position of the bearer in a given relation and arguing against the existence of the correlative right with the unacceptability of the duty which would be on the state (for examples see: Markovich [2015a] and Markovich [2016]). But if we fail to see clearly who the involved parties, the agents, are, for example with putting the state wrongly in one of the positions, our argument will also fail. Ensuring a human right in its complexity might, of course, affect the duties of the state at different points and levels, depending especially, for instance, on which generation of human rights\(^2\) we consider. Therefore, we cannot—and I do not—say that an argument refuting rights due to refuting the duties of the state \textit{ab ovo} cannot be right. What I will say—and show—is that in the case of simple rights, the rights have been systematized by Hohfeld, the role of the state can be clearly restricted to a well-definable role, a position in which the state really becomes an agent of the given rights relation.

\textbf{6.1.6 Counterpartyness}

As it has been clarified, correlativeity puts agents in specific positions: each agent at each side of a given rights relation. And it has also been discussed above, the special role has been specified in the "paradigmatic case" of a

\(^2\)It is a common way to refer to different types of human rights according to the different times and historical circumstances they arose in, calling theses groups first-, second-, or third generation rights.
relation involving a right and a duty: if we consider the first correlative pair, the agent having the duty is the duty-bearer, while the other, who has the correlative claim-right is usually referred to as the counterparty. The fact that being involved in a rights relation means a special relation—and also, that this speciality defines in a way the rights relation itself—has been found fundamental by the reception of Hohfeld, and as we also could see above, it is one of the main points Marek Sergot considered as shortcomings of the most notable formalizations of the Hohfeldian conceptions (if we consider them as such).

As we saw above, Sergot [2013] introduces two directions of how to interpret the role of counterparty: as the claimant and as the beneficiary. I will use both in the proposal. The counterparty being beneficiary will be used on the level of the notation introduced by Herrestad and Krogh [1995]. The substance, whose absence was pointed out by Sergot, will be implemented by the interpretation as a claimant. Let’s see again how he introduces the claimant interpretation of counterparty: "counterparty as a claimant notion is associated with ‘power’. Thus a commonly expressed view of what it means to be a counterparty is in the terms of a conditional power". Sergot first cites Wellman [1990]: "A relative duty in the law is owed to the party who has the legal power to initiate proceedings to enforce that duty", then cites Makinson [1986] giving a definition for obligation we already discussed above:

\[
x \text{ bears an obligation to } y \text{ that } F \text{ under the system } N \text{ of norms } \\
\text{ iff } \\
\text{ in the case that } F \text{ is not true then } y \text{ has the power under the code } N \text{ to initiate legal action against } x \text{ for non-fulfillment of } F
\]

As I wrote above, Sergot does not discuss the difficulties that Makinson himself laid down as problems to face when formalizing, but points out another difficulty of this—intuitive-sounding—definition of counterparty (which is also a definition of a duty—and therefore of a claim-right) having in its right-left direction of the biconditional: \( y \) can initiate a legal action against anyone without having a claim-right. It’s just that he won’t win the case. Maybe we should involve some kind of expectation of success in the definition, a point Sergot raises.

An enrichment like this sounds a valid approach to what happens in courts, a legal realist one. Sergot himself refers to litigation’s uncertainty: assigning some expectations of success to initiating a legal action if we have a claim-right and it has not been fulfilled would reflect to the legal reality properly, indeed, to the legal systems’ frailness and contingency coming from being operated by, and dealing with, humans.
In my opinion, however, an expectation of success in itself might point correctly to the counterparty, but doesn’t reveal the nature of the duty or its correlative, the claim-right. It shows some correlation in a statistical sense—being a good approximation to legal reality—but nothing more in the philosophical, conceptual sense. In my formalization, I pursue to demonstrate how this expectation comes about and what it means regarding the type of the claim-right as a right (and a duty as an obligation). This will happen with a solution that also put a light on why my approach differs from what Sergot writes about claimant interpretation of counterparty: I do not associate it with Power, I am convinced the key concept is Claim-right. I will argue for this in detail when providing the formulas.

Even if departing from the Makinsonian definition above in leaving Power aside, I am going to build my formal proposal on a crucial point being already grasped by him—one of the most (if not the most) important attribution(s) of legal rights from a legal theoretical viewpoint: the possibility of seeking remedy in court.

6.1.7 First Group’s Formal Representation: Enforceability by the State

This point is worth spending some time on. As it already has come up, one of the contexts in which the question of rights arises is the context of human rights, while the question of human rights practically automatically involves the natural law–positive law debate. Therefore, binding the existence/essence of rights to state enforcement might sound a radical and divisive step. It is well to clarify some issues in its defense.

- In this dissertation, only legal rights and duties are considered. The Hohfeldian theory has a relevance in ethics, since obligations and rights can be moral ones; it also find applications, as Sergot [2013] writes, "in other areas, such as the specification of aspects of computer systems, as a contribution to the formal theory of organisations in the analysis of notions such as responsibility, entitlement, authorisation and delegation, and in the field of multi-agent systems." But the original, and, therefore, primary context of the Hohfeldian conceptions is law. If, without going into details (since it would take another dissertation), we restrict the human rights’ issue to those questions that consider human rights’ legal representation, the principle I am going to use might seem less harmful. One can regard the theory to be presented as a normative one: in this case what I will show about rights is the way how they should be. We will come back to this point later.
The possibility of seeking remedy in court—and binding the very existence of the rights to it—actually is a well-grounded, fundamental expectation in western legal culture. The Declaration of the Rights of Man and of the Citizen, being approved by the National Assembly of France, August 26, 1789, is not just the crucial document of the French Revolution, but also the establishing pioneer of the European chartal constitutionalization. Its articles\(^3\) list the rights we still consider as the most important ones regarding ourselves as humans and citizens. Some of these are general liberty, the right to property (Article 2), equality before the law (Article 6) or free speech (Article 11). But there are two articles in which the reader finds some "meta-requirements", or "meta-rights":

Article 12
To guarantee the Rights of Man and of the Citizen a public force is necessary. (...)

Article 16
Any society in which no provision is made for guaranteeing rights (...), has no Constitution.

Article 12 declares the necessity of state enforcement of rights listed above and below it, while Article 16 clarifies this necessity’s level: not ensuring the existence of state enforcement of rights has no constitution: a society in which the observance of the law is not assured has no constitution at all. Meanwhile the Declaration considers human rights and political rights, which are usually regarded as complex rights of the sui generis ones described by Hohfeld, we have no reason to debate that the notion of a right as such involves this possibility of enforcement as a requirement of being a legal right.

Let’s consider this issue a bit more generally: among norms, the possibility of seeking remedy in court is the differentia specifica of legal ones. At first glance, it might seem intuitive to regard sanction as the hallmark of a legal norm, but would not be true: for Catholic people, going to Hell constitutes a sanction in the case of religious norms; also, being ostracized can be highly unattractive, which is a reason behind following social norms. But none of these norms has the state behind them. And while rights and, especially, duties as notions can be associated with morality and ethics also, Hohfeld himself was a jurist and called these notions fundamental legal conceptions as applied in judicial reasoning. Therefore it seems natural, reasonable, and justified to

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\(^3\)the English translation presented here is from the Digital Public Library of America: https://dp.la/primary-source-sets/sources/889
capture their essence by building on their specificity making them legal rights and duties, just like Makinson did. In my proposal, though, as written in the previous section, not Power, but Claim-right plays the crucial role in describing the possibility of seeking remedy in court.

6.1.8 Power’s Formal Representation: Duty-generating Potential with Constitutive Rules behind it

Beside handling counterparties, Sergot [2013] declares formalizing Power as the other limitation of the theory of normative positions. As it has also been introduced, Kanger in his theory uses right-names that seem to be in line with the Hohfeldian ones, but the Kangerian power and immunity have nothing to do with the Hohfeldian Power and Immunity.

As discussed in the first chapters, the higher-order property of the second group of rights and duties (Power, Liability, Immunity and Disability) comes from the fact that, as Sergot [2013] says "this group is concerned with changes of legal/normative relations". We also discussed findings of Fitch [1967] and Makinson [1986], that tell more, though, on how these second-order rights work: while Fitch—with his formal representation, tacitly—emphasized that these rights embody a strong relation to capacity, we also already know that this capacity has to be differentiated from the physical capacity, as it was stressed by Makinson (remember the example of the priest who has power to wed a couple, but might lack of the physical capacity to do so—or the other way around). Makinson also called attention to the fact that Power has to be distinctly separated from permission, with which, albeit it is in a parallel, there are important structural differences. For example, the consequences of their respective lacks: doing something without permission involves a threat of a punishment, while doing something for which we do not have the power means that we are actually not doing that thing.

The formal representation of Power and the related rights, thus, has to give an account of the special capacity involved in Power and the incapacity to actually do the given act when it lacks; with explaining or showing somehow the way it differs from physical or practical capacity. That is why Sergot mentions his paper with Jones in which they argue that "power’ in this Hohfeldian sense is to be understood as a special case of a more general phenomenon, whereby in the context of a given normative system or institution, designated kinds of acts, performed by designated agents in specific

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4 Again: this choice might seem at first glance a strict legal positivist’s approach, but actually this restriction is exactly the step through which we avoid such a commitment: we do not describe rights in general in this way, we only describe legal rights in this way.
circumstances, *count as* acts that create specific kinds of institutional relations and states of affairs. This switches attention from the formalisation of permission to the formalisation of the *counts as* relation more generally. They were the first ones but not the only ones who found this connection plausible. In Grossi and Jones [2013] (which is a chapter in the *Handbook of Deontic Logic and Normative Systems*) an obvious reason to discuss the possible formal representation of ‘counts-as’ relation is to clarify the nature of Power, which is named in the chapter’s abstract as a benchmark problem. We already investigated the existing formal attempts shortly, now let’s delineate some considerations on the philosophical issues of constitutive norms and counts-as relations.

Depending on one’s background or the area within which the issue arises, typically two authors come to people’s minds referring to the notion of constitutive rules: Searle, if we work within the philosophy of language or the theory of institutions, and Rawls, if we are dealing with the philosophy of law. But they are not the only ones who substantially dealt with the conception to which we refer as constitutive rules or counts-as relations: regarding the theory of norms, one can easily argue for the relevance of Wright [1963] and Alchourrón and Bulygin [1971].

Let’s stick at this point to the first two, though, and let’s have a look at their theory to understand why it is so obvious that there is some relation between constitutive rules, counts-as relations and the Hohfeldian Power. Sergot [2013] puts the emphasis on the relevance of counts-as when saying: "in the context of a given normative system or institution, designated kind of acts, when performed by designated agents in specific circumstances, *count as* acts that create or modify specific kinds of institutional relations and states of affairs. This switches attention from the formalisation of permission to the formalisation of the count as relation more generally." I prefer to stress Power’s strong connection to the notion of constitutive rules, instead. Why do I feel so?

I think one of the—if not the—most important property of the Hohfeldian Power is what Makinson [1986] says discussing the difference between permission and Power (and therefore the deontic and the capacitative modalities) when acting without the given position: doing something that is not permitted comes (well, at least may or should come) with the consequence of punishment; while doing something without Power comes with the consequence that the given thing is actually not done: "if a person tries, say, to

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5 Since constitutive rules and counts-as relation are highly connected, the diverse emphasis might seem unessential, but I think it helps the reader understand what essential point to be found behind power.
celebrate a marriage or issue a passport without having the power to do so, then we say that he has not in fact celebrated a marriage or issued a passport (for emphasis: has not issued a valid passport) but has only gone through the motions or given the appearance of doing so." Formally:

$$\neg P'(x \text{ do } F) \rightarrow \neg (x \text{ do } F)$$

the contraposition of which is

$$x \text{ do } F \rightarrow P'(x \text{ do } F)$$

This property seems quite familiar with good reason: it invokes what Searle [1996] suggests when he differentiates between two types of rules. According to Searle, *regulative rules* regulate pre-existing activity, whose existence is independent of the rules; these rules can be stated typically as imperatives. While *constitutive rules* are the ones constituting an activity, the existence of which is logically dependent on the rules. Searle says that constitutive rules can be expressed as imperatives in certain cases, but they can also be expressed as non-imperative, "counts as" rules. As we can see, what Searle says about constitutive rules is something very similar to what Makinson says about the Hohfeldian Power. But if this similarity in features is not convincing enough, let’s consider what John Rawls said about practice rules—a few years earlier: in Rawls [1955] we find a similar distinction to the one made by Searle. On *summary rules* Rawls says they indicate that we should not do certain things because they lead to bad results, and to do certain things because they produce good results. Meanwhile, we can talk about *practice rules* that define an action: if the rule is not followed, then we are not engaged in the defined activity. With this last clause, it is quite obvious that there must be a strong connection between constitutive/practice rules and Hohfeldian Power (the expression ‘constitutive’ I find much more expressive than ‘practice rules’, therefore, in spite of the chronological priority, I am going to use Searle’s terminology).

In my approach I am going to build on this strong connection involving the nature of constitutive rules in the formal definition of Power. The reason to prefer emphasizing Power’s relation to constitutive rules rather than that to counts-as relations, although they go together in the Searlian definition, is that if we talk about ‘counts-as’, I feel more weight on the context provided by the rules to the action defined by them. As Sergot himself phrased it too: "in the context of a given normative system or institution, designated kind of acts, when performed by designated agents in specific circumstances". I agree that when we look for the meaning of Power, we have to build our structure on the nature of the action for which we have or do not have the

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6 About the specific—Hungarian—legislative language use depending on the rule type and the modality, see Markovich [2015b].
power to do so, but I think the nature or structure of (the given) constitutive rules explain more how and why this strong connection between them is there.

What can we use from technical representations of this notion we have considered? We have checked Grossi and Jones [2013] introducing the formal contributions to the theory of counts-as. As I argued above, I conceive that structure of constitutive rules is what is interesting, and not the fact that ‘counts-as’ rules always draw a context: we are within a legal context, we already talk about a given legal system’s rules. Therefore, I do not tend to involve any of those formalizations discussed in Grossi and Jones [2013], since all of them are created on the Searlian scheme:

\[ X \text{ counts as } Y \text{ in the context } C. \]

It does not mean that my formal representation of Power, which, as I promised, will be built on the notion—and especially on the structure—of constitutive rules won’t somehow rely on this scheme. To understand how—and why—, we need to have a look at some further informal contributions of the theory of counts-as, or, more precisely on constative rules.

Grossi and Jones [2013] investigate the ‘raison d’être’: "the constitutive rules seem to be a pervasive feature of legal systems, but why is it so? Or, said otherwise, what are constitutive rules actually good for?" And they quote the famous example of Ross [1957]:

"On the Nosulli islands in the South Pacific lives the Noît-cif tribe, generally regarded as one of the more primitive peoples to be found in the world today (...). This tribe (...) holds the belief that in case of infringement of certain taboos—for example, if a ma encounters his mother-in-law, or if a totem animal is killed, or if someone has eaten of the food prepared for the chief—there arises what is called tû-tû. The members of the tribe also say that the person who committed the infringement has become tû-tû. It is very difficult to explain what is meant by this. (...) tû-tû is conceived as a kind of dangerous force (...) a person who has become tû-tû must be subjected to a special ceremony of purification."

According to Ross, ‘tû-tû’ is a word without reference, playing an important role, though: the role which can be called ‘interpolant’ from a logical point of view, a bridge to connect facts creating, thus, an inference like the following:

(i) If a person has eaten of the chief’s food, she is a tû-tû.
(ii) If a person is a tû-tû, she has to be subjected to a ceremony of purifica-
(iii) If a person has eaten of the chief’s food, she has to be subjected to a ceremony of purification.

With the Searlian terminology, tû-tû is an institutional fact and (i) connects it to a specific brute fact. According to Ross, conceptions like, for instance, ownership are also like tû-tû, allowing us to connect a set of concrete circumstances to a set of legal consequences. This is a very important thesis upon which I am going to create the formalization of power. What is happening with a kind of tû-tû is actually an abbreviation. Without it, connecting brute facts $F_1, ..., F_n$ with normative consequences $C_1, ..., C_n$ could happen as displayed below:

$$
\begin{align*}
F_1 &- C_1 \\
F_2 &- C_2 \\
\vdots & \vdots \\
F_n &- C_n
\end{align*}
$$

But with tû-tû, let’s call it $Y$, connecting brute facts with normative facts can happen as in the following scheme:

$$
\begin{align*}
F_1 & \quad C_1 \\
F_2 & \quad C_2 \\
\vdots & \quad \vdots \\
F_n & \quad C_m
\end{align*}
$$

What happens here is called ‘a manageable and effective "technique of presentation" for a system of norms’ by Ross. I strongly agree, thus, I will pursue to point at this function—which is actually build in constitutive rules—when formalizing Power.

The other crucial feature of Power and related rights (the ‘second group’ of rights and duties) that we already have discussed a lot is their scope: they "affect" the first group of rights (and, actually, on other second group rights, too). As I mentioned in the chapter discussing Hohfeld’s paper, the most important, or at least most "guideline-providing" sentence of him about the nature of the second group is when he says that "it is a liability to have a duty created", which—because of Power’s and Liability’s correlativity—also means that it is a power to create a duty (on someone to have). As I explained when discussing this sentence first, what Hohfeld means but literally does not unfold here is that the power (and the liability) is already there.
before being used to create that duty. The point is exactly that legal capacity, which enables the power-holder to create that duty—and the exposure of the liable person, is there even before having that given duty. Thus, the duty that can be created is a good point to grasp the essence of Power, but only if we show its "threatening" nature or (by) conditionality. In my approach, I call this capacitative feature of power ‘potential’ and I capture it as a feature borne by power and actions together.

6.2 Formal Tools, Formal Language

As I already laid down above, the axiomatic background from which I start to develop the formalization of Hohfeldian conceptions is basically the same as in Sergot [2013] when rephrasing the Kanger-Lindahl theory, that is:

- **Propositional Calculus**
  I use the language of propositional logic, with the usual sentential operators, and propositional letters. That is, we start from the following axiom schemes:
  
  (1) \((A \rightarrow (B \rightarrow A))\)
  
  (2) \(((A \rightarrow (B \rightarrow C)) \rightarrow ((A \rightarrow B) \rightarrow (A \rightarrow B)))\)
  
  (3) \(((\neg A \rightarrow \neg B) \rightarrow (B \rightarrow A))\)
  
  and the derivation rule modus ponens:
  
  \[\frac{A, (A \rightarrow B)}{B}\]

  In the language we have variables \(x, y\) to represent agents, and \(j\) to represent a special agent with the intended meaning ‘a judge’. Within the text, I am going to refer to the judge with the definite article ‘the’ expressing that in the given case there is one judge, or even if there is a panel of judges, we consider them as one agent, but still: (at first) ‘\(j\)’ is a variable, not a constant—only a special one. (After a while, I will modify its character and I will refer to it as a constant denoting ‘judiciary’.) These agent-variables will only be used as indices.

  There will be modal operators referring to actions and to the rights and duties we would like to talk about. All of them will be indexed with agent-variables.

- **Action Logic: ET**
  To represent actions, we will use the modal—stit—operator \(E\) in an
indexed way to denote the agent who sees to it that, that is, \(E_x\) stands for ‘x sees to it that’. The logic behind the operator is the one Chellas [1980] called ET having two axioms: (E.RE) which is the usual derivation rule about the interchangeability of equivalent expressions within the argument of the operator, and the axiom scheme (E.T) in order to speak about successful actions with laying down that whenever \(x\) sees to it that \(F\) is the case, \(F\) will be the case. Respectively:

\[
\begin{align*}
(E.RE) & \quad A \leftrightarrow B \\
\quad & \quad \frac{E_xA \leftrightarrow E_xB}{E_xA \leftrightarrow E_xB}
\end{align*}
\]

\[
(E.T) \quad E_xA \rightarrow A
\]

Being a unary sentential operator, \(E_x\) can have a proposition in its argument, for which we usually will use the propositional letter \(F\) to refer to a given state of affairs. But it can and will happen that other, more complex sentences get into the argument of \(E_x\), for instance because it is iterable, that is \(E_jE_xF\) is a well-formed expression with the intended meaning ‘the judge sees to it that \(x\) sees to it that \(F\) is the case’.

\section*{Deontic Logic: SDL/EMCP}

Regarding the modal operators I am going to deal with, my intentions require an axiomatic system which practically works like Standard Deontic Logic (SDL). It can be SDL, which is (above the propositional axioms) constituted of the following: Axioms K and seriality

\[
\begin{align*}
(K) & \quad O(A \rightarrow B) \rightarrow (OA \rightarrow OB) \\
(Ser)/(D) & \quad OP \rightarrow PA
\end{align*}
\]

with derivation rules of modal generalization:

\[
(MG) \quad \frac{A}{OA}
\]

But as Sergot [2013] wrote about what to use for rephrasing the theory of normative positions the system Chellas [1980] called EMCP is enough for these intentions. EMCP is slightly weaker than SDL (or as Sergot refers to it following Chellas [1980] KD) missing only the modal generalization derivation rule (which is referred in this system as rule of necessitation, (O.RN)), or equivalently the axiom scheme \(OT\). This is because the existence of absence of this rule does not influence our formalization. What we do have in EMCP is the following set of axioms and the derivation rule on the interchangeability of equivalent expressions:

\[
(O.M) \quad O(A \land B) \rightarrow (OA \rightarrow OB)
\]
\[(O.C) \quad (OA \land OB) \rightarrow O(A \rightarrow B)\]
\[(O.P) \quad \neg O \bot\]
\[(O.RE) \quad \frac{A \leftrightarrow B}{OA \leftrightarrow OB}\]

The typical axiom scheme of SDL, (D) is not in the list above because it follows from (O.C) and (O.P).

• **Notations**

According to what has been explicated above, we introduce notations into our language where rights positions are denoted (mostly) with the rights’ initial letters: CR is for Claim-right, O is for Duty, PR is for Privilege, NC is for No-claim, P is for Power, L is for Liability, I is for Immunity, and D is for Disability. In order to express their being assigned to agents, we use agents (variables or constants) as indices denoting the right-owner/duty-bearer. We lean on the notion of directedness that Herrestad and Krogh [1995] introduced, but use the notation of it Sergot [2013] uses (denoting ‘x has a directed obligation (that is, a duty) toward y’ with ‘O_{x \rightarrow y}’). As we will see, it will be needed not merely in the case of Duty, but also for the other active right positions, that is, in cases where the agent whose right position we consider and the one whose action we consider are the same.

• **Syntactic constraints**

This language enables us to express some special properties of rights relations discussed above. In order to ensure that these operators play the roles we (Hohfeld) intend(s) them to play, we can create syntactic constraints. From a semantic point of view, these can be regarded as meaning postulates, with which we at the same time express the difference between active and passive rights:

- Having claim-right is a passive situation, therefore as an argument of the claim-right operator there can be only a stit operator with a different agent-index. This means that CR_x E_y F is syntactically wrong, since we can have a claim-right only to someone else’s act.
- Having privilege is an active situation: it pertains to our own act. Therefore, as an argument of the privilege operator, there can be only a stit operator with the same agent-index as the freedom operator itself. This means that PR_x E_y F is syntactically wrong (if \( x \neq y \)).
• Definitions of Formal Semantics

Definition 1. The basis of our semantics is a finite set $A$ of agents.

For a set $W$ of possible worlds and the set $A$ of agents write

$$\mathfrak{F} = \langle W, f_a, R_{a,b}\rangle_{a,b \in A}$$

where $f_a : \wp(W) \rightarrow \wp(W)$ is a function and $R_{a,b} \subseteq W^2$ is a binary relation.

Models are structures

$$\mathfrak{M} = \langle W, f_a, R_{a,b}, v\rangle_{a,b \in A}$$

where $v$ is a valuation function for atomic propositions: $v : \Phi \rightarrow \wp(W)$

Definition 2. Our modal language is given by

$$p \in \Phi \mid \psi \land \phi \mid \neg \phi \mid \bot \mid E_a \phi \mid O_{a \rightarrow b} \phi$$

for $a, b \in A$, where $\Phi$ is the set of propositional letters.

Definition 3. For $\mathfrak{F} = \langle W, f_a, R_{a,b}\rangle_{a,b \in A}$ and evaluation $\| \cdot \| : \Phi \rightarrow \wp(W)$ we let

- $w \models p \iff w \in \|p\|$ for propositional letters $p \in \Phi$.
- $w \models \phi \land \psi \iff w \models \phi$ AND $w \models \psi$.
- $w \models \neg \phi \iff w \not\models \phi$.
- $w \models O_{a \rightarrow b} \phi \iff \forall w'(wR_{a,b}w' \Rightarrow w' \models \phi)$
- $w \models E_a \phi \iff w \in f_a(\|\phi\|)$
- $w \models O_{a \rightarrow b} E_a \phi \iff \forall w'(wRw' \Rightarrow w' \models E_a \phi)$

As it can be seen, the operator (the modality) of Duty is defined with the relation with which obligation is defined in SDL—but because of the indices, we are concerned with as many of them as ordered pairs of agents we have. Also, we assign a function to each agent. These functions show which states of affairs are seen to it by the given agents—which set of states of affairs will be clearly a subset of the truth set of the formula describing the given state of affairs. This feature and the seriality of the relation structure given by the directed obligation are shown with constraints restricting the set of models to those coming with evaluations satisfying the axioms we need (T in the case of $E_a$ and D in the case of $O_{a \rightarrow b}$):

Constraints

- constraint on $f$: $f_a(\|\phi\|) \subseteq \|\phi\|$
- constraint on $R_{a,b}^O$: $\forall w \exists w' wR_{a,b}w'$
Let’s begin with the first pair of rights and duties: the right that Hohfeld called in the same way, using the word in a narrow sense, and the duty which also bears the name of the bigger category—just in a narrower sense. These solutions of Hohfeld’s terminology may suggest that these two are prototypes, or core notions of what we call right and duty. And indeed, as it will be shown soon, I have to agree that they are.

What have we found to be the crucial features of this correlative pair?

- We already know that correlativity is intended to be equivalence: this has been expressed by Hohfeld, and this is what the reception always regarded as self-evident, I will make the same assumption.
- I have already laid down that I build on the state enforcement, and, therefore, capture what Makinson captured with the description of who the counterparty is, which is practically a definition of a directed obligation (that is, a duty—and, therefore, also a definition of a claim-right) when saying that

\[ x \text{ bears an obligation to } y \text{ that } F \text{ under the system } N \text{ of norms iff } \]

in the case that \( F \) is not true then \( y \) has the power under the code \( N \) to initiate legal action against \( x \) for non-fulfillment of \( F \)

but I do not use Power to grasp the agent being the \textit{claimant} in the given rights relation, but make Claim-right playing the crucial role.
- I also declared that I would like to reflect on how that expectation of success comes into the picture that Sergot mentioned to add to the Makinsonian definition above.

The reason that this Makinsonian description of who the counterparty is can be considered as a definition of a directed obligation (Duty) is because it is a biconditional having the Claim-right-Duty rights relation on
the left (definiendum), and a conditional situation description on the right (definiens), which does not contain the Claim-right-Duty relation, and, therefore fulfills the requirements of a proper definition. I, however, do not use the notion of Power, but do use Claim-right again, with different agents, though:

\[ x \text{ has a claim-right that } y \text{ see to it that } F \text{ is the case if and only if } \]

in the case that \( y \) does not see to it that \( F \), \( x \) will have a claim-right that the judge see to it that \( y \) sees to it that \( F \) is the case

Formally:

\[ O_{x \rightarrow y}E_x F \leftrightarrow \neg E_x F \rightarrow CR_y E_j E_x F \]

That is, \( x \) has a claim-right that \( y \) see to it that a given state of affairs is the case if and only if in case of not fulfilling this duty of \( y \) (which follows from the claim-right of \( x \) since those two things are equivalent) \( x \) has a claim-right toward the judge (the judiciary) to make \( y \) see to it that the given state of affairs is the case—since this is what state enforcement means. Before defending this position having only Claim-right in the description and omitting Power, let’s revoke Makinson’s considerations on his own informal proposal.

Makinson called attention to the fact that the biconditional’s right-hand side is problematic if we would want to formalize the case of unfulfillment as a material conditional. In our case—just as in Makinson’s case—it would mean that whenever \( y \) would see to it that \( F \), making the conditional’s antecedent false, by which the conditional itself becomes true, it would follow—because of the right to left conditional part of the biconditional—that \( x \) had the claim-right that \( y \) saw to it that \( F \). So whenever I sneeze, I had the duty toward the reader to sneeze. This is obviously something we do not want to accept. We want to make it sure that whenever a duty stays unfulfilled, a new claim-right arises: this time against the judiciary, to enforce the original duty’s fulfillment. We want to make sure this holds since this is something that surely is this way in law; it cannot be otherwise in the case of legal rights. The obvious logical tool to express this legal certainty is a necessity operator, which, therefore, serves as a legal necessity operator:

\[ (CR) \quad CR_y E_x F \leftrightarrow \Box(\neg E_x F \rightarrow CR_y E_j E_x F) \]

Do we need to indicate that this necessity is a legal necessity and not a
metaphysical one? No. We are within the law, all of our propositions are legally relevant, legally interpreted propositions, and within the law\(^7\) it is (simply) necessary that unfulfillment involves Claim-right to enforcement.

We extend, therefore, our modal language with this operator:

For a set \(W\) of possible worlds and the set \(A\) of agents write

\[
\mathfrak{F} = \langle W, f_a, R_{a,b}^O, R_{\Box} \rangle_{a,b \in A}
\]

where \(f_a : \varphi(W) \to \varphi(W)\) is a function and \(R_{a,b}^O, R_{\Box} \subseteq W^2\) are binary relations.

Models now are structures

\[
\mathfrak{M} = \langle W, f_a, R_{a,b}^O, R_{\Box}, v \rangle_{a,b \in A}
\]

where \(v\) is a valuation function for atomic propositions: \(v : \Phi \to \varphi(W)\)

Our modal language is given now by

\[
p \in \Phi \mid \varphi \land \psi \mid \neg \varphi \mid \bot \mid E_{a\varphi} \mid O_{a\rightarrow b\varphi} \mid \Box \varphi
\]

for \(a, b \in A\), where \(\Phi\) is the set of propositional letters.

The truth conditions for \(\Box\):

\[
w \models \Box \varphi \iff \forall w' (w R_{\Box} w' \Rightarrow w' \models \varphi)
\]

The legal necessity operator—enabling us to speak about legal metaphysics—comes with a logic called S5, containing the K axiom scheme (which makes it a normal modal logic) and the ones that make its alternative relations an equivalence relation:

\[
(K) \quad \Box (A \rightarrow B) \rightarrow (\Box A \rightarrow \Box B)
\]

(reflexivity) \(\Box A \rightarrow A\)

(transitivity) \(\Box A \rightarrow \Box \Box A\)

(symmetry) \(A \rightarrow \Box \Diamond A\)

and the derivation rule of modal generalization.

\(^7\)What is more, we are in a given legal system (as von Wright calls attention to the relativity of the rules at the end of Wright [1963]), within which the necessity operator is to interpreted.
With formula (CR) I tacitly describe something from how a system of rights works: it describes formally what effect a claim-right—together with a refraining—has to the legal system: a new claim-right arises.

The relevance of choosing a general and simple action operator that we can iterate becomes visible above: with iterating we can express the enforcement. We are not interested in how this enforcement happens, we are not interested in the details of what the judge actually does: the point is only that \( x \) has a claim-right toward her to see to it that the original duty of \( y \) is fulfilled. We will come back later to the question whether it always makes sense (or if it makes sense anytime), and if not, how it influences our formal description.

As it has been clarified quite early on: \( x \)'s claim-right that \( y \) see to it that something is the case is equivalent with \( y \)'s duty to do so, which duty is a directed one:

\[
(C\text{ReqvD}) \quad \text{CR} x E y F \leftrightarrow O_{y \rightarrow x} E y F
\]

Notice that the directedness is obvious on the claim-right side: \( x \) is the right-owner and \( y \) is the agent of the action, but has to be indexed by the notion of the directed duty, formally introduced by Herrestad and Krogh [1995], using the notation Sergot [2013] uses, since there the duty-bearer and the acting agent is the same: it has to be indicated that there is another party, the counterparty toward whom the duty is standing.

This equivalence, of course, means that the biconditional of the Hohfeldian Claim-right can be given to the Hohfeldian Duty:

\[
(D) \quad O_{x \rightarrow y} E x F \leftrightarrow \Box(\neg E x F \rightarrow \text{CR} y E x F)
\]

And, of course, we can also put duty at the place of the arising claim-right on the right-hand side:

\[
(Dd) \quad O_{x \rightarrow y} E x F \leftrightarrow \Box(\neg E x F \rightarrow O_{y \rightarrow x} E x F)
\]

While in the case of Claim-right we cannot speak about a definition since the definiendum arises in the definiens, in the case of Duty, formula (D) seems working like a definition. Not the formula (Dd), though, since in that one Duty is there on the right-hand side, too; and since CR as a modality can be defined by O, actually, (D) is not a proper definition either. However, all of

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8 Classical STIT logics usually engage with an assumption called independence of agents, which implies the theorem \( \neg E x E y \phi \quad (x \neq y) \), that is, we could not express enforcement.

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them work in a definition-like way in the sense that they give the necessary and sufficient conditions of talking about a Hohfeldian Duty or a Claim-right.

Does this mean that we can only talk about a duty if it is not fulfilled? Of course not. This definition-like biconditional does not tell everything about Duty. As saying that a pear is a fruit having a shape of a lightbulb does not tell anything about the pear’s sweetness. But it is enough to recognize when we see a pear and to describe it in a (good enough) way to be identified.

6.3.1 Why without Power?

I promised above to discuss why it is better to leave Power aside from the definition of a claim-right/a duty.

The problem is half shown in Sergot’s critique on Makinson’s informal definition: the fact that someone has a legal power to initiate a legal action cannot mean that he has (had originally) a right, because having a power to initiate a legal action is general, not bound to an original claim-right. The thing for which we all\(^9\) have a kind of right—which is a power, indeed—when we initiate a legal action, is to ask the judiciary to decide a case (justly\(^10\)).

What state enforcement means, however, is that in the case of non-fulfillment a new claim-right arises where the right-owner is the same as the original was, but the agent, against whom he has it, is different: not the original duty-bearer but the judiciary/a given judge. The action about which the new claim-right is also differs "a bit": it is not directly seeing to it that the given state of affairs is the case, but seeing to it that the original duty-bearer see to it that that is the case.

One could question whether the (Dd) or the following formula, which is also true because of (CReqvD), sound as plausible as (CR) or (D):

\[(\text{CR}_d) \quad \text{CR}_x E_y F \leftrightarrow \square (\neg E_y S \rightarrow O_{j \rightarrow x} E_j E_y F)\]

The reason these biconditionals might sound strange at first is because we usually think to the state enforcement as a possibility that we can ask for. But in this formula we see that if the original duty is not fulfilled, the judiciary’s duty arises without any further step or happening. What could or should be an intermediate step? Obvious candidate to say that the right-

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\(^9\)not exactly ‘all’ of us, explanation of this comes soon

\(^{10}\)At least in some languages, like Hungarian, it is common to refer to judiciary with terms describing it as the one which serves justice (‘igazságszolgáltatás’); also the English word ‘justice’ for judiciary. This issue of the connection of law and justice takes us far from the topic of this dissertation, therefore, we do not discuss it further.
owner initiates that legal action in which he has the claim-right towards the judge, that is, put the power back to the conditional:

\[(\text{CRwithP}) \quad \text{CR}_x E_y F \leftrightarrow \Box (\neg E_y F \rightarrow P_x \text{CR}_x E_j E_y F)\]

or\(^{11}\)

\[(\text{CRwithP'} \quad \text{CR}_x E_y F \leftrightarrow \Box (\neg E_y F \rightarrow P_x E_x \text{CR}_x E_j E_y F)\]

This version may seem more natural at first sight by saying that in the case of non-fulfillment one has the power to create a situation in which the judge has the duty to enforce the original duty. Especially, that this version of biconditional does not suffer from the problem Sergot points at in the case of Makinson’s version: here the formula really identifies the counter-party, since others, while having the power to initiate a legal action, won’t have the power to have a claim-right toward the judge (that is, to put the duty on her) to enforce the original duty. However, there are at least two problems with this version including power that might tone the picture:

- The first problem is that it may seem quite natural to us to regard rights as ones whose enforcement depends on our decision: we have the power to decide whether we put a duty on the judiciary of the enforcement in the case of claim-rights in contract law.\(^{12}\) But if we consider fundamental human rights or rights defended by criminal law we might require a more direct defense. We will come back to these right-contexts to discuss them in detail.

- The other reason to reconsider whether this could be the good solution is tougher—and in my opinion, decisive. If we "define" what having a claim-right means with Power, we lose a crucial differentiation between two kinds of feature an agent can have. This differentiation is crucial in the legal tradition and systems of civil law countries, but not that sharp in common law, therefore in English both are often referred to as ‘legal capacity’, while there are two different notions behind: the one is that an agent can have rights, the other one is that an agent has the legal ability to act: to do legal actions—this latter is sometimes referred to as ‘legal competence’ or ‘capacity to act’. In Hungarian these conceptions are sharply distinct in the terminology. ‘Jogképesség’ refers to the ability of having rights—all the human agents have this ability, even unborn ones (conditionally), also companies, organisations, etc.; and ‘cselekvőképesség’, to the ability to act in a legal sense—these actions are the ones we need to have power to conduct—, and this is something that a lot of people lack: for instance minors under 14, obviously.

\(^{11}\)The choice depends on whether we want to let the iteration of the deontic/capacitative modalities.

\(^{12}\)This regard would strengthen the approach of the will theory.
unborn ones, but there are other reasons to diminish this capacity, too, typically the lack of mental soundness. An expressive phrasing can be to say that these two notions are about our ability to have a right and to exercise it, respectively. The same sharp distinction can be found in the terminology of other civil law systems, for instance: in German they are ‘Rechtsfähigkeit’ and ‘Handlungsfähigkeit’; in Spanisch they are ‘capacidad de obrar’ and ‘capacidad de goce’; in Polish ‘zdolność prawna’ and ‘zdolność do czynności prawnych’, all respectively. If we describe what a claim-right means with a power to ask for the enforcement we lose this fundamental difference. Binding the notion of a claim-right to Power would therefore be shallow from a legal theoretical point of view.

One could raise here that this only means that we have to recant the option of describing Claim-right—and therefore Duty—in terms of state enforcement. I disagree. We only have to disjoint its notion from Power—this is what I am going to argue for. Or one could develop a system including Power to Claim-right’s description, and involving the notion of acting on behalf of someone. It does not seem unfeasible, but at this point I think it would take the light from the essence of what state enforcement means as an inherent feature of legal rights.

- One can view the meaning of the description of what a claim-right is as formalized in (CR) from another viewpoint. We can consider the antecedent of the conditional: \( \neg E_y F \) as we are already in the courtroom, that is, it means that it has been proved there that \( y \) did not fulfilled his duty. If we are already in the courtroom, the power to initiate a legal action has no business there anymore. I already said that we are within the law: all sentences in this logical setup are legal sentences in the sense that we do not deal with facts with which the law has nothing to do (if there are such sentences at all). The view that even those facts that do not seem legal ones at first sight can depend on court decision is general: we often phrase about, for instance, murders or other felonies in a way that the fact whether they have been committed (by a given person) is decided (not learnt, acknowledged or anything else) by the judge/court.\(^\text{13}\) This everyday phrasing reflects on that these facts are not extra-law: murdering someone, or committing any felony is a

\(^{13}\)For example, at the end of *Twin Peaks* series one, there is a short dialogue between Agent Cooper and Audrey Horne about the presumed murderer of Laura Palmer:

Audrey: Did you arrest him?
Cooper: Yeah.
Audrey: Did he do it?
Cooper: That’s for a court to decide.
legal fact, since murder and felony are legal, or legally relevant, legally recognized notions. We will come back to this point later.

The deep structure of state enforcement can be drawn without Power, what is more, I am convinced that is the proper way of it.\textsuperscript{14} It might be difficult to accept how a judge can have any duty without even knowing about it—as it has to be since there is no way to learn about all the non-fulfillments right in the moment of their coming about. So maybe we should emphasize that these biconditionals describing what a claim-right—and a duty—means have nothing to do with epistemology, and they do not consider a special judge, just \textit{a judge}, where the agent is embodied by the role not by the person itself, so the epistemic notions probably would not even make sense. My considerations are at another level, a level that can be called the level of \textit{legal metaphysics}:	extsuperscript{15} this is what the structure of a right looks like, independently of who finds out what and when. The use of the (legal) necessity operator strengthens this view.

This way of formal representation provides some explanation how the claimant’s expectation of success comes when he initiates a legal action against the duty-bearer, the expectation that Sergot [2013] suggested to add somehow to the definition. The claimant has this expectation because he does not only ask the judiciary to decide his case—what he actually does with initiating a legal action (for which he has the power), but this claimant, beside being the counterparty in the original right-relation, is also the counterparty in another claim-right-duty type of rights relation having, in this case, with the judiciary as the duty-bearer. As he probably had an expectation when creating the original rights relation (or simply while having it) that the duty-bearer would fulfill her duty, he is reasonable to have the expectation that even if the original duty-bearer did not fulfill her duty, the new duty-bearer will. What is more, from a socio-psychological viewpoint this expectation might be higher since the new duty-bearer is an official operator of law. And, if he considers the new duty-bearer being only another human after all, the possibility of appeal is still there to strengthen the expectation, which is, at the end of the day, goes toward the \textit{rule of law}.

\textsuperscript{14}There are some terminological considerations, too, at our side worth mentioning (even if we see the restricted confirming power of etymology). One is the word used by Sergot [2013] when describes the two interpretations of what a counterparty is. He uses the word \textit{‘claimant’} and introduced Makinson’s informal definition. ‘Claimant’ means the agent who initiates the legal action, the plaintiff, and in general, its meaning is bound in processes to the agent who \textit{claims}—why would we involve Power then? The other confirmation comes from Hungarian legal terminology: in the Hungarian legal language the state in which one’s fundamental right gets in the case of infringement is (already) called ‘claim’ (\textit{‘alapjogi igény’}) and it describes one seeking remedy in court.

\textsuperscript{15}I would like to thank it to John F. Horty for propounding this description of my way of grasping these notions.
6.3.2 Refinements on the Formal Representation of Claim-right and Duty

Private Law, Law of Contracts

At one point above, I promised to discuss whether it makes sense in every case—or in any—that judges enforce the original action. I emphasized that we are not interested in how this enforcement happens, we are not interested in the details of what the judge actually does: the point is only that $x$ has a claim-right toward her to see to it that the original duty of $y$ is fulfilled. But one may wonder, with good reason, whether it is (always) possible to see to it that the original duty-bearer see to it that $F$—being $F$ the same given state of affairs as the original duty concerned. What if $F$ is like ‘$y$ is at Trafalgar square at 5 pm on 2nd of June in 2017’? Without time-machine this given state of affairs cannot be seen to it after the indicated time. And then no judge can see to it after the date that $y$ see to it that he was there.

When discussing what a given state of affairs is, I made a reflection on the difference between a type and a token of a given state of affairs: each time when we talk about a contract, in which parties agree about its matter—what duties each undertakes, they describe a token of a given state of affairs since they specify all the circumstances they care of the specific state of affairs they agree about. That is, the duty and claim-right arising with their contracting will be about a token about a given state of affairs. In these cases it is conceptually (well, physically) excluded that the original duty’s fulfillment can be enforced once it has not been by the date/deadline specified in the contract. What do we then refer to in law using the phrase ‘enforcing a right/duty’? What does then the judiciary enforce? What happens in law is that another token of a given state of affairs can only be enforced, but this token either similar enough to the originally described one, or—if it is not possible—a new one created to compensate the original right-owner for his loss caused by the non-fulfillment.

Should this problem somehow be shown in the formula and if yes, how? Surely, since it can cause a problem formally too: if $\neg E_x F$ is true then $E_j E_x F$ cannot be true, and we do not want a system having duties on impossible actions (see the axiom $(O.P)$). I introduce to the language a sentential operator, $C$, standing for compensation: a compensation is done for a given state of affairs in a sense that ‘$C F$’ is to be interpreted as ‘another state of affairs is the case which is similar enough to or compensates $F$’. That is, we understand ‘compensation’ in a broader sense: not just the "classic" compensation (which we typically understand as monetary), but also the similar enough state of affairs is included in the intended meaning. The original state of affairs goes to the operator’s argument indicating that this is the state the
compensation has to be similar enough to, or whose backlog the compensation has to stand for. Formally it looks like the following:

\[(\text{CR}_{\text{priv}}) \quad \text{CR}_x E_y F \leftrightarrow \Box(\neg E_y F \rightarrow \text{CR}_x E_y C F)\]

How should we interpret the working of the new modal operator? What we need here is another state of affairs to be seen to it, which is similar enough to \(F\). This similarity as a relation cannot be given formally, since it highly depends on the state of affairs, the given case, the judge and several other things what other states of affairs can be considered as a compensation. What we can do is to require some properties of this relation which seem to satisfy our intuitive notion of such a kind of "similarity" relation. (I use quotation marks around similarity because similarity relation in its mathematical sense is an equivalence relation, which is not good for us because of its reflexivity: we would end up at the same problem we want to solve by using something else in state enforcement instead of '\(F\)'.) Seriality is obviously needed since we do not want a state of affairs of which there is no alternative in sense of compensation: no judgement says that there was a claim-right of \(x\) to "have" \(F\) done by \(y\), but since there is no other state of affairs like \(F\) would have been, the court does not oblige \(y\) to realize any compensation. The other property that seems intuitive to require in the case of such a similarity is that it is transitive: if \(F'\) is similar enough to \(F\), and \(F''\) is similar enough to \(F'\) then \(F''\) should be similar enough to \(F\). What we do not want, though, is reflexivity. The system that is serial, transitive but not reflexive is OS4, so we can consider the compensation operator coming with OS4 axioms.\(^{16}\)

\(^{16}\)It is not obvious, though, what we can provide from this nature at the level of semantics. With the formal tools we have, we can make the following extension of the language: For a set \(W\) of possible worlds and the set \(A\) of agents write

\[
\tilde{\mathfrak{S}} = \langle W, f_a, R_{O_{a,b}}, R^C, R^{\text{comp}} \rangle_{a,b \in A}
\]

where \(f_a : \varphi(W) \rightarrow \varphi(W)\) is a function and \(R_{O_{a,b}}, R^C, R^{\text{comp}} \subseteq W^2\) are binary relations. Models now are structures

\[
\mathfrak{M} = \langle W, f_a, R_{O_{a,b}}, R^C, R^{\text{comp}}, v \rangle_{a,b \in A}
\]

where \(v\) is a valuation function for atomic proposition: \(v : \Phi \rightarrow \varphi(W)\)

Our modal language is given now by

\[
p \in \Phi \mid \varphi \land \psi \mid \neg \varphi \mid \bot \mid E_{a \rightarrow b} \varphi \mid O_{a \rightarrow b} \varphi \mid \Box \varphi \mid C \varphi
\]

for \(a, b \in A\), where \(\Phi\) is the set of propositional letters.

Truth conditions to \(C \varphi\):

\[
w \models C \varphi \iff \exists w'(w R^{\text{comp}} w \Rightarrow w' \models \varphi)
\]
With this refinement we get the paradigmatic claim-right case of private law from the area of contracts. Not this is the only area of law, though, where we consider (claim-)rights. Let’s see another one.

**Criminal Law**

One may wonder whether the rights like right to physical integrity can be formalized in this way, since the compensation there does not really sound adequate. Indeed, it doesn’t. The notion we have to deal with in criminal law instead of compensation is *sanction* or *sentence* (or *penalty*—still we are going to use an ‘S’ to denote). However, this is not the only challenge in formalizing the rights protected by criminal law.

The biggest challenge\(^\text{17}\) is provided by the right we simply usually mention as the right to life. In this case it seems rather problematic to say in terms of state enforcement that the counterparty of a duty not to kill is the person whom the murderer kills, since it would mean that the murdered person—being already dead—is the one who has a claim-right toward the judge. Even if law admits that some inherent rights make sense after the death (like the right to reputation), after the death of the given person, the owner of these rights are the relatives, and the subject of this right is the testamentary heir’s *right in memoriam*. In case of right to life it would sound strange that after the death of the person there are other people who have *that* right of the dead person to life. We need to notice that the *right in memoriam* is a different notion than the original *right to reputation* was: it is a new right which can be violated in the same way as the original could be, but it is the right of the relatives connected to the memory of the deceased person, protected from being violated. Therefore, in principle, it could be the case that there is a notion of a right of the relatives after the given person has been murdered, but what would it be a right to? If it is a right to their relative’s life then it was there before his death too, so it is not a new one. There is no obstacle to consider such a right but it still does not solve the problem of right to life. Unless we consider the right to life as only the right of the relatives to their relative’s life. It would mean that someone’s right to life is not his or her right at all. Sounds plausible, but rather bizarre from the viewpoint of ethics. Fortunately, there is another way out of the puzzle of right to life. For this solution we need to look at the picture as

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It is not straightforward, though, whether this extension reflects on our intuitions properly. Since we want to talk about the relation between F-s, it might be the case that we would need a two-sorted language to serve properly our intuitions on what is needed to be told here. I come back to this point at the very end of the dissertation.

\(^\text{17}\) At least this is an often arising counterexample to the Hohfeldian theory’s (primarily, the correlativity’s) generality
a whole: why and how the matters of facts in the criminal code differ from other cases where some rights are hurt? If the answer is not straightforward, it is worth to consider the role of the public prosecutor. They represent the State, actually they represent the whole society. This suggests—and this is what I suggest too—that the right which is violated in case of felonies is a claim-right of everyone: a claim-right to live in a society where there is no felony. And since the correlativity must be present here too, who is the other agent having the duty? Well, the answer here is also ‘everyone’. With these agents we can express formally the rights handled in criminal law as versions of the Hohfeldian claim-right:

\[
\bigwedge_{x \in A} \text{CR}_x \rightarrow \bigvee_{y \in A} E_y F \leftrightarrow \Box (E_y F \rightarrow \bigwedge_{x \in A} \text{CR}_x E_j (\neg E_y F \land S_y F))
\]

where \( A \) is the—arbitrarily large but finite—set of agents. What the formula intends to "say" is, in words: all of us have the claim-right that none of us see to it that a felony is committed. If this is the case regarding a given action then whenever any of us commits that act, all of us have the claim-right towards the judge to see to it that the perpetrator does not see to it that the given felony is committed and inflict a punishment, or more precisely, see to it that the perpetrator is punished. And the other way around: if seeing to it that a given action is committed involves such consequences, then that action is something whose not-commitment all of us have a right to. The intended meaning of \( S_x F \) is that \( x \) is sanctioned\(^\text{18}\) because of (seeing to it that) \( F \). It seems from the parentheses that the conjunction goes to the judge's action. And even if original claim-right is about refraining something, the new one is about doing something by the judge—just as it has been said in Holmes and Sunstein [1999]: "in the context of citizens' right to sate enforcement, all rights are positive".

Some questions can be raised. In the case of seeing to it that the perpetrator does not commit the felony one could wonder easily whether it makes sense at all, once the felony has already been committed. This problem has the same source as the one we already discussed connected to types and tokens of states of affairs. It is worth to consider some examples from the criminal code to see how they phrased. Let's stay at homicide: in this case \( F \) would sound—in a significantly simplified version—as "a person is dead" or "someone dies", not a specific person, anyone. That is, the claim-right is on that no one see to it that someone dies. It is a type of a state of affairs. And a token is what can happen: when any of us sees to it that someone (anyone) dies. If any of us saw to it that someone died then all of us have a

\(^{18}\)That is, here lower index indicates the subject of the sanction, not the agent who conducts.
claim-right toward the judge to see to it that the perpetrator do not see to it that someone (else) dies—and also that he is sentenced. Since the criminal code does not order to punish a given person’s murder but any person’s murder, putting \( F \) in the conditional’s consequent would make sense. The problem is that we would like to use the same \( F \) to denote the type and the token, which causes problems. For example, one could raise of course that from a general prohibition on homicide we can also derive that all of us have a claim-right that none of us kill the person \( a \). That is, \( F \) can be "\( a \) dies". In this case of course our claim-right toward the judge would also go for seeing to it that \( y \) do not see to it that \( a \) is dead. Considering a specific person this requirement does sound less plausible, still not senseless and actually feasible—considering that \( a \) is already dead, it cannot be seen to it that he dies again. Since what we require here is not that the judge see to it that \( y \) see to it that \( \neg F \) is the case—this is something different: not to kill someone and to manage someone is not killed are pretty distinguishable actions, that is why we do not have an axiom saying \( E_x \neg F \iff \neg E_x F \), it is simply not true in the majority of the cases. Seeing to it that \( y \) does not kill that person again, or anyone else does not seem an implausible requirement for which the society claim from the judge. But the problems that can be solved with an intended reading of \( F \) can be solved in another way, too. One of the aims of imprisonment is making it physically impossible (at least, a bit more difficult) to kill another person, as well as to "teach" the murderer—and the other members of the society—that it is not worth doing it, so he hopefully won’t kill again when he comes out of prison. That is, the fact that the murderer is punished is supposed to serve the aim of that he do not see that \( F \) is the case (again), too. This can mean for us that we do not need to include the formally questionable part in the formula:

\[
(CR_{crim}) \bigwedge_{x \in A} CR_x \neg \bigvee_{y \in A} E_y F \leftrightarrow \Box (E_y F \rightarrow \bigwedge_{x \in A} CR_x E_{S_y} F)
\]

Rights protected by criminal law (at least in an overwhelming majority—crimes by omission are obvious exceptions) are claim-rights requiring refraining from given acts—acts the criminal code declares as felonies.\(^{19}\) The fact that here we have to put a refraining of an act into the argument of the modal operator of the claim-right (and duty) is not a speciality of course, it absolutely makes sense someone having a duty to refrain from seeing to it that a given state of affairs is the case in private law, too: the other typical

\(^{19}\)Guido Governatori suggested to me to consider whether this solution is a real solution of defining Claim-right and Duty in societies like in the movie Minority Report, where there is a system called pre-crime through which they prevent all murders that are going to be committed—therefore there is no violation of the right to life. Modelling a system like that could probably be a solution if we understood \( F \) as realized by planning or deciding to commit the given felony.
branch of law where it has paradigmatic occurrence comes after discussing the Hohfeldian privilege.\textsuperscript{20}

**Appeal**

There is something which plays a crucial role in the rule of law and in state enforcement by aiming at minimizing the fallibility and contingency coming with the human factor in it. This is Appeal. The system of appeal can be involved into our formal representation of what a claim-right means in the following way:

\[
\text{(CRwA)}
\]

\[
\text{CR}_x E_y F \leftrightarrow \Box (\neg E_y F \rightarrow \text{CR}_x E_j E_y C F) \land \Box (\neg E_j E_y C F \rightarrow \text{CR}_x (E_j E_y C F \lor E_j E_y C F))
\]

where \( J \), as a restricted variable, stands for the appellate court (or judge).

The disjunction in the consequent of the added conditional shows the two possible types of decision of the appellate court: overwriting the original sentence or remanding the case to the original court for new trial can be the tools for ending up with a decision in accordance with the claim-right.

Usually the appeal systems are multi-level systems, but if the nature of the appeal and the possible decisions of different levels of appellate courts do not differ then it does not have to be indicated separately. To provide a formally detailed and proper representation of how a system of appeal works one would need to interpret a relation on the proper subset of agents containing judges. This relation should be irreflexive and should indicate which one is the "maximal" element of the subset in order to have some kind of ordering on the subset which shows the hierarchy within it. This relation could be involved into the notion of appeal as a condition that \( J \) should be "higher" than \( j \) was, whose decision we do not agree with.

Another option is, and from now on I am going to apply this one, to simply handle \( j \) as a constant to ‘judiciary’ as I often referred to it in words, and

\[
\bigwedge_{x \in A} \bigwedge_{y \in A} E_y \neg E_y F \leftrightarrow \Box (E_y F \rightarrow \bigwedge_{x \in A} \text{CR}_x E_j S_y F)
\]

\textsuperscript{20}If we choose to be consequent in putting \textit{action} into the argument of the deontic operator instead of any kind of proposition, formally we need to represent refraining from something as an action. In this case we need to follow Belnap [2001] (and actually Sergot [2013]) in representing refraining with using plus one action operator expressing it as one seeing to it that (s)he do not see to it that the given state of affairs is the case. This change of course will affect the disjunction and turn into a conjunction (since the negation goes inside the formula):
then there is no need to add the possibility of appeal since it is already included.

What has been shown is that even if we can (and maybe feel necessary to) refine the formal representation of what we mean by the notion claim-right according to the different areas—or levels—of law, there is a basic structure that can be formally drawn. This structure is built on how doing or refraining from an action affects one’s right position: what other claim-right this effect result, which other claim-right is actually the core of state enforcement.

6.4 Formal Representation of Privilege and No-claim

While Hohfeld devotes a much longer discussion to Privilege and No-claim (at least to Privilege) than he does to Claim-right and Duty, the discussion of Privilege’s and No-claim’s formal representation is going to be shorter. One of the reasons is: what is difficult to justify in words when conceptualizing, is not necessarily challenging when it comes to formalize the already conceptualized notion. What is more, in this case, it will be relatively, since Hohfeld already gave the key to what logical connection with the right positions we have already formalized is to be grasped. This connection is represented with blue arrows on the drawing above, indicated by the word ‘opposites’. This will be—obviously\(^21\)—a negation in the formal representation. This seems to be in perfect harmony with what Hohfeld said emphasizing that his notion of Privilege is simply the lack of a duty doing otherwise. Therefore, we can say the following formally:

\[
PR_x E_y F \iff \neg O_x \to y \neg E_y F
\]

There are more salient points in this short formula worth discussing—and,\(^21\)Once we understood what Hohfeld means, it is obvious; not so obvious from the expression ‘opposite’, though. As we saw in the previous chapters, according to Kocourek [1920], for instance, it should be called ‘contradictories’.
due to some of them, we will need to refine it. First, it is worth mentioning, that the negation has to be put at two places, otherwise the equivalence would not hold—this is what Hohfeld referred to explaining that: "always, when it is said that a given privilege is the mere negation of a duty, what is meant, of course, is a duty having a content or tenor precisely opposite to that of the privilege in question."

Another thing to notice is about the other agent: while its role is obvious from the right-hand side of the equivalence, since we put a duty, that is, a directed obligation from which our agent $x$ is free, this directedness is not visible at all on the left-hand side, suggesting this way that there is no duty at all. If we would like to use a separate notation, an abbreviation for having privilege, we have to indicate that it is really—only—a privilege: missing a duty (not necessarily all the possible duties), the duty, which is directed toward the person who has therefore no claim toward $x$, as it shortly will be shown formally. When introducing Hohfeld’s paper, I mentioned that it may seem strange how a lack of something could bear the property of being correlative, since if I do not have a duty to refrain from something, that privilege of mine sounds quite general and rather not correlative. Hohfeld's example is revealing: "Suppose that X, being already the legal owner of the salad, contracts with Y that he (X) will never eat this particular food. With A, B, C, D and others no such contract has been made. One of the relations now existing between X and Y is, as a consequence, fundamentally different from the relation between X and A. As regards Y, X has no privilege of eating the salad; but as regards either A or any of the others, X has such a privilege. It is to be observed incidentally that X's right that Y should not eat the food persists even though X's own privilege of doing so has been extinguished." I said back then, and I say now, too, that this ‘as regards a given agent’ is crucial in understanding the notion of correlativity and, thus, is crucial in formalizing these correlative notions. As the reader might remember, Hohfeld is clear-cut in this issue: "A right is an affirmative claim against another, and a privilege is one’s freedom from the right or claim of another".

My proposal to the notation from whose claim $x$ is free by having the given privilege is the following:

\[(\text{PReqvNotD}) \quad \text{PR}_{x \rightarrow y} \exists_x F \iff \neg \text{O}_{x \rightarrow y} \neg E_x F\]

It does not mean that a "complete" privilege could not been interpreted in the Hohfeldian system: if there is no duty obliging us to act differently, then we have the privilege to do something. I will show this connection and its formalization in a separate section on directedness later.
What about No-claim? As its name and the blue arrow clearly indicate, it is the negation of one having a claim-right. Thus, the obvious way of formal definition is:

\[(\text{NCeqvNotCR}) \quad \text{NC}_y E_x F \iff \neg \text{CR}_y E_x F\]

In this formula, having a no-claim being an active right—even if the point is exactly having no right, the two parties are visible at both sides, therefore there is no need to introduce specific notations: the difference between the deontic operator’s and the action operator’s indices tell what has to be told.

And of course what we already know is that Privilege and No-claim are a correlative pair, thus, they are equivalent:

\[(\text{PReqvNC}) \quad \text{PR}_{x \rightarrow y} E_x F \iff \text{NC}_y \neg E_x F\]

which of course also means:

\[(\text{PReqvNotCR}) \quad \text{PR}_{x \rightarrow y} E_x F \iff \neg \text{CR}_y \neg E_x F\]

that is, having a privilege to do something means that the other party, whose claim-right we are free from has no claim-right toward us not to do that something;

and

\[(\text{NCeqvNotO}) \quad \text{NC}_y E_x F \iff \neg \text{O}_{x \rightarrow y} E_x F\]

that is, the fact that we have no claim toward the other party to do something means that he has no (directed) obligation to do that something.

As I wrote about it earlier, the Hohfeldian Privilege is practically the same as Permission in Standard Deontic Logic where we also define it as a negation of an obligation not to perform the given act:

\[\text{PA} \iff \neg \text{O}\neg A\]

This sameness is well-known, however, I will insist to use the expression 'privilege' to stress being within the Hohfeldian frame.

Privilege and No-claim have no "definition" in the sense Claim-right and Duty have. The reason why we cannot introduce that kind of biconditionals is that Privilege, as it has been said several times already and as it is visible from being equivalent with No-claim—is a lack: a lack, that is, a negation of
a given duty. The only way an action or its lagging would trigger the occurrence of another claim-right if we considered the negation of a privilege—but we already have considered that case when defining Duty. Therefore, there are no biconditionals for Privilege and No-claim in their affirmative form, as they are only the negations of those right positions which can be "defined" by biconditionals.

What we can do, though, is that we put them in the definitions of Claim-right or Duty. This is how (D) looks like:

\[(D) \quad O_{x \rightarrow y} E_x F \leftrightarrow \Box (\neg E_x F \rightarrow CR_y E_j E_x F)\]

If we want to put \(x\)'s privilege in the formula, what we have to do is, because of (PReqvNotD), to put the negation in front of his duty:

\[\neg O_{x \rightarrow y} E_x F \leftrightarrow \neg (\neg E_x F \rightarrow CR_y E_j E_x CF)\]

that is

\[\neg O_{x \rightarrow y} E_x F \leftrightarrow \Diamond (\neg E_x F \wedge \neg CR_y E_j E_x CF)\]

that is

\[(PR) \quad \neg O_{x \rightarrow y} E_x F \leftrightarrow \Diamond (\neg E_x F \wedge \neg CR_y E_j E_x CF)\]

that is, in the case of \(x\) having a privilege (from \(y\)) not to see to it that \(F\) is the case it can happen that \(x\) does not see to it that \(F\) and \(y\) has no claim-right toward the judiciary that it see to it that \(x\) see to it that \(F\) is compensated. Why not sure? Does this diamond bother us? It does not, first of all because a conjunction without the diamond would mean for instance that—in case of there being no duty of \(x\) toward \(y\)—\(x\) does not see to it that \(F\). This is obviously not something we want to say, since it would mean we only do things that we have to.\(^{22}\)

\(\neg O_{x \rightarrow y} E_x F \leftrightarrow \Box (\neg E_x F \rightarrow \neg CR_y E_j E_x CF)\)

since it can be the case, for instance, that the state of affairs in which \(F\) is compensated coincides with another state of affairs \(F'\)'s compensation—which the judiciary has a duty to enforce, since \(x\) had a duty to see to it that \(F'\) but he did not. That is \(CF \iff CF'\), which means, because of (E.RE), that \(E_x CF \iff E_x CF'\). For example, let \(F\) the case of a (particular) book being given to \(y\). \(x\) has a privilege not to give this book to \(y\), that is, \(y\) has no claim toward \(x\) to give it to her. The value of this book is $100, so if there was a duty of \(x\) to give it to \(y\), and he would not, then \(y\) would have a claim-right.
The other direction of the biconditional, though, does not need justification: if it can happen that in the case of non-fulfillment there is no claim-right of the "original counterparty" to enforce the given state of affairs (its compensation), then it means there was no such duty, that is, the agent had a privilege not to do so. At least, this is what we require about how things should be in law.

6.4.1 Vested Liberties/Civil Liberties

There is a type of freedom, which—together with some action—can trigger a new right and thus can be described by a conditional. This type is the one Hohfeld declared different from Privilege that he wanted to deal with—and therefore kept aloof from discussing. These are protected freedoms, in law the paradigmatic cases are those that we usually call civil liberties, or as Bentham called vested liberties (and what Wright [1963] called simply ‘right’). In the case of these freedoms—and I think this is the most common use of the word ‘freedom’—we mean that I have a freedom to do something if and only if as soon as someone precludes me from acting according to my freedom—and this is what I will represent as a prohibition on interference—I will have a claim-right toward the judiciary to stop him from precluding me.

As the reader might remember, in Makinson [1986] we find a few paragraphs about interference and the difficulties of its formal representation. Makinson is not satisfied with the way Kanger and Lindahl bypassed the issue by formalizing counterperformance instead, but admits that there are some things to clarify because of which its formula representation is not that straightforward. One of the first questions Makinson raises is what exactly it should be there is a prohibition to interfere with in case of vested liberties: "suppose that $x$ is permitted to see to it that $F$, and $y$ is forbidden to interfere. Interfere with what? With it coming to be the case that $F$, or specifically with $x$’s seeing to it that $F$? It can happen, for example, that $y$ interferes with $x$’s seeing to it that $F$ not because he has anything against the realization of $F$, but rather because he wants himself to be the one who toward the judiciary to see to it that $x$ pay $100$. There is no such duty of $x$ giving the book to $y$, but this does not mean that there is no duty of him to pay $100$ to $y$, for example, because she lent this amount last month to him. In this case what happens is that $x$ does not give the book to $y$ and there is a claim-right of $y$ toward the judiciary to see to it that $x$ pay $100$ to $y$. It will be a compensation of the "original" state of affairs (in which $x$ pays when it’s due). If one feels that a compensation in such a case would be $110$ (because of the delay) then the example can be made up with a book of value of $110$—or can be left in a value of $110$, since the delay would matter in the case of the book, too. That is, it can happen, so it is not counterintuitive that we do not exclude this possibility.

If there was no duty, there was no counterparty
brings it about, or has something against \(x\)’s bringing it about." Later he tacitly chooses the second version using consequently in the text that "\(y\) is forbidden to interfere with \(x\)’s seeing to it that \(F\)". In deep agreement with that, I argue that this is the right way to grasp the prohibition of interference, and the following seems a legitimate way of formalizing with our formal tools:

\[
\text{CL}_x E_x F \iff \Box (E_y \neg E_x F \rightarrow \text{CR}_x E_j \neg E_y \neg E_x F)
\]

As we can see in the case of political liberties the trigger is an active action: the action of the other party by which he sees to it that I do not see to it that the given state of affairs be the case, whose "existence" I have the freedom to (that is, he interferes). The most practical example is probably the freedom of speech: my freedom to speak freely means that if someone does not let me do so, I have a claim-right toward the judiciary to see to it that he do not see to it that I do not see to it that I have said what I wanted to. That is, if someone stops me speaking freely, the judiciary has the duty to stop him from stopping me.

There are two issues that can be raised regarding the formula above. One is the problem of ‘\(F\)’ we already discussed. Since in cases when a civil liberty is abridged there is some legal consequence, we can use the same solution as the one we did in the case of rights in criminal law: the aim of this legal consequence (sanction) is to retain people seeing to it that someone cannot exercise their freedoms, that is, in our example, that \(\neg E_y \neg E_x F\) is the case. Using this notion we do not lose what we want to express because of the difficulties with \(F\):

\[
\text{CL}_x E_x F \iff \Box (E_y \neg E_x F \rightarrow \text{CR}_x E_j S_y (\neg E_x F))
\]

Note that what is written on the right-hand side, is a special case of Claim-right:

\[
\text{CR}_{x \neg E_y \neg E_x F} \iff \Box (E_y \neg E_x F \rightarrow \text{CR}_x E_j S_y (\neg E_x F))
\]

But this would also mean that:

\[
\text{CL}_x E_x F \iff \text{CR}_{x \neg E_y \neg E_x F}
\]

but this is not completely true: there is no other agent indicated in the formula \(\text{CL}_x E_x F\), meanwhile \(y\) is involved on the right-hand side. The reason is that Freedom (Civil liberty) is usually thought as a general situation: when we have this type of requirement not to interfere toward everyone else. That is, we have to make the formula on the right-hand side up to express this "generality":

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It also could be raised that this description above does not contain the permission (with Hohfeldian terms, the privilege) of \( x \) to see to it that \( F \). Makinson [1986] described a vested liberty as a conjunction of these two (just like Bentham did). We have two options to express this connection: the obvious one is to include this conjunction in the formula, the other is to say that the permission is implied from a civil liberty regarding a given action.

What we see is that Civil liberty is a special case of Claim-right in having a claim-right against all the other parties to refrain from something—namely refrain from interference. What does it tell us about the Hohfeldian fundamental legal conceptions? The fact that we can express the protected freedoms/political or civil liberties as a special case of Claim-right confirms that Hohfeld was right in not dealing with these and not listing them as fundamental. And that he was absolutely right to list Claim-right as fundamental: so far it seems, we can express other right types with it, even ones that—from a legal theoretical viewpoint—we consider as basic types.

### 6.5 Formal Representation of Power and Liability

Let’s list briefly what we know about power and what we want to use for its formalization. The features to grasp all come from the way the rights and duties at this second group differ from the ones in the first one. Here the state enforcement cannot be a base to build on: the point in Power and Liability (and Immunity and Disability) is something very different. In the case of this group of rights and duties, I won’t provide model theoretical interpretation but will concentrate on showing how the working of them can be explained. There are two main properties we already have analyzed and have to keep in mind to find the way of formal representation we would like to.
One is about having a constitutive rule "in the background" of Power, since these are the type of rules which by definition "behave" like Power behaves: constitutive rules define an action, which is, therefore, not actually done if one does not follow the rules constituting it. And this corresponds perfectly with what Makinson said about Power: if we do something without Power (which would require to do it), we actually do not do it. (As I argued above presenting my underlying considerations, I conceive that the structure of constitutive rules is interesting, rather than the fact that the 'count as' conditionals always draw a context: we are within legal context, we already talk about a given legal system’s rules.)

If we would like to start form this Makinsonian observation a very intuitive approach would be to say that this feature of power should be formalized in the following way:

$$\neg P_x E_x F \rightarrow (E_x F \land \neg F)$$

but this solution of course won’t work since we have the the following axiom for $E_x$:

(E.T) $E_x A \rightarrow A$

It seems then reasonable to grasp another feature to start with.

First of all we know that the correlativity between Power and Liability is an equivalence. This formally, at first thought, looks like this:

$$P_x E_x F \Leftrightarrow L_y E_x F$$

but just the second thought is whether we face a directedness like we did in the case of the rights and duties in the first group. The answer is provided by the formula’s right-hand side: the formula of Liability contains two agents, so it is a directed right position, too—all of Hohfeldian right poistions are. That is, Power too. The "equivalence" above suggests a general power, meanwhile the right-hand side shows a right position which refers to a rights relation between two given agents. To have the proper equivalence, we need to add the notation of directedness we used in case of active duties and rights so far:

(PeqvL) $P_{x \rightarrow y} E_x F \Leftrightarrow L_{y} E_x F$

In the separate section on directedness we will come back to this issue to clarify why it might sounds counterintuitive that power is directed, too. But now let’s see how to provide some formalization on what Power and Liability
The feature upon which we can start the formalization is the way the second group rights affect the first group’s ones (also the ones in the second group): they can change them. The most direct hint comes from Hohfeld when saying: "it is a liability to have a duty created". As said above, this more precisely means that the possibility of creation of such a duty is already there before the very creation, the point is that if the person whose power implies our liability acts then we will have the duty he creates. Getting this hint our first intuition leads us to formalize Liability in the following way:

\[ L_y E_x F \leftrightarrow E_x F \rightarrow O_{y \rightarrow v} E_y F' \]

The immediate questions raising in the reader are probably who v is and what F' is, and we are going to come back to this in a moment, but before that we need to give it a longer thought whether the structure of this formal representation of Liability is proper. If this is what Liability looks like and we know (PeqvL), then we also already know that:

\[ P_{x \rightarrow y} E_x F \leftrightarrow E_x F \rightarrow O_{y \rightarrow v} E_y F' \]

which sounds—at first—a plausible approach. Now let’s jump a bit forward in the discussion. We know Power’s relation to Disability—its lack or negation, so we already know we will need this form also working, therefore, we can try whether this solution keeps its plausibility when we would like to have the negation of Power:

\[ \neg P_{x \rightarrow y} E_x F \leftrightarrow \neg (E_x F \rightarrow O_{y \rightarrow v} E_y F') \]

which is

\[ \neg P_{x \rightarrow y} E_x F \leftrightarrow (E_x F \land \neg O_{y \rightarrow v} E_y F') \]

This is something we obviously do not want to say, since it can happen that y has the duty toward v because of a different reason. What we do want to say is that in case x is lack of power, it can happen that he acts but there will be no duty on y—which could not happen if x had the power. The straightforward formal solution is to put a diamond into the formula standing for the ‘could’ in the sentence above:

\[ \neg P_{x \rightarrow y} E_x F \leftrightarrow \Diamond (E_x F \land \neg O_{y \rightarrow v} E_y F') \]

which of course means that the "original" affirmative definition of power also has to have a box added:
\[ P_{x \rightarrow y} E_x F \leftrightarrow \Box (E_x F \rightarrow O_{y \rightarrow v} E_y F') \]

That is, the fact that \( x \) has a power on \( y \) (saying ‘on’ sounds more fitting in case of power than to say ‘toward’) to see to it that \( F \) if and only if it is necessary that in case of \( x \) seeing to it that \( F \) \( y \) has a duty to see to it that \( F' \). This necessity operator is the same legal necessity operator we already used. Using this operator also solves the problems of the material conditional on the right hand side of the biconditional “threatening” the whole formula in the same way we saw in the case of Claim-right.

Let’s see now the agents of such a rights relation, that is, the variables in the formula above. It is important to see that regarding the agents’ coincidence there is hardly any restriction: \( x \) can have a power to change his rights relation by which he can undertake duties, so it can happen that \( x = y \). All the people who has legal competence/capacity to act (the notion we analyzed excluding Power from Claim-right’s definition), have the power to change their rights relations: this happens when we make contracts or when we marry someone. We can do these because we have the power to actions by which we change our rights. It also can happen—probably it often does—that \( x \) puts such a duty on \( y \) of which he is the counterparty, that is \( x = v \), but it is not necessary. For example, if I lend someone my car and we agree that she can use it as far as I ask it back, then what I have is a power to put a duty of giving it back on her. Till I do not say so, there is no duty of her to give the car back, but when I say, the duty of her arises, and the counterparty of this duty is me: I will have the claim-right toward her, that is, if she does not give the car back by the time I said, I will have a new claim-right toward the judiciary to make her give my car back. Or let’s take the most obvious example: the legislator. The legislation is something for which one clearly needs power, and the legislator can create such a duty on me that I have to allow my neighbour to my land if he wants to pick up the fruit of his tree falling into my land. One could ask whether \( y \) and \( v \) can be the same. In ethics it might make sense to say that I have a duty toward myself, but in legal sense it does not: it would mean that in the case of not fulfilling I had a claim-right towards the judiciary to make me fulfill the given duty of mine. No court would accept such a scenario as a legally valid (relevant) one. But something important has to be noticed: an agent’s power always involves two agents’ liability in the Hohfeldian theory. Since a rights relation always involves two agents, and it is a rights relation that can be changed in order to change someone’s rights (position), having power is always means the liability of the given rights relation’s two agents.

\[ ^{24} \text{It might be worth mentioning that these claims about agents are sentences of the meta-language: we did not introduce equality sign into the language.} \]
We can ask of course what we had asked in the case of Claim-right: does this mean that I only have a power if I use it? The answer here is also ‘no’ just like it was in the case of Claim-right to the question whether we only have it when it is not fulfilled. But using Power makes it visible in the legal system: the action of seeing to it that a given utterance is made on rights and duties (that I eftsoon show)—together with the existence of the power—creates an event of the given legal system of rights as making a new duty (or right) of a given person arise.

At this point we can get back to the big question of what is $F'$ in the formula above which expresses how one puts duty on someone—who is liable. It is purposely shown with choosing $F'$ that there is some connection with $F$. But what connection is that? In order to answer this question, first we need to call up some examples that require power: acts which we can say one has the power (authority) to perform. Hohfeld mentions transferring one’s interest, extinguishing of legal interest, agency relations and options, we can add for example eviction, search (of premises), garnishment or other types of execution. Should we use these as values for $F'$? Sounds reasonable since—knowing the Hohfeldian system—saying things like ‘one has the power to execute’, ‘one has the power to transfer his interest’ or ‘one has the power to evict someone else’ makes sense. But then what is $F''$? If $F$ is eviction then $E_y F'$ should stand for ‘y leaves her house’. But it sounds a little bit strange that if $x$ sees to it that the eviction is done (if $x$ evicts) then $y$ has the obligation to leave her house. The source the strangeness is that if $x$ has already done the eviction then $y$ has already left her house, so there is no obligation of her to leave it. Of course we can say that she has the duty to keep away, but when we talk about eviction, we primarily think her duty to leave as the one which comes due to the eviction. That is how we imagine the eviction itself: to order her to leave her house. Accordingly, my proposal is to consider $E_x F$ as something more simple, more elementary instead of executing the eviction. It should rather be saying something—in case of eviction saying that ‘y has to leave the house’. That is, $F$ is an utterance, it is that something is said (or written). The something that is said is not independent from $F$. A proposal formally could be the following:

\[ F^c : U(O_{y \rightarrow v} E_y F' \lor O_{y' \rightarrow v'} E_{y'} F'' \lor O_{y'' \rightarrow v''} E_{y''} F''' \lor \ldots ) \]

that is, $F^c$ (where $c$ shows that this $F$ is a special state of affairs created by a constitutive rule) is an utterance of $y$’s obligation to see to it that $F'$ and $y'$’s obligation to see to it that $F''$ and $y''$’s obligation to see to it that $F'''$ and so on (the notation ‘…’ does not mean that the formula is infinite, only that we cannot tell how long it is.)
One could raise here that maybe in the case of eviction the sentence ‘y has to leave the house’ is really said, but if x wants to sell his car, he will say only ‘I sell it’ and says nothing literally about others’ (or own) duties. But actually saying ‘I sell it’ is just an abbreviation: it contains a set of changes in rights—which changes are listed in the sales contract, or actually already in the civil code. Just as the duty of y to leave is written in the eviction order. These abbreviations are the names or labels of the legal or institutional constructions (which can be different in the different legal systems). This is the technique of presentation Ross [1957] talked about. These legal constructions are constituted by constitutive rules—constitutive rules that have the structure described in the formula above.

Being these notions exploited in the contract, or in the civil code, or any other legal norm means—and it is important to lay down that—these relations, these abbreviations of the given legal system are given by legal, professional axioms, not by logical ones, so it is not a task of the logical calculus to say more about them.

What is the situation with axiom T? Does not it derange this way of formalizing Power? The axiom T for $E_x$ does not render the formula of Power vacuous since it cannot be the case that someone sees to it that $F$ and not $F$, but it can be the case that someone utters what comes after $U$, but it won’t be the case that is in the argument of $U$: to be able to have the consequent (of making the utterance $U$) that y has a duty, x needs to have the power.

Standing next to the Trump Tower I can utter the sentence that ‘I transfer it to an Ethiopian orphan’, but—unfortunately—nothing will happen. The reason for this lack of consequence is that I don’t have the power, that is, I have a disability considering the selling of the tower.

One of the major theses I claim here is that we only can have power on special actions, actions that are utterances and constructed as it is visible in the formula above. It is important that ‘utterances’ is to be understood in a broad sense: not just oral, but obviously written too, as well as implicit conduct. With this formal system of description we can express what making a legal statement is:

$$E_x F_c$$

if x has a power to see to it that $F_c$ is the case—not without: seeing to it that an utterance is said does not mean that the claims contained on rights and duties in it will have legal binding, that requires power.

Before proceeding, we need to ask an important question: is it only a duty that can be arisen by a power? Of course not: it also can be a claim-right of
someone, what is more, whenever a duty arises, also does a claim-right. But
given that y’s duty by virtue of correlativity involves v’s claim-right (who,
as we have just written above, can be the same as x is), therefore there is
no need to indicate it separately. But what about Privilege? Can a privilege
also be a result of using a power? Of course it can: it makes sense to say
that I have the power to absolve her from the commitment to see to it that
F—which means that she has a privilege to not see to it that F. Should we
indicate this somehow formally? It seems there is an important making up
we need to do. On the one hand, we always have referred to the sentence of
Hohfeld saying that "liability is a having a duty created"—and this is what
we formalized. But on the other hand, we also referred to Power and related
right-position as second-order ones affecting other rights—on claim-rights,
duties, privileges and no-claims, and actually even on other second-order
rights. Which means that Power can concern creating any type of rights.
There is no need to indicate Claim-right separately since it follows (and act-
ually can be read) from the created duty. But it seems we need to indicate
that it can be a privilege that is created—with which we will have referred
to the correlative no-claim too. And it seems we need to do the same with
the second-order rights that can be changed: two of them are need to be
indicated (with which the other two—their correlative pairs—are indicated
as well.) The constitutive rule behind Power amounts to the following:

(ConsRuleP)
\[ F^c : U(O_{y \rightarrow v} E_y F' \lor \neg O_{y' \rightarrow v'} E_{y'} F'' \lor P_{y'' \rightarrow v''} E_{y''} F''' \lor \neg P_{y'''' \rightarrow v''''} E_{y''''} F''''}) \]

What about the definition of power? It, of course, also has to be constructed
according to the structure of constitutive rules behind it in order to receive
the proper description of what having a power means:

(P)
\[ P_{x \rightarrow y} E_x F^c \leftrightarrow \square(E_x F^c \rightarrow (O_{y \rightarrow v} E_y F' \lor \neg O_{y' \rightarrow v'} E_{y'} F'' \lor P_{y'' \rightarrow v''} E_{y''} F''' \lor \neg P_{y'''' \rightarrow v''''} E_{y''''} F'''')) \]

and for liability:

(L)
\[ L_y E_y F^c \leftrightarrow \square(E_y F^c \rightarrow (O_{y \rightarrow v} E_y F' \lor \neg O_{y' \rightarrow v'} E_{y'} F'' \lor P_{y'' \rightarrow v''} E_{y''} F''' \lor \neg P_{y'''' \rightarrow v''''} E_{y''''} F'''')) \]

It is important that while in (ConsRuleP) there are (possibly, not neces-
sarily) different y-s, since it can be the case that F^c means an utterance
about changes in different agents’s right positions; in the case of (P) and (L)
it is only y whose right position is concerned: this is the point in Power’s
directedness—that x can change y’s right position and not that he can change
It is also very important to notice that the $F'$-s are different within each formula above: if they weren’t, what we could derive would be a tautology.

What can we tell about (ConsRuleP)? What does it mean that there is a constitutive rule with this structure behind Power? This role of it can be described by saying that it works like a function: it assigns a set of $F'$-s to each $F^c$. But that is all what we can tell about it formally at this point.

### 6.5.1 Brief Detour about Speech Act Theory

The description of utterances given above obviously evokes the theory of speech acts. Even if for the purposes we are aiming at it seems dispensable to use its theoretical framework, it is nice to have a short look to see where the correspondences are.\(^{25}\)

The theory of speech acts is primarily attributed to John L. Austin, who provided its foundation in Austin [1988], and his student, John R. Searle, who, according to the general reception, refined his mentor’s theory, mainly in Searle [1969]. The basic idea (of Austin) was distinguishing uttering sentences having truth value (constatives—about which Austin later claimed they were a subset of performatives since we have the action of claiming that we say\(^{26}\)), and performative acts by which we also perform an act and not just say something. The classical examples of performatives are promising, ordering, asking, apologizing, etc. What is common in these acts is that they happen by saying the words ‘I promise’, ‘I order’, ‘I apologize’, or other verbal expressions (for example using imperative mood in case of ordering) creating these very actions. The acts are the *speech acts* which have three features or levels: locution—the very act of saying something (this is the one all utterances have); illocutionary act and its force—this is the most prominent feature of speech acts, this is the very action one performs by a speech act (for example naming a ship by saying ‘I name it’), and the force is what it realizes; and perlocutionary effect—this is the effect the speaker’s utterance results on the side of the listener (for instance convincing, scaring, encouraging, etc.).

There are conditions to be met for a given speech act to be performed felici-

\(^{25}\)I have no intention to go into details since it would take another dissertation, only to highlight how one could fit this theory of mine into the theoretical framework of speech acts, or vice versa.

\(^{26}\)Austin himself dismissed then the whole differentiation, but it is not necessary since there are still differences between constitutives and performatives—but as I indicated above, we do not go into details here.
ously: authority is an obvious one of them—without it, for instance, no one can declare a couple married, that is, the speech act fails to be performed as such. Its lack causes misfire—as Green [2015] puts it: "If I utter, before the QEII, "I declare this ship the Noam Chomsky" I have not succeeded in naming anything simply because I lack the authority to do so. My act thus misfires in that I've performed an act of speech but no speech act." Another type of misfire is when the addressee fails to respond with the appropriate uptake, which can happen for instance in case of a bet. There is also another type of mistake by which the speech act will be less than felicitous: this is abuse. For example in case of a promise: if I promise something with saying the words 'I promise' without having the intention to keep it then there was a speech act but its performance lacking sincerity was not felicitous.

Bach and Harnish [1979]—among others—differentiated four groups of speech act types:

- **Constatives**: affirming, alleging, announcing, answering, attributing, claiming, classifying, concurring, confirming, conjecturing, denying, disagreeing, disclosing, disputing, identifying, informing, insisting, predicting, ranking, reporting, stating, stipulating

- **Directives**: advising, admonishing, asking, begging, dismissing, excusing, forbidding, instructing, ordering, permitting, requesting, requiring, suggesting, urging, warning

- **Commissives**: agreeing, guaranteeing, inviting, offering, promising, swearing, volunteering

- **Acknowledgments**: apologizing, condoling, congratulating, greeting, thanking, accepting (acknowledging an acknowledgment)

The theory of speech acts is a rich direction of research with a lot of contemporary contributions in a long list of particular issues (for instance on differentiating explicit and implicit speech acts), but instead of introducing these I prefer to focus on what this theory can tell about law and how its theoretical framework possibly fits what I would like to tell about Power.

Considering the law as a system it is appealing to regard it as a mere set of speech acts, since it is quite common to think that the whole of law is prescriptive, that is, a set of orderings, that is, directive speech acts. Without opening the question whether it is really the case that law consists only speech acts, I might lay down with no danger of being impetuous that speech acts have a great role in law—and concentrate only what I would like to investigate.

The utterances we described with (ConsRuleP) are quite plausible to find as
speech acts. For instance selling something happens with a verbal utterance like saying or writing it. Rarely without: in institutionalized environment like shops are it can happen also only with implicit conduct by putting down the money on the counter when receiving the goods. To include them when declaring all the power-required actions as speech acts it needs a broad sense of speech acts—for which it can be an argument that in these cases the given gestures substitute verbal utterances. And it would be difficult not to see the obvious parallel between what was is said by Makinson about Power—when we do something without the required power we do not really do it); and what is said in speech act theory on authority—without is we cannot perform the given speech act. Therefore it is obvious that Power—if it differs from the thing speech acts theory calls ‘authority’ at all—is a necessary condition of the speech act that people having the power can perform. What we can say thus is that actions on which one can have a power are speech acts (in a broad sense including the implicit conduct, too).

6.6 Formal Representation of Immunity and Disability

This is the shortest part of the formalization. We can start with one of the blue arrows and say that the obvious relation between being liable and having immunity is a negation: whenever I have a liability that someone sees to it that $F^c$, it means that I do not have immunity regarding that action done by that given person. So the way I formalize Immunity in terms of Liability is pretty straightforward:

$\text{I}x E_y F^c \iff \neg L_x E_y F^c$

(This equivalence obviously can be phrased with putting the negation to the other side)
As the arrows show and as we discussed when investigated Hohfeld’s text, Immunity is the legal situation we have when the other agent has no power to change our rights relations. It does not mean that they do not have such a power to change someone’s rights relations, but toward us, they do not have this capacity. This situation reminds us very much the way we found in case of privilege, but now we do not need to introduce the specific directedness notation to indicate who the other party is from whose action I have the immunity since its formula shows both the right-owner and the agent of the action:

\[(\text{InotP}) \quad \neg \Box_y E_y F^c \leftrightarrow \Box_x E_y F^c\]

And of course this means that this person has a disability: a disability, since, as I have just said, it is not sure that he has no power toward other people—who are liable regarding his action. This means, in the case of Disability, we have to indicate in the formula who the other agent is toward whom he has no power to see to it that \(F^c\):

\[(\text{DnotP}) \quad \Diamond_{y\rightarrow x} E_y F^c \leftrightarrow \neg \Box_{y\rightarrow x} E_y F^c\]

(Here again we can phrase this equivalence in a way putting the negation to the other side and thus the light on the power:

\[(\text{PnotD}) \quad \neg \Diamond_{y\rightarrow x} E_y F^c \leftrightarrow \Box_{y\rightarrow x} E_y F^c\]

And of course \(y\) having the disibility to see to it that \(F\) toward \(x\) means that \(x\) has the immunity against this very action done by \(y\)—as it already follows from the equivalences above:

\[(\text{DeqvI}) \quad \Diamond_{y\rightarrow x} E_y F^c \leftrightarrow \Box_{x} E_y F^c\]

It is well to show that the formula that has been provided to describe how Power works stays intuitive if we apply it for what happens when someone lacks this power. As it happened in the case of Privilege and No-Claim, a separate formula dedicated to this pair specifically cannot be provided, since, as it has been shown, are right positions that are negations of the ones that can be described with a conditional. But we can negate those conditionals to show something about the nature of these positions; this is what for instance the negation of Power looks like:
that is, $x$ has a disability toward $y$ regarding seeing to it that $F^c$ if and only if it can happen that $x$ sees to it that $F^c$ and no change in $y$’s right position arises.

6.7 No Loss of Direction

As in the reception of Hohfeld we saw, discussing relationality and directedness of Hohfeldian rights and duties usually stops after considering the correlative pair of Claim-right and Duty, meanwhile other Hohfeldian rights and duties are not even really discussed in being or not being directed, so in this way we lose the directions that Hohfeld strictly assigned to his fundamental legal conceptions. The Claim-right-Duty pair indeed can be considered even more fundamental than the others, Hohfeld is very consequent about this relationality, though: all four pairs of legal positions are described strictly as relational ones. This is crucial in understanding them. Therefore, in the previous sections I used the notion of directedness introduced by Herrestad and Krogh [1995] (as its relevance has been shown in Makinson [1986] previously). This is how we got the following correlative pairs:

Claim-Right and Duty\[ CR_{x}E_{y}F \Leftrightarrow O_{y \rightarrow x}E_{y}F \]

Privilege and No-claim\[ PR_{x \succ y}E_{x}F \Leftrightarrow NC_{y}E_{x}F \]

Power and Liability\[ P_{x \rightarrow y}E_{x}F^c \Leftrightarrow L_{y}E_{x}F^c \]

Immunity and Disability\[ I_{x}E_{y}F^c \Leftrightarrow D_{y \rightarrow x}E_{y}F^c \]

Discussing Power, I mentioned that it may seem strange at first glance that Power is relational. This feeling might come from this: in everyday life, we usually refer to Power as something which is not relational: we usually say the someone has the power to do something, that is, the only thing we mention is the right-owner. But—as I emphasized a few times—the Hohfeldian system is consistently relational: with the power we can change someone’s rights or duties. In the US, a state has the power to summon each citizen registered in the given state—which sounds like a general Power, but this is a superficial considering. On the one hand, conditions excusing from being a juror can be indicated in a form previously sent to prospective jurors, which
is actually reference to their immunity, that is, that the given citizen is not liable. And on the other hand, no US state has the power to put a duty on me—since I am a Hungarian citizen. Most of the given state’s citizens are liable regarding this action of their state, though: each of them has a situation in which their duty of serving as a juror can be created.

The other reason that Power’s directedness is practically never discussed is the way the reception handles it: for instance Jones and Sergot [1996] argue that Hohfeldian Power is to be understood as falling within the more general phenomenon of ‘counts as’: "in the context of a given normative system or institution, designated kind of acts, when performed by designated agents in specific circumstances, count as acts that create or modify specific kinds of institutional relations and states of affairs. This switches attention from the formalisation of permission to the formalisation of the count as relation more generally." The formal representations I mentioned in the previous chapter all focus on the counts-as relation behind Power, talking about the special context by which some "normal" actions or utterances (like saying something) realize another actions—a "special", legally relevant one (like declaring, ordering, etc.). The legal relevance means that the new action or its result state of affair is something that all of us recognize in its role. If a registrar declares a couple married, we all recognize that arising marriage as a legal construction (institution). This generality takes the attention away from what was important in Power to Hohfeld: that it changes legal positions, and, therefore, legal relations. And this is where directedness or relationality comes into the picture: who can change whose legal position. This is what I represented in its formalization.

Does this mean that in the Hohfeldian theory there are no undirected rights and duties? No, it not necessarily does—but it might depend on what we mean undirected. We can find some hints even within the Hohfeldian theory about how to fit that kind of right conceptions into his framework that, at first sight, seem quite different—for example, because of being undirected.

As it was mentioned in the introduction, the Fundamental Legal Conceptions essay has a second part. It is much less cited or discussed, but contains an analysis on the difference between paucital and multital rights.27 (Hohfeld uses these terms instead of the common terms ‘relations in personam’ and ‘relations in rem’, respectively. He does so because he feels the classical expressions misleading—whose clarification takes a remarkable part of the essay, that we won’t discuss at all, though.) Simmonds [2001] picks two

27This is not the only pair of conceptions connected to rights that Hohfeld wanted to analyze, he lists seven pairs at the beginning of the essay section, but his early death stopped him in conducting; unfortunately we can read only the first pair’s analysis.
picturing examples of each category: "Suppose that I have a contract with you whereby you are obliged to manufacture a quantity of widgets. I have a claim-right against you and you have a correlative duty to manufacture the widgets. I might have a similar contract with another widget manufacturer, with similar consequences in terms of our claim-rights and duties. However many such contracts I have, however, my claim-rights are essentially limited to a definite number of persons. These are what Hohfeld calls ‘paucital’ claim-rights. (...) Suppose on the other hand, that I am the owner of Blackacre. I have a claim-right that you should not enter the land without my consent. I have the identical claim-right against your mother, my employer, the Bishop of Ely, and anyone else that you care to mention. Each of these claim-rights is a consequence of my ownership of Blackacre. These are ‘multital rights’." Hohfeld gives a short summary description of each type’s features: "A paucital right, or claim, is either a unique right residing in a person (or group of persons) and availing against a single person (or single group of persons); or else it is a one of a few fundamentally similar, yet separate, rights availing respectively against a few definite persons. A multital right, or claim, is always one of a large class of fundamentally similar yet separate rights, actual and potential, residing in a single person (or single group of persons) but availing respectively against persons constituting a very large and indefinite class of people." There is another good example of Simmonds [2001]: "my claim-right that you should not assault me is a multital right, since it is only one member of a large class of similar rights holding against an indefinite number of people (i.e. I have a right that your mother should not assault me, a right that the Bishop of Ely should not assault me, and so on)", for which Hohfeld would add the example of a patentee’s right that any other person shall not manufacture articles covered by the patent.

Why this analysis of Hohfeld is important to us? Because this shows that the way I expressed formally what happens with the Hohfeldian basic notion of Claim-right in criminal law or in the case of civil liberties actually fits perfectly with the way Hohfeld was thinking about his conceptions.

At first glance, though, this does not explain the naturality of how Power’s directedness perfectly fits to the image we have about power. But only at first glance, since meanwhile the examples and descriptions are only about claim-rights, privileges, powers and immunities can also appear in both multital and paucital versions. As Simmonds [2001] adds: "on the Hohfeldian analysis, the owner of Blackacre possesses not a single right over an area of land, but a complex aggregation of claim-rights, powers and immunities. For example the owner has a series of multital claim-rights that persons should not trespass on the land; a series of multital privileges to enter upon and exploit the land himself; a series of multital powers to transfer title to the land or create lesser interests in it, such as leases or easements; and a series of
multital immunities against having his title affected by act of other persons."
This description of ownership shows wonderfully that rights we usually think
as absolute and not relative can also be expressed in Hohfeldian terms.

How does this series of power look like formally if \( F^c \) is that the title to
the land is transferred?

\[
\bigwedge_{y \in A} P_{x \rightarrow y} E_x F^c
\]

The operation Makinson [1986] called ‘decountering’, that is, the way duty
as a directed obligation can be formally changed into an undirected one looks
like the following in this language:

\[
(O_{\text{und}}) \quad \bigwedge_{y \in A} O_{x \rightarrow y} \neg E_x F \Leftrightarrow O_{x \rightarrow y} \neg E_x F
\]

The same can be shown in cases of active rights and duties (where the
right-owner or the duty-bearer is the same as the person whose action is
considered):

\[
(PR_{\text{und}}) \quad \bigwedge_{y \in A} PR_{x \rightarrow y} E_x F \Leftrightarrow PR_{x \rightarrow y} E_x F^c
\]

\[
(P_{\text{und}}) \quad \bigwedge_{y \in A} P_{x \rightarrow y} E_x F^c \Leftrightarrow P_{x \rightarrow y} E_x F^c
\]

\[
(D_{\text{und}}) \quad \bigwedge_{y \in A} D_{x \rightarrow y} E_x F^c \Leftrightarrow D_{x \rightarrow y} E_x F^c
\]

In the case of passive rights and duties the formal representation of the
relation between directed and undirected versions needs a different modifi-
cation since the other party appears as the index of the action operator. We
introduced it in this indexed form in our language, but if we let it be unin-
dexed with the interpretation of ‘\( EF \)’ as ‘\( F \) is seen to it’ than the formulas
are straightforward:

\[
(CR_{\text{und}}) \quad CR_x \bigwedge_{y \in A} E_y F \Leftrightarrow CR_x EF
\]

\[
(NC_{\text{und}}) \quad NC_x \bigwedge_{y \in A} E_y F \Leftrightarrow NC_x EF
\]

\[
(L_{\text{und}}) \quad L_x \bigwedge_{y \in A} E_y F^c \Leftrightarrow CR_x EF^c
\]

\[
(I_{\text{und}}) \quad I_x \bigwedge_{y \in A} E_y F^c \Leftrightarrow I_x EF^c
\]
6.8 The Role of the State

As we could see, judiciary has a crucial role in rights relations. The way I modelled them formally shows that a legal right (in the first Hohfeldian group) is a conditionally three-sided relation, since non-fulfillment involves judiciary into it. But there is another power (now in the Montesquieunian sense) of the state that is often touch by arguments on rights and duties: the legislation. What role does legislation have, if any? The role of legislation is to create a legal system which can be described with a logic in which the biconditionals I provided are valid. At this point I have to refer to what already has been mentioned about the Hohfeldian theory’s and my formalization’s status. If these are descriptive theories then they tell how things with rights and duties are in the law. If we can imagine that they can be otherwise, then what this theory and its formalization describe is how things with rights and duties should be in the law: what legal rights and duties should mean, how their system should work. This requirement is the one that has been included in the Declaration of Rights of Man and of the Citizen when saying that "Any society in which no provision is made for guaranteeing rights (...), has no Constitution." One can accept that (human) rights exist without this effort of the state, but what they expect is that the state create a legal system which makes these right legal rights. What I have been pursuing with this formal representation is showing how a system of legal rights works. And what is needed to have this system is having the biconditionals valid in a logic that describes the given legal system.

Could it be otherwise? Well, the usual comment here—concerning the general relation between logic and law—is that legal validity is actually insensitive to logical validity. A legal system can be legally valid without maintaining consistency among rights and duties. But to say a legal system really contains a right means that the legal system obeys a logic having these biconditionals valid. And this is what is expressed in paragraph cited from the Declaration of the Rights of Man and of the Citizen.

This dilemma on being descriptive or normative is on the meta-level, though, therefore, it does not harm the adequacy of the expression ‘legal metaphysics’ as describing the right-system account provided in this dissertation.

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28 and this is how far we will go into discussing this issue, since its detailed investigation would take another dissertation(s)...

29 Here I consider legal validity as the existence of a legal norm: it has been created in the prescribed way (procedure), by the agency who has the power to create it. If the prescribed procedure doesn’t say anything specific about logical requirements then legal norms can exist (which means that they are valid—as von Wright assumes too from the very beginning) without being consistent.
Chapter 7

Summary and Conclusions

There are some theses I would like to some up, and to point to directions where further continuation of this project I see worthwhile.

- All the Hohfeldian rights and duties are directed. This should not be surprising considering that Hohfeld described them as parts of rights relations, ordering them into correlative pairs. Still, the only pair that the reception clearly handles as containing directed right and duty is the first one: the correlative pair of claim-right and duty. But all pairs have this property. The formal language we used automatically shows the involved agents—and, therefore, the directedness of the given right position—in the case of passive rights and duties (Claim-right, No-claim, Liability, Immunity); while in the case of active rights and duties (Duty, Privilege, Power, Disability) we needed to use the notation of directedness involving this way the other agent concerned by the given right position into its formal representation. In this way we could show clearly the equivalences according to which Hohfeld described these conceptions: the correlativity and the opposition.

- The first group’s right positions can be expressed by Duty, while the second group right positions can be expressed by Power. As we saw in the reception, there are some crucial differences between the two dimensions, as Makinson calls them—that is, the first group’s and the second group’s right positions. Still, in this dissertation an approach has been presented by which we can provide a description of what right positions mean. We did not define them in the strict sense, since Hohfeld himself refused to reduce them to something else; but we described each in a way that makes them identifiable by assigning different conditional consequences to them. The conditions we presented are actions—or their absence. This way we could delineate how legal rights and duties "behave" in the legal system: under which condition they result in other right positions’ arising. This is of course a static
picture of how this system works; to have a dynamic picture of the
system we need to change the language—in what follows, I will reflect
on this direction briefly.

- State enforcement is the institution we based the formalization of the
first group’s right positions on. The starting point was Makinson’s
definition that intended to define who the counterparty of a duty is
as the claimant, that is, as he says, the one who has the power to
initiate a legal action in the case of non-fulfillment. This definition of
the counterparty happened to be a definition of a duty-claim-right pair,
too. I showed that while I think that the state enforcement is the proper
starting point of defining what a legal duty (or a legal claim-right) is,
Power should not be involved since then we lose an important difference
between the ability of having rights and the ability to change them;
explaining Claim-right with Power would mean for example we cannot
explain the rights of a child. If we take the state enforcement as the
crucial factor—which, again, I think is reasonable to take—the main
point of having a claim-right is the new claim-right which arises in the
case of non-fulfillment: the one which is against the judiciary to enforce
that the original duty-bearer fulfill his duty. That is, what happens in
the case of the first group’s rights relations—if we consider law—is
that they are two-sided with the threatening/potentiality (depending
on which position is ours) to become three-sided.

- It has been shown that the description I gave of what Claim-right
is can be used to explain rights in cases where usually we see them
undirected, like in criminal law. What we need to see, and what Hohfeld
himself explained in his second essay, is that directedness does not
exclude the interpretation of positions we feel (and in legal theory
usually call) absolute: just like in the case of property, where the owner
has "a series of multital claim-rights that persons should not trespass
on the land; a series of multital privileges to enter upon and exploit
the land himself; a series of multital powers to transfer title to the
land or create lesser interests in it, such as leases or easements; and
a series of multital immunities against having his title affected by act
of other persons"; right to life and right to physical integrity also can
be described as going against every other agent—but being directed
toward everyone does not mean that it is not directed. Realizing this
important feature of the Hohfeldian theory and the passage between
directedness and "undirectedness" helps us show that Hohfeld was right
in not taking vested liberty (protected freedom) fundamental: what this
kind of freedom means can be expressed—conceptually and formally—
with claim-right. This also shows that Hohfeld was right, though, in
taking claim-right to be fundamental.
It has been shown that—as Hohfeld’s reception found, too—the notions of constitutive rules and counts-as relations have a strong connection to how Power works. I argued, though, that in order to understand what was crucial to Hohfeld in his system we need to concentrate on the structure of constitutive rules that create those actions that require power to be done, and not primarily the count-as feature of these actions that creates the special context: when analyzing Hohfeld, the relationality of Power is more important to see than the generality resulted by an action which counts as something institutional. Power is directed, too: someone has a power to change someone’s legal position. It is an important difference between the two groups of rights that in the case of Power these two someones can be the same, while in law the Claim-right-Duty pair only makes sense involving two different agents.

I argue that Makinson [1986] is right in referring to an action requiring power as a "performative utterance or inscription or some other conventionally recognized gesture or procedure" and showed what this utterance is about: how and why it results in altering one’s right position—if the utterer has a power to see to it that the constituted action be done. A description which is not affected by the axiom T has been provided. I sketched a connection between power-required utterances and speech act theory arguing that it seems an obviously adequate conceptual frame to describe what happens in the case of Power, but I also argued that ‘utterance’ has to be understood in a broad sense when used in the explanation of the Hohfeldian theory: legal statements can be made with implicit conduct, too—where ‘implicit’ shows that they substitute an oral or written utterance.

Meanwhile my starting point was to use the same formal tools that have been used in the classical formalization, there was a need to extend the language with a (legal) necessity operator, ending up in this way in describing legal metaphysics. This approach is not far from the original Hohfeldian intentions: he provided his analysis to bring terminological and conceptual clarity to the area of rights in judicial reasoning and, therefore, in law. Such an analysis practically realizes metaphysics of law since it is about its ontology is and the nature of things within the realm of law. Or, how they should be: a study engaging in conceptual clarity starts out as a descriptive study, but eventually acquires, if it is any good, a normative element. This duality, however, does not take the metaphysical character away.

To sum up, my aim has been to present a comprehensive formal analysis of the Hohfeldian fundamental legal conceptions, proposing that rights and duties have descriptions in a uniform manner according to their conditional legal/logical consequences. The formal tools I used are basically the same as
the logics that have been used in he classical formalizations, but instead of
the computational features, I focused on the internal relations of a right sys-
tem describing what the differences are between the various rights and duties
if we consider them from the same viewpoint of potential consequences. With
this analysis we could shed light on several crucial points of the Hohfeldian
theory, refusing some of its counterarguments at the same time.

To have a perfect and complete formalization of the Hohfeldian fundamental
legal conceptions there are many steps ahead, of course. The truth conditions
of propositions with Power has not been shown: the structure has been pre-
sented and its dependence on the constitutive rule which creates the actions
requiring power to be done. It seems to me now that the given background
(constitutive) rules should be formalized in order to make Power’s formal
representation more precise in model theoretical sense, or we might need a
two sorted language where we have a set of states of affairs in the semantics
so we can define relations on it. Such a solution would influence other points
of the formal theory, too: as it has been shown, the representation of states
of affairs are much more complex than it seems at first sight. Statutes de-
scribe types, while the ones described in contracts rather can be considered
as tokens—and it is surely a token of a state of affairs that actually happens.
It is not obvious how we should formally represent the relation between these,
but it seems that it requires a separate sort of formulas representing states
of affairs.

More-sorted language can be useful in other extensions of the formal the-
ory, too. One point is describing duties and rights in the way that has been
presented is to get a picture of how a system of rights works: what other
rights are generated. Showing dynamism has more direct ways, though. One
is applying transitions—which requires a two-sorted language when one sort
is for formulas of transitions, the other is for states. The logic presented in
Sergot [2014], for instance, seems adequate to describe this dynamism more
directly: the actions are transitions between states representing one’s right
position. For example, not y seeing to it that F takes the state x having a
claim-right against y to see to it that F into a state where x has a claim-right
against the judiciary to see to it that y sees to it that F. And one obvious
feature of a system of rights is also embodied in this language: that seeing
to it that F takes an agent from the state of having to see to it that F to a
state in which the agent no longer has this duty. It is, after all, noteworthy to
reach a state of accomplishment: once a duty (like completing a dissertation)
is fulfilled we are free from it.
Bibliography


