Finite size matrix elements and thermal correlators in integrable quantum field theories

István M. Szécsényi
Ph.D. Thesis statement

Advisor: Gábor Takács, professor, DSc

Eötvös Loránd University, Budapest
Physics Doctoral School
Astronomy and Particle Physics program
Department of Theoretical Physics

Director of doctoral school: Dr. Tamás Tél
Head of doctoral program: Dr. László Palla

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My research concerned finite size matrix elements and thermal one- and two-point functions, in particular in 1+1 dimensional integrable quantum field theories. The results below are published in [ST12],[STW13] and [PST15].

Introduction

In my PhD thesis, I study the evaluation of finite size matrix elements and thermal correlators in integrable quantum field theories (QFT). Correlation functions are fundamental quantities in QFTs, since the knowledge of all correlator of local operators contains all the information in the QFT. Thermal correlators play important role describing finite temperature quantum systems, e.g. understanding their phase transitions or relating theoretical predictions to experimental results. Finite size quantum systems appear many times as a way of regularisation, or numerical method in physics, e.g. in lattice QCD calculations. Understanding the volume dependence of matrix elements is of great importance to the infinite volume extrapolation process. In two dimensional relativistic theories, the finite size and finite temperature setup are related. For example, the thermal expectation value of a local operator is the same as the expectation value of the operator in the finite size vacuum state.
Thesis statements

1 Numerical verification of the LeClair-Mussardo series

LeClair and Mussardo (LM) conjectured an expansion for thermal one-point functions that uses form factors and the thermodynamic Bethe ansatz (TBA) pseudoenergy [LM99]. They showed the validity of their conjecture for the trace of the stress-energy tensor that in this case coincides with the TBA equations, derived by Zamolodchikov in [Zam90]. Saleur extended the argument and proved the equivalence of the TBA and the LM series for one-point functions of conserved charge densities [Sal00].

For general operators, where a TBA construction is not known, the LM series was demonstrated to be true up to (and including) three particle terms in [PT08b], and a proof to all orders was presented in [Poz11]. These results rely on the finite volume form factor formalism introduced in [PT08a, PT08b], which is not proven from first principles yet.

I implemented the numerical evaluation of the LeClair-Mussardo series in the so-called $T_2$ model. This included numerical evaluation of the corresponding TBA equations and the connected form factors of the model, in a way fast enough to make the numerical integration practically possible for the first 8 terms in the series. I compared the result to an independent numerical method, the renormalisation group improved truncated conformal space approach (TCSA). I found perfect agreement between the two numerical evaluations, and the LM conjecture has good convergence for large volume parameters, i.e. low temperature. For small volumes, the terms in the LM series have the same magnitude, hence the convergence breaks down and the series is not an efficient way of numerical evaluation anymore.

These results were published in [STW13].
2 Proof of the excited LeClair-Mussardo series for the trace of the stress-energy tensor

As mentioned before, the finite temperature and finite volume description are related in two dimensional relativistic theories. As a consequence of this relation, the LM conjecture describes the expectation value of local operators in the finite size vacuum state. By analytic continuation in the volume parameter, Pozsgay generalised the LM series for expectation values in excited finite size states [Poz13]. In my thesis, I stated the general form of these finite size matrix elements. In the infrared limit, i.e. large volumes, the general formula reproduces previously known results for finite volume diagonal form factors. Moreover, I proved the conjecture for the trace of the stress-energy tensor operator.

These results were published in [PST15].

3 Numerical verification of the excited LeClair - Mussardo series

To numerical verification of the excited LM series, I improved the speed of the evaluation of connected form factors. This made it possible to achieve similar precision as with the original LM series. I evaluated the series in the $T_2$ model and compared it to RG improved TCSA data.

In the domain where the series converges, there was perfect agreement between the form factor series and TCSA results. Similarly to my investigation with the LM conjecture, the generalised series is not convergent for small volume parameters. Moreover, for states where the TBA equations have a nontrivial transition at some critical volume, the convergence of the generalised conjecture breaks down exactly at the critical volume. In general, this can happen for states with zero total momentum, and for volumes where the TBA active singularities, specifying the state, coincide. Above the critical volume, the generalised conjecture is convergent, and it is an efficient way to evaluate the finite size diagonal matrix element.

These results were published in [PST15].
4 Constructing the spectral expansion for thermal two-point functions at second order

The spectral expansion of thermal two-point functions is ill-defined in infinite volume due to the presence of singularities. In [PT10], Pozsgay and Takacs developed a method using finite volume regularisation that leads to a well-defined finite spectral expansion. I reproduced their previous calculation and also corrected the $D_{22}$ term in the expansion. I supported the analytic calculation by numerical evaluation of the finite volume regularised expression in the sinh-Gordon model. Furthermore, I showed the symmetry property of the $D_{22}$ term. The calculated terms give the correct spectral expansion at second order.

These results were published in [ST12].

5 Clustering property of the spectral expansion for thermal two-point functions

Thermal two-point functions in massive theories are expected to obey the cluster property, meaning for large spatial separation of the operators present in the correlator, the two-point function factorises to the product of two thermal one-point functions. I showed the cluster property of the thermal two-point functions up to second order in the spectral expansion, which is a further nontrivial consistency check of the results.

This results was published in [ST12].
References


