The application of perturbative unitarity in the extensions of the Standard Model

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Ph.D. Thesis

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## Contents

1 Introduction 1

2 Perturbative unitarity 4
   2.1 Effective and simplified models 4
   2.2 Perturbative unitarity in partial wave expansion 5
   2.3 Formulation of the electroweak interaction 7
      2.3.1 Four-fermion theory 8
      2.3.2 Intermediate vector boson theory 8

3 A simplified model of dark matter 14
   3.1 Doublet-singlet model of dark matter 16
   3.2 Dark matter constraints 19
   3.3 Regions of small couplings 24
      3.3.1 Small dark matter-Z coupling with \( \tan \beta \approx -1 \) 25
      3.3.2 Small dark matter-Z coupling with \( \tan \beta \approx 1 \) 26
      3.3.3 Small dark matter-Higgs coupling with \( \sin(2\beta) \approx -\frac{m_\chi}{m_h} \) 28
   3.4 Perturbative unitarity constraints 29
      3.4.1 \( \Psi^- \Psi^+ \rightarrow W^- W^+ \) scattering 30
      3.4.2 \( \Psi^- \chi_i \rightarrow W^- h \) scattering 32
      3.4.3 \( \Psi^- \Psi^+ \rightarrow \Psi^- \Psi^+ \) scattering 33
      3.4.4 \( \chi_i \chi_i \rightarrow \chi_i \chi_i \) scattering 34

4 Extension of the Standard Model with a singlet scalar 36
   4.1 Effective approach 37
      4.1.1 \( \gamma \gamma \rightarrow S \rightarrow \gamma \gamma \) scattering 38
      4.1.2 \( Z_L Z_L \rightarrow S \rightarrow Z_L Z_L \) scattering 41
      4.1.3 \( W_L^- W_L^+ \rightarrow S \rightarrow W_L^- W_L^+ \) scattering 43
   4.2 Renormalizable extension 45
      4.2.1 Resolving the \( S \rightarrow B^\mu B^\nu \) interaction 46
4.2.2 $SU(2)_{\text{weak}}$ doublet vector-like $T$ 

5 Conclusion

A Scattering amplitudes with Mathematica

B Experimental bounds on $\kappa_{\gamma}$ and notation conversion
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Chapter 1

Introduction

The Standard Model of particle physics is one of the most successful models to this day. It can describe the fundamental particles and their interactions with high accuracy as it is observed in particle colliders, like the Large Hadron Collider (LHC) at CERN which is the highest energy collider experiment at the present time. The Standard Model is a renormalizable spontaneously broken gauge theory based on the symmetry group $SU(3)_{\text{color}} \times SU(2)_{\text{weak}} \times U(1)_Y$. All the particles of the Standard Model have been discovered. The last missing piece, the Higgs boson responsible for the symmetry breaking was discovered in 2012 [1]. Nevertheless, the Standard Model is thought to be an effective model that is not valid up to arbitrary high energies, as suggested by theoretical considerations and astrophysical observations.

From astrophysical observations as WMAP [2] and Planck [3] that map the content of the universe by its gravity, it turned out that baryonic matter that contains all known particles makes up only 5% of our observable universe. The so-called dark matter takes up 24%, while the remaining 71% is dark energy. The dark components are mostly unknown. The dark matter density in the universe is measured and if it has particle origins then it is very weakly interacting. Its existence suggests that the matter content of the Standard Model should be extended.

Another open question is the hierarchy problem that concerns the Higgs boson. If the Higgs boson is a fundamental scalar, than it gets large quantum corrections to its mass that is proportional to the cut-off scale of the model. The Standard Model can be valid up to the Planck scale $m_{\text{Planck}} \approx 10^{19}$ GeV that is where gravity should be incorporated. In comparison, the measured mass of the Higgs boson is very small, $m_h = 125$ GeV. If we accept the fine-tuning of the parameters, then it is only a problem of aesthetics. Therefore, it would be comforting to solve the hierarchy problem with some new mechanism beyond the Standard Model that can protect scalar masses from the large quantum corrections and resolve the conflict between the scales as chiral symmetry protects the masses of the fermions or gauge symmetry protects the masses of gauge bosons.

In addition, a complete theory should not have Landau poles, where a coupling becomes
infinite and should be asymptotically free. The weak interaction and quantum chromodynamics are asymptotically free, however, the $U(1)_Y$ abelian part of the Standard Model gauge group is not and has a Landau pole, similarly as the $\phi^4$ theory that describes the Higgs scalar potential.

Finally, the Standard Model combines three interactions, the electromagnetic, weak and strong, but the fourth one, gravity, is not included. It is still an open question how quantum field theories and classical gravity can be merged.

An important goal of fundamental particle physics would be to find an ultraviolet (UV) complete theory that is valid up to arbitrary high energy scales and is an extension of the Standard Model. There are candidates such as supersymmetric models (SUSY) [4] that introduce new symmetries that protect the scalar Higgs mass from quantum corrections. Others such as composite Higgs models [5] avoid the hierarchy problem with suggesting that the Higgs boson is not a fundamental scalar, rather a composite bound state like mesons. Again a different approach is the grand unification, that tries to merge the three gauge groups into one that is asymptotically free. All these completions of the Standard Model predict new particles that might account for the dark matter content of the universe. Studying these models, however, has a difficulty that they contain so many new particles that are not observed, yet and parameters that makes it rather demanding to compare them with the experimental results.

The high energy extensions of the Standard Model cannot be studied directly by experiments. Currently, the highest energy collider is the LHC at CERN that is running with center-of-mass energy $\sqrt{s} \approx 13$ TeV which allows for examining phenomena around a few TeV. One approach to incorporate physics at higher energies is to introduce effective theories. These are valid up to a given energy scale and all particles heavier than this cut-off are integrated out. The resulting effective lagrangian contains non-renormalizable interactions that are induced at loop-level in a complete renormalizable theory. This allows us to concentrate on phenomena that occur at a given energy scale. Beside, it is a model independent framework that can provide information on any theory where these effective operators appear at lower energies. Since effective theories have a natural cut-off scale, where they lose their predictive power, one can take a different approach. Extend the Standard Model with a complete new sector from larger theories that can explain the phenomena in question, such as dark matter. These are called simplified models. They make the comparison with experimental data simpler and as they contain a complete sector from a larger theory, they provide constraints on larger theory as well.

Complementing the experimental searches, there are theoretical approaches to examine effective models. One technique is using perturbative unitarity to establish the validity range of the model and constrains its parameters [6]. Unitarity of the scattering matrix in field theories ensures that the probabilities of events add up to one. When unitarity breaks down in a model, i.e. it predicts probabilities bigger than one, some new mechanism or particles are expected to appear and provide a way to fix unitarity. So examining processes in effective models with per-
turbative unitarity helps establish not only the validity range of the model, but also determines which sector needs completion. As it was in the early formulation of the electroweak sector of the Standard Model. The first description by Fermi was a four-fermion interaction between fermions with dimensionful coupling and it was successful at low energies. Then examining fermion two-to-two scatterings led to the intermediate vector boson picture with the heavy $W$ boson, which still had unitarity violating processes that suggested the presence of the $Z$ boson and finally with the Higgs and the joint description with the electromagnetic interactions become a renormalizable theory.

In this thesis, I introduce two models that I study using perturbative unitarity as a tool. The dissertation is organized as follows. In chapter 2, I give an overview on perturbative unitarity constraints and how they apply to effective models. I show how these constraints coming from the unitarity of the $S$ matrix and how they apply to two-particle scattering amplitudes, using the partial wave expansion. In addition, as an example, I show how considering unitarity helped in the formulation of the Standard Model starting from four-fermion interactions. Then, I examine two models using the introduced technique.

In chapter 3, I introduce a simplified theory for dark matter [7–9]. It extends the Standard Model with a pair of vector-like fermions that are doublet under the weak $SU(2)_{\text{weak}}$ gauge group and a singlet under the Standard Model gauge group, they can be thought as the higgsino-bino sector of supersymmetric models. Due to the Yukawa couplings between the singlet and the doublet, their neutral part mixes. Similarly to matter parity in SUSY, a $\mathbb{Z}_2$ symmetry is assumed that protects the direct coupling between the Standard Model fermions and the new leptons. That makes the lightest new neutral particle stable and a good dark matter candidate. After presenting the experimental constraints on the model from dark matter searches, I derive bounds from perturbative unitarity on the parameters of the model.

An excess over the background in the diphoton searches were reported recently by both the ATLAS [10] and CMS [11] experiments at CERN, that can be explained with a new scalar resonance [12–14]. I introduce an effective model in chapter 4 that extends the Standard Model with a singlet scalar. I present the perturbative unitarity constraints from vector boson scatterings and give the validity range of the effective description. Then, I consider a possible model that can result in this effective one after integrating out an extra heavy fermion that has renormalizable interactions with the new scalar. The previously calculated limits from unitarity now translate to constraints on the new parameters as the fermion mass, helping to discriminate between the otherwise experimentally allowed possibilities.

In chapter 5, I close my thesis with a conclusion. A one-page summary is attached after the bibliography, both in English and Hungarian.
Chapter 2

Perturbative unitarity

2.1 Effective and simplified models

Ultraviolet complete theories that may describe the observed phenomena beyond the Standard Model in high energy particle physics introduce a large number of new particles and new parameters. The large parameter space results in comparing the theory prediction with the experimental observation rather difficult. In addition, we do not observe these new particles, making it hard to put meaningful bounds on the parameter space of the theory. There are ways to overcome this problem, such as taking out only a piece of the larger theory that is relevant for the phenomena we would like to study.

Effective theory description works below a chosen energy scale $\Lambda$ that associated with the phenomena that it describes. Anything that is above this scale are explicitly integrated out in the field theory. This provides a model independent way to study and compare results from different theories. The effective lagrangian contains all, even non-renormalizable operators and can be expanded in powers of $\frac{1}{\Lambda}$ so we do not have to deal with infinite number of operators. However, there are processes with cross sections growing with the the center of mass energy that violates unitarity above the energy scale $\Lambda$, where the effective description brakes down. Studying these processes therefore helps to determine the validity scale of the effective model and constrain its parameters. That makes perturbative unitarity a powerful theoretical tool in analyzing effective theories.

However, the effective description has limited predicting power. If we are interested in phenomena close to or above the cut-off scale of the the theory, then it loses its reliability. In this case, simplified models are a convenient choice. Simplified models can be thought of as phenomenological sketches of a complete theory. They extend the Standard Model with a new sector from a larger theory. For example, the minimal supersymmetric extension of the Standard Model (MSSM) contains a lot of new particles and parameters that are not easy to handle, but if one only interested in describing dark matter that is often a neutralino, then taking out only the
neutralino sector in addition to the Standard Model makes the analysis easier and more relevant. The smaller parameter space is convenient to handle, to compare with the experimental data and to make predictions. On the other hand, it is also simpler to connect simplified models with the larger theory and express the findings in the bigger parameter space.

2.2 Perturbative unitarity in partial wave expansion

The unitarity of the $S$-matrix can be translated to the perturbative scattering amplitudes at every order. After expanding the scattering amplitudes in total angular momentum, the constraints from unitarity provide practical limits on the real and imaginary part of the partial wave coefficients that depends on the center of mass energy $\sqrt{s}$ of the actual process [16]. In practice, I will look at two-particle scatterings, where the scattering amplitude depends on the scattering angle beyond the energy. Therefore, the partial wave amplitudes will only depend on center of mass energy and the unitarity constraints can be directly written for $\sqrt{s}$, giving the energy scale where the effective operator description is valid.

The scattering matrix can be separated into the identity, where no scatterings happen and a non-trivial part $T$ that describes the actual interactions,

$$S = I + iT$$

The fact that all possibilities add up to one, can be expressed with the unitarity of the $S$-matrix, $S^\dagger S = I$ that translates to $T$ as follows,

$$T - T^\dagger = iT^\dagger T = iTT^\dagger$$

$$T_{fi} - T^*_{if} = i \sum_n T^*_{nf} T_{ni} = i \sum_n T_{fn} T^*_{in}$$

where $T_{fi} = \langle f | T | i \rangle$ is the matrix element between $|i\rangle$ initial and $|f\rangle$ final states. On the right hand side, $n$ goes through all possible intermediate states. Introducing the Lorentz invariant scattering amplitude, $\langle f | \mathcal{M} | i \rangle = \mathcal{M}_{fi}$,

$$T_{fi} = (2\pi)^4 \delta^{(4)} (p_f - p_i) \mathcal{M}_{fi}$$

$$\mathcal{M}_{fi} - \mathcal{M}^*_{fi} = i \sum_n (2\pi)^4 \delta^{(4)} (p_i - p_n) \mathcal{M}_{fn} \mathcal{M}^*_in$$

Now, consider only two-to-two scatterings that will be important for the further study. For the kinematic description, I choose the center-of-mass frame of the incoming particles, their momenta and masses are $p_1, m_1$ and $p_2, m_2$. The outgoing particles have $p_3, m_3$ and $p_4, m_4$.
momenta and masses. The scattering can be described with two parameters, the center of mass energy $s = (p_1 + p_2)^2 = (p_3 + p_4)^2$ and the scattering angle $\theta$. The process is illustrated in figure 2.1.

$$
p_1 = \left(\frac{s + m_2^2 - m_1^2}{2\sqrt{s}}, 0, 0, p\right) \quad p_3 = \left(\frac{s + m_4^2 - m_3^2}{2\sqrt{s}}, k\sin \theta, 0, k\cos \theta\right)
$$

$$
p_2 = \left(\frac{s + m_2^2 - m_1^2}{2\sqrt{s}}, 0, 0, -p\right) \quad p_4 = \left(\frac{s + m_4^2 - m_3^2}{2\sqrt{s}}, -k\sin \theta, 0, -k\cos \theta\right)
$$

(2.6)

where the incoming and outgoing momenta can be expressed with the center of mass energy as follows,

$$
p = \frac{\sqrt{(s - m_1^2 - m_2^2)^2 - 4m_1^2m_2^2}}{2\sqrt{s}}
$$

$$
k = \frac{\sqrt{(s - m_3^2 - m_4^2)^2 - 4m_3^2m_4^2}}{2\sqrt{s}}
$$

(2.7)

In this case of two-particle scattering, the scattering amplitude depends only on the center of mass energy $\sqrt{s}$ and the scattering angle $\theta$, $\mathcal{M}_{fi} = \mathcal{M}_{fi}(\sqrt{s}, \cos \theta)$.

To put the unitarity condition in equation (2.5) in a more useful form, it is practical to use the helicity formalism of Jacob and Wick [15] and project the scattering amplitude onto partial waves of total angular momentum $J = 0, 1, 2, \ldots$. The expansion coefficients called the partial wave amplitudes are the following

$$
a_J^{fi} = \frac{\sqrt{\lambda_f \lambda_i}}{32\pi s} \int_{-1}^{1} d\cos \theta \mathcal{M}_{fi}(\sqrt{s}, \cos \theta) P_J(\cos \theta)
$$

(2.8)

where the helicity of the initial and final state is $\lambda_l = \lambda(s, m_{i_1}^2, m_{i_2}^2)$ with $l = i, f$ in general is the following,

$$
\lambda_l(s, m_{i_1}^2, m_{i_2}^2) = (s - m_{i_1}^2 - m_{i_2}^2)^2 - 4m_{i_1}^2m_{i_2}^2
$$

(2.9)
for the two-particle scattering, it is related to \( p \) for the incoming and \( k \) for the outgoing case,

\[
\lambda_i(s, m_1^2, m_2^2) = 4sp^2, \quad \lambda_f(s, m_3^2, m_4^2) = 4sk^2
\]  

(2.10)

In the high energy limit as \( s \to \infty \), the masses of the particles can be neglected and \( \lambda_i \to s^2 \)
and \( \sqrt{\lambda_f \lambda_i} \to s \). With these considerations, the unitarity condition of equation (2.5) for the partial wave amplitudes follows,

\[
\frac{1}{2} (a_{fi} - a_{f*}^{i}) = i \sum_n a_{fn}^{J} a_{in}^{J*}
\]

(2.11)

In the case of identical initial and final states \( i = f \), the unitarity condition further simplifies into a more useful inequality for the partial wave amplitudes.

\[
\text{Im} a_{ii}^{J} \geq |a_{ii}^{J}|^2 = \text{Re}^2 a_{ii}^{J} + \text{Im}^2 a_{ii}^{J}
\]

(2.12)

\[
(\text{Re} a_{ii}^{J})^2 + \left(\text{Im} a_{ii}^{J} - \frac{1}{2}\right)^2 \leq \frac{1}{4}
\]

(2.13)

That implies for the real and imaginary part of the \( J \) partial wave amplitude separately the following constraints,

\[
|\text{Re} a_{ii}^{J}| \leq \frac{1}{2} \quad \text{and} \quad |\text{Im} a_{ii}^{J}| \leq 1
\]

(2.14)

Taking the zeroth order or spherical partial wave amplitude, noting simply \( a_0 \), where \( P_0 = 1 \),

\[
a_0 = \frac{1}{32\pi} \int_{-1}^{1} d(cos \theta) M
\]

(2.15)

Now, equation (2.14) translates as following for the zeroth partial wave amplitude \( a_0 \) in two-to-two scatterings,

\[
|\text{Re} a_0| \leq \frac{1}{2}
\]

(2.16)

This is the most practical form of the unitarity condition applied to the zeroth partial wave amplitude that I will use later to study specific models. To show the importance of unitarity in model building, in the next section, I present the formulation of the electroweak interaction in the light of unitarity, to arrive at the \( SU(2)_{\text{weak}} \times U(1)_{Y} \) description that became part of the Standard Model.

### 2.3 Formulation of the electroweak interaction

Perturbative unitarity played an important role in the first field theory formulation of the weak interaction that is now the \( SU(2)_{\text{weak}} \) part of the Standard Model. Starting from the effective
four-fermion interactions and studying processes that appeared to occur higher than one proba-
bility, came the heavy intermediate vector boson theory. With the introduction of vector boson
mediators, the weak interaction could get its group theory description with integrated in the
electromagnetic interactions, born the electroweak $SU(2)_{\text{weak}} \times U(1)_Y$ that is used in the Higgs
mechanism.

2.3.1 Four-fermion theory

After the postulation of the neutrino, Fermi proposed a four-fermion theory for the $\beta$-decay,
$n \rightarrow p e^− \bar{\nu}_e$, 
$$L_F = -\frac{G_F}{\sqrt{2}} (\bar{p} \gamma_\mu n) (\bar{e} \gamma^\mu \nu) \tag{2.17}$$
where $G_F = 1.17 \times 10^{-5} \text{ GeV}^{-2}$ is the Fermi constant.

After discovering more decays with relatively long lifetime, like the $\beta$-decay, the concept
of a new interaction emerged. The effective lagrangian for the weak interaction was similar to
the one of the $\beta$-decay in equation (2.17), in the current-current form.

$$L_{\text{eff}} = -\frac{G_F}{\sqrt{2}} J^{\dagger}_\mu J^\mu + \text{h.c.} \tag{2.18}$$

with weak current, $J^\mu$ written as vector-minus-axial (V-A) vector and separated into the lep-
tonic and hadronic currents, $J^\mu = J^{\mu}_l + J^{\mu}_h$.

The V-A lagrangian successfully describes the low energy weak interaction to the leading
order in $G_F$. However, it is not a self-consistent quantum field theory. This interaction is a
dimension-six operator, which is not renormalizable, as the coupling constant $G_F$ has dimen-
sion mass$^{-2}$. Closely related to renormalizability, there are processes that violates unitarity, as
the one $e\bar{\nu}_\mu \rightarrow \mu \bar{\nu}_e$, shown in 2.2a. Its high energy cross-section grows with the center of mass
energy $\sqrt{s}$ as

$$\sigma_{e\bar{\nu}_\mu \rightarrow \mu \bar{\nu}_e} \sim G_{F}^2 \sqrt{s} \tag{2.19}$$

Applying the unitarity bounds the limits on the center of mass energy is $\sqrt{s} \lesssim G_F^{-2} \sim
300 \text{ GeV}$. Before reaching this energy, some new mechanism or particles should appear that
provides a way to these processes to become unitary.

2.3.2 Intermediate vector boson theory

Similarly as in QED, introducing a new massive gauge field $W^\mu$, the effective weak interac-
tion lagrangian can be replaced by the following with renormalizable interactions, where the
Figure 2.2: The process $e\bar{\nu}_\mu \rightarrow \mu \bar{\nu}_e$ in 2.2a the four-fermion theory is the low energy approximation of the $g^2$ order diagram in 2.2b the intermediate vector boson theory with the identification $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W}$.

Figure 2.3: The process $\nu_e \bar{\nu}_e \rightarrow W^+W^-$ in 2.3a the intermediate vector boson theory that has an amplitude growing with the center of mass energy $\sqrt{s}$, violating unitarity. Assuming a new gauge boson in s-channel exchange in 2.3b makes the process unitary.

previous contact interaction is mediated by this new boson as shown in figure 2.3a.

$$\mathcal{L}_{\text{int}} = g \left( J_\mu W_\mu + \text{h.c.} \right) \quad (2.20)$$

The four-fermion interaction now is a low energy effective theory generated by $\mathcal{L}_{\text{int}}$ at $g^2$ order with the identification $\frac{G_F}{\sqrt{2}} = \frac{g^2}{8m_W}$. The lagrangian for the massive gauge boson is the following,

$$\mathcal{L}_W = -\frac{1}{4} W^{\mu\nu} W_{\mu\nu} + m_W^2 W_\mu^+ W^\mu \quad (2.21)$$

with $W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu$. Now even though the coupling $g$ is dimensionless, the problem of unitarity remains in other processes that contain longitudinally polarized $W^\mu$ bosons, as $\nu \bar{\nu} \rightarrow W^+W^-$, transferring the problem from the contact interaction to the massiveness of the gauge boson.

The energy dependence in the amplitude can be canceled with a new heavy lepton in a u-channel exchange diagram, or with a new neutral vector boson in an s-channel exchange diagram as in figure 2.3b [17]. After demanding the cancellation of all bad high energy behavior
\[ e^- + e^- \rightarrow W^+ W^- \]

Figure 2.4: The process \( e^- + e^- \rightarrow W^+ W^- \). Its amplitude in the intermediate vector boson model is growing with the center of mass energy \( s \). After the introduction of the new heavy gauge boson \( Z \), the leading order in \( \frac{m_W^2}{s} \) is canceled by the s-channel Z exchange graph, still leaving the second order \( \sim m_e \sqrt{s} \). Then after the electroweak symmetry breaking induced by the Higgs boson, which coupling to the electron is related to the electron mass, the s-channel Higgs exchange graph compensates the for the \( m_e \sqrt{s} \) growth, leaving the whole process unitary.

in all processes, one ends up with a renormalizable lagrangian with the choice of gauge group \( SU(2) \times U(1) \), containing four massless gauge bosons.

\[
\mathcal{L}_{EW} = -\frac{1}{4} F^\mu_\nu F^\mu_\nu - \frac{1}{4} A^{a \mu \nu} A^a_{\mu \nu} \tag{2.22}
\]

where \( F^\mu_\nu = \partial^\mu B^\nu - \partial^\nu B^\mu \) and \( A^{a \mu \nu} = \partial^\mu A^a_\nu - \partial^\nu A^a_\mu + g e^{abc} A^b_\mu A^c_\nu \) (\( a, b, c = 1, 2, 3 \)).

The introduction of the neutral weak current the new \( Z \) boson exchange graphs compensates the worst growing part in the amplitudes \( \mathcal{M} \sim s \), but it still leaves some process growing with \( \mathcal{M} \sim \sqrt{s} \). One of these, is the \( e^+ e^- \rightarrow W^+ W^- \) scattering shown in figure 2.4. Solving the problem of how the weak gauge bosons and fermions get their masses via the Higgs mechanism a new scalar, the Higgs boson is introduced. This adds a new s-channel graph shown in figure 2.4d with an amplitude that is also growing with the center of mass energy and the higgs-electron coupling that is proportional to the electron mass \( y_e \sim m_e \), the amplitude has the form \( \mathcal{M}_{e^+ e^- \rightarrow h \rightarrow W^+ W^- } \sim m_e \sqrt{s} \), and exactly cancels the remaining unitarity violating part of the other three graphs [6]. The four scattering amplitude is the following

\[
i\mathcal{M}_\gamma \left( e_{s_1} (p_1) e_{s_2}^+ (p_2) \rightarrow \gamma (q_s) \rightarrow W^- (k_3) W^+ (k_4) \right) = (-ie)^2 (\bar{v}_{s_2} (p_2) \gamma^\mu u_{s_1} (p_1)) \frac{-ig^\rho}{s} \]

\[
\cdot \left( (q_s + k_4)_\mu g_{\rho \nu} + (-k_4 + k_3)_\rho g_{\nu \mu} + (-k_3 - q_s)_\nu g_{\mu \rho} \right) \epsilon^\nu (k_3) \epsilon^{\nu \rho} (k_4) \tag{2.23}
\]
\[ i \mathcal{M}_{\nu_e} (e_{s_1}^- (p_1) e_{s_2}^+ (p_2) \rightarrow \nu_e (q_1) \rightarrow W^{-\mu} (k_3) W^{+\nu} (k_4)) = \]
\[ = \left( \frac{ig}{2\sqrt{2}} \right)^2 \left( \bar{v}_{s_2} (p_2) \gamma_{\nu} (1 - \gamma_5) \frac{i (q_1 + m_e)}{t - m_e^2} \gamma_{\mu} (1 - \gamma_5) u_{s_1} (p_1) \right) \epsilon^{\mu} (k_3) \epsilon^{*\nu} (k_4) \quad (2.24) \]

\[ i \mathcal{M}_Z (e_{s_1}^- (p_1) e_{s_2}^+ (p_2) \rightarrow Z (q_s) \rightarrow W^{-\mu} (k_3) W^{+\nu} (k_4)) = \]
\[ = \left( \frac{ig}{4 \cos \theta_w} \right)^2 \left( \bar{v}_{s_2} (p_2) \gamma_{\nu} ((-1 + 4 \sin^2 \theta_w) + \gamma_5) u_{s_1} (p_1) \right) \frac{-i}{s - m_Z^2} \left( g^{\mu\nu} - \frac{q_3^{\mu} q_3^{\nu}}{m_Z^2} \right) \]
\[ \cdot ((q_s + k_4)_\mu g_{\rho\nu} + (-k_3 + k_3)_\rho g_{\nu\mu} + (-k_3 - q_s)_\nu g_{\mu\rho}) \epsilon^{\mu} (k_3) \epsilon^{*\nu} (k_4) \quad (2.25) \]

\[ i \mathcal{M}_h (e_{s_1}^- (p_1) e_{s_2}^+ (p_2) \rightarrow h (q_s) \rightarrow W^{-\mu} (k_3) W^{+\nu} (k_4)) = \]
\[ = \left( -\frac{igm_e}{2m_W} \right) (i em_W) \left( \bar{v}_{s_2} (p_2) u_{s_1} (p_1) \right) \frac{-ig_{\mu\nu}}{s - m_h^2} \epsilon^{\mu} (k_3) \epsilon^{*\nu} (k_4) \quad (2.26) \]

For calculating the amplitudes in all fermion helicity channel, I use the $\pm$ helicities in the wave function factors as the follows

\[ u_{\pm} (p) = \sqrt{\frac{m}{p^0}} \left( \frac{\sqrt{p \cdot \sigma}}{\sqrt{p \cdot \bar{\sigma}}} \right) u_{\pm} (0) \]
\[ v_{\pm} (p) = \sqrt{\frac{m}{p^0}} \left( \frac{\sqrt{p \cdot \sigma}}{\sqrt{p \cdot \bar{\sigma}}} \right) v_{\pm} (0) \quad (2.27) \]

where $\sigma^\mu = (1, \sigma^\mu)$ is the four-vector of the Pauli matrices and $\bar{\sigma}^\mu = (1, -\sigma^\mu)$. The zero-momentum wave function factors are the following,

\[ u_{\pm} (0) = \frac{1}{\sqrt{2}} \begin{pmatrix} \xi_{\pm} \\ \xi_{\pm} \end{pmatrix} \]
\[ v_{\pm} (0) = \frac{1}{\sqrt{2}} \begin{pmatrix} \pm \xi_{\mp} \\ \mp \xi_{\mp} \end{pmatrix} \quad (2.28) \]

with the two component vectors are $\xi_{\pm} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and $\xi_{-} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$. In the scattering amplitude matrix the fermion helicities are in the following order,

\[ \mathcal{M} (s_1 s_2) : \begin{pmatrix} -- & ++ \\ +-- & +++ \end{pmatrix} \quad (2.29) \]

After calculating the scattering amplitude matrix, without the $Z$ boson in the intermediate
vector boson theory, the amplitude is growing with the energy square in the helicity reserving channels.

\[ M_{\gamma+\nu_e}(s_1s_2) = \left( \frac{g^2 \sin^2 \theta_w \sin \theta}{2m_W^2} s + \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 \right) \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^{-1} \left( \frac{m_W}{\sqrt{s}} \right)^0 \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 + \frac{g^2 \sin^2 \theta_w \sin \theta}{2m_W^2} s + \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 \right) \]

(2.30)

Applying the unitarity constraint for the partial wave expansion (2.16),

\[ a_{0,\gamma+\nu_e}(\pm) = \frac{g^2 \sin^2 \theta_w}{128m_W^2} s \leq \frac{1}{2} \] (2.31)

\[ \sqrt{s} \lesssim \frac{8m_W}{g \sin \theta_w} \approx 2.1 \text{ TeV} \] (2.32)

This shows that the intermediate vector boson description breaks down around 2 TeV energy and we can expect new physics to show up before reaching this limit. Then came the introduction of the neutral weak gauge boson, its mass is \( m_Z = 91.2 \text{ GeV} \), well below the above limit.

After introducing the neutral \( Z \) boson, the growth with the energy square cancels between the \( \gamma, \nu_e \) and \( Z \) exchange graphs, leaving an amplitude growing with the energy in the helicity changing channels.

\[ M_{\gamma+\nu_e+Z}(s_1s_2) = \left( \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 \right) \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 + \frac{g^2 \sin^2 \theta_w \sin \theta}{4m_W^2} m_e \sqrt{s} + \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)^0 \right) \] (2.33)

Again, applying the unitarity constraint,

\[ a_{0,\gamma+\nu_e+Z}(\pm) = \frac{g^2 m_e}{64\pi m_W^2} \sqrt{s} \leq \frac{1}{2} \] (2.34)

\[ \sqrt{s} \lesssim \frac{32\pi m_W^2}{g^2 m_e} \approx 3.2 \times 10^6 \text{ TeV} \] (2.35)

As the amplitude now only grows with the energy, the validity range increased. From the \( e^+e^- \rightarrow W^+W^- \) scattering this constraint is not strong to expect new particles soon, similar limits can be derived from the other fermion to \( W^\pm \) scatterings and as the amplitude is proportional to the fermion mass, the strongest bound will come from the \( t\bar{t} \rightarrow W^+W^- \) scattering, \( \sqrt{s} \lesssim \frac{32\pi m_t^2}{\sqrt{6}g^2 m_e} \approx 3.5 \text{ TeV} \) [18]. So the new mechanism that solves the problem of the particle masses should introduced below the TeV scale relying only fermion to \( W^\pm \) scatterings. The Higgs boson mass turned out to be \( m_h = 125 \text{ GeV} \) that is again well below the theoretical
Finally, taking into account the Higgs mechanism, the new Higgs exchange graph is growing with the energy, too, that compensates exactly the other three, making the whole process unitary.

\[
\mathcal{M}_h(s_1 s_2) = \begin{pmatrix}
0 & -\frac{g^2}{4m_W^2} m_e \sqrt{s} + \mathcal{O}\left(\frac{m_W}{\sqrt{s}}\right)^0 \\
-\frac{g^2}{4m_W^2} m_e \sqrt{s} + \mathcal{O}\left(\frac{m_W}{\sqrt{s}}\right)^0 & 0
\end{pmatrix} \quad (2.36)
\]

Similarly, all other previously unitarity violating processes turn out compensated by the new graphs containing the Higgs boson. As illustrated through the formulation of the electroweak interaction in the Standard Model, considering perturbative unitarity in studying new models is very instructive, not just in at which scale to expect new physics, but from the unitarity violating processes one can suggest what to look for exactly, too.
Chapter 3

A simplified model of dark matter

As mentioned in chapter 1, there is no particle or mechanism in the Standard Model that can explain the observed dark matter in the universe. If it has a particle origin, than the Standard Model needs to be extended. There are several features that one can consider. Is it thermally created, is the energy spectrum of dark matter particles like in thermal equilibrium? Is it cold or hot, meaning that when the expansion of the universe reached the point of the decoupling of dark matter particles, were they still relativistic or rather non-relativistic, $v_{\text{dm}} \approx \frac{c}{3}$? Non-thermally created dark matter candidate is the axion, that is the vector boson of a new spontaneously broken global symmetry. Thermally created hot dark matter can be a light active neutrino, that is coupled to the Standard Model, while thermally created cold dark matter for example the WIMP, weakly interacting massive particles.

There are several experiments going on to observe dark matter, illustrated in figure 3.1. There are direct searches for dark matter particles. These detectors are huge tanks of noble gases waiting for dark matter particles to pass by and recoil from the nuclei, like PICO [19], LUX [20] and XENON100 [21]. They address the question 'How does dark matter couples to the Standard Model particles?', especially to quarks. Indirect searches can be done with telescopes like Fermi-LAT [22], or neutrino experiments as IceCube [23], where the question is 'How does the dark matter annihilate?'. Finally, collider bounds from the LHC and earlier from the LEP [25] are important, too, where they can study 'How the dark matter is created?'. In figure 3.2 the various experimental constraints are illustrated for the mass of dark matter particles.

WIMP dark matter is still a favorite solution within particle physics. The relic density of dark matter from astrophysical observations, most recently from Planck [3], is $\Omega_{\text{dm}} h^2 = 0.12$, this is the density ratio of the dark matter to the critical matter density that would make our universe flat and $h$ is the dimensionless Hubble parameter. For a thermally created particle to give the correct relic density abundance, its annihilation cross section should be $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle \approx 10^{-26} \text{cm}^3\text{s}^{-1}$, where $v_{\text{rel}}$ is the velocity of the dark matter particles. This annihilation cross
(a) Direct detection: 'How does dark matter couple to the Standard Model?'

(b) Indirect detection: 'How dark matter annihilates into the Standard Model?'

(c) Collider experiments: 'How is dark matter created from the Standard Model?'

Figure 3.1: There are three main approaches to dark matter detection that address different questions about the properties of dark matter particles as illustrated in the figure.

Figure 3.2: Constraints on the mass of dark matter particles from various searches [26].
section is what is expected from a new particle with mass $\sim 100$ GeV that interacts via the weak interaction. This observation is known as the "WIMP miracle", making WIMP dark matter attractive in particle physics.

As chiral fermions are more constrained by electroweak precision tests of the Standard Model, vector-like fermions are popular candidates to explain new phenomena such as the dark matter. 'Vector-like' in the sense of that their left- and right-handed projections are in the same representation under the Standard Model gauge group, so their couplings are the same, allowing for a mass term in the lagrangian and they do not need only the Higgs mechanism to get their mass. They appear in various beyond the Standard Model theories such as supersymmetric models [4], composite Higgs models [5, 27] and little Higgs models [28, 29], extra dimensional scenarios [30]. Vector-like fermions help gauge coupling unification, but lower the unification scale, that is in conflict with proton decay constraints.

Extending the Standard Model with a sector of vector-like fermions is now a popular simplified model for dark matters. Here, I study one, where the new sector consists of a pair of weak doublet and a singlet lepton [31]. This can be thought of as the higgsino-bino part of supersymmetric theories, this neutralino sector contains the supersymmetric fermion partners of the neutral electroweak gauge bosons and the Higgs. Taking only the doublet is already excluded, due to the too large dark matter-$Z$ boson coupling, while including only the singlet that has no direct couplings to the Standard Model cannot be tested with experiments. Alongside with the new particles, a matter parity-like symmetry is introduced to avoid the direct couplings of the Standard Model fermions to the new sector, this will result that the lightest new fermion become stable and if electrically neutral, a good dark matter candidate. After introducing the model, I show the constraints coming from dark matter searches. These are constraining the parameterspace of the model as whole, not the individual parameters. To complement these constraints, I use perturbative unitarity considerations to explore the limits of the parameters separately where it is possible.

### 3.1 Doublet-singlet model of dark matter

Here, I introduce the doublet-singlet model of dark matter where the Standard Model is extended with a pair of $SU(2)_{\text{weak}}$ doublet Weyl-fermion, $\psi_1 = \begin{pmatrix} \psi_0^0 \\ \psi_1^- \end{pmatrix}$ and $\psi_2 = \begin{pmatrix} \psi_2^+ \\ \psi_0^0 \end{pmatrix}$, they can acquire a Dirac mass term together and a weak singlet Weyl-fermion $\chi^0$ with a Majorana mass term, all the new fields are color singlets. Their quantum numbers are listed in table 3.1 alongside with Higgs boson $h$. Also, the model assumes a parity-like $Z_2$ symmetry that prevents the new fermions to directly couple to the Standard Model fermions, only through loop-induced couplings by the gauge bosons and the Higgs. So the lagrangian separates into the
Table 3.1: The electroweak quantum numbers of the new fermions, the weak doublet $\psi_1$, $\psi_2$ and singlet $\chi^0$, and the Higgs boson $h$. The hypercharge is normalized as $Y = Q - T_3$.

<table>
<thead>
<tr>
<th></th>
<th>$\psi_1^0$</th>
<th>$\psi_1^-$</th>
<th>$\psi_2^+$</th>
<th>$\psi_2^0$</th>
<th>$\chi^0$</th>
<th>$h$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_3$</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>$0$</td>
<td>$-\frac{1}{2}$</td>
</tr>
<tr>
<td>$Q$</td>
<td>$0$</td>
<td>$-1$</td>
<td>$+1$</td>
<td>$0$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
<tr>
<td>$Y$</td>
<td>$-\frac{1}{2}$</td>
<td>$-\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$0$</td>
<td>$\frac{1}{2}$</td>
</tr>
</tbody>
</table>

already known Standard Model one and in addition, the lagrangian for the new doublet-singlet sector, $\mathcal{L} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{DS}}$. Where $\mathcal{L}_{\text{DS}}$ contains the kinetic and mass terms of the new fermions and their Yukawa-type coupling to the Higgs doublet. In the unitary gauge $H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix}$ where the Higgs VEV is $v = 246$ GeV.

$$\mathcal{L}_{\text{DS}} = \frac{i}{2} \left( \chi^0 \sigma^\mu \partial_\mu \chi^0 + \psi_1^0 \sigma^\mu D_\mu \psi_1 + \psi_2^0 \sigma^\mu D_\mu \psi_2 \right) - \left( m_d \psi_1 \psi_2 + \frac{1}{2} m_s \chi^0 \chi^0 + y_1 \psi_1 H \chi^0 + y_2 \psi_2 \tilde{H} \chi^0 + h.c. \right)$$ (3.1)

There are four new parameters, two of them are the dimensionful mass parameters $m_s$ and $m_d$. The other two are the dimensionless Yukawa couplings $y_1$ and $y_2$. Three of them can be chosen to be positive and real, the remaining one can have a CP violating phase [8], though here, I consider only real parameters. An other widely used parametrization, that makes the relation with the MSSM higgsino-bino sector more apparent,

$$y_1 = y \cos \beta, \quad y_2 = y \sin \beta$$ (3.2)

where $\beta$ translates directly to the $\beta$ mixing angle used in the MSSM and $y$ is related to the $U(1)_Y$ gauge coupling there as $y = \frac{g'}{\sqrt{2}}$.

The mass spectrum of the model is the following. The electrically charged parts of the doublets form a Dirac fermion, $\Psi^- = \begin{pmatrix} \psi_1^- \\ \psi_2^+ \end{pmatrix}$ with mass $m_d$. At energies above the electroweak symmetry breaking, the neutral parts of the doublets also form a Dirac fermion, $\Psi^0 = \begin{pmatrix} \psi_1^0 \\ \psi_2^0 \end{pmatrix}$ with mass $m_d$, while $\chi^0$ remains a Majorana fermion with mass $m_s$. Below electroweak symmetry breaking, due to the Yukawa couplings, a mixing occurs in the neutral sector between $\psi_1^0$, $\psi_2^0$ and $\chi^0$.

$$\mathcal{L}_{\text{DS}} \supset -\frac{1}{2} \begin{pmatrix} \chi^0 & \psi_1^0 & \psi_2^0 \end{pmatrix} M_n \begin{pmatrix} \chi^0 \\ \psi_1^0 \\ \psi_2^0 \end{pmatrix} + h.c.$$ (3.3)
with the neutral mass matrix
\[
M_n = \begin{pmatrix}
    m_s & \frac{y_1 v}{\sqrt{2}} & \frac{y_2 v}{\sqrt{2}} \\
    \frac{y_1 v}{\sqrt{2}} & 0 & m_d \\
    \frac{y_2 v}{\sqrt{2}} & m_d & 0
\end{pmatrix}
\] (3.4)

To find the masses and mass eigenstates, \( M_n \) should be diagonalized. That can be achieved by using the Takagi factorization of symmetric complex matrices [32], that is in the following form
\[
M_n^{\text{diag}} = \tilde{U} M_n \tilde{U}^T
\] (3.5)

where \( \tilde{U} \) is a unitary matrix. The resulting eigenstates can still be negative, the physical masses are the absolute values of the eigenstates. For the negative eigenstates, the corresponding Takagi vector can be multiplied by \( i \) then used to get directly the positive masses, I will denote this modified version of the matrix used for the diagonalization with \( U \). The masses in growing order \( m_1, m_2, m_3 \) and the corresponding mass eigenstates \( \chi_1, \chi_2, \chi_3 \) then follows,
\[
\begin{pmatrix}
    \chi_1 \\
    \chi_2 \\
    \chi_3
\end{pmatrix} = U \begin{pmatrix}
    \chi_0 \\
    \psi_0^1 \\
    \psi_0^2
\end{pmatrix}
\] (3.6)

Else, another way to find the mass eigenstates is to solve the corresponding characteristic equation which is the following,
\[
(m_s - \lambda) (\lambda^2 - m_d^2) + m_d y_1 y_2 v^2 + \lambda \left( \frac{y_1^2 + y_2^2}{2} \right) v^2 = 0
\] (3.7)
\[
(m_s - \lambda) (\lambda^2 - m_d^2) + \frac{y^2 v^2}{2} (m_d \sin(2\beta) + \lambda) = 0
\] (3.8)

Solving this third order equation is complicated analytically beside some special cases when \( |y_1| = |y_2| \) or \( |m_\chi| = m_d \). Surprisingly these are the preferred regions from dark matter coupling measurements.

In general, the spectrum will contain three Majorana fermion \( \chi_1, \chi_2 \) and \( \chi_3 \). The lightest particle from the neutral sector \( \chi = \chi_1 \) with mass \( m_\chi = m_{\chi_1} \) will be stable due to the introduced \( \mathbb{Z}_2 \) matter parity and becomes an ideal dark matter candidate, if it is lighter than the charged doublet \( m_\chi < m_d \).

\[
\chi = U_{11} \chi_0 + U_{12} \psi_0^1 + U_{13} \psi_0^2,
\]
\[
|U_{11}|^2 + |U_{12}|^2 + |U_{13}|^2 = 1
\] (3.9)

\( U_{11}^2 \) characterizes the amount of the singlet in \( \chi \). When \( |U_{11}|^2 > 0.5 \) the dark matter is more singlet- or bino-like, consequently when \( |U_{11}|^2 < 0.5 \), the dark matter is more doublet- or
higgsino-like.

For the dark matter searches, the important couplings are the dark matter coupling to the $Z$, $c_{Z\chi\chi}$ and Higgs boson, $c_{h\chi\chi}$. The coupling to the Higgs is related to the mass, $c_{h\chi\chi} = \frac{\partial m_x(v)}{\partial v}$, that can be determined by differentiating the the characteristic equation (3.7) with respect to $v$.

$$c_{h\chi\chi} = \frac{(2y_1y_2m_d + (y_1^2 + y_2^2) m_\chi) v}{m_d^2 + (y_1^2 + y_2^2) \frac{v^2}{2} + 2m_s m_\chi - 3m_\chi^2} \quad (3.10)$$

The $Z$ coupling can be also expressed from the characteristic equation (3.7) with the parameters and the mass $m_\chi$,

$$c_{Z\chi\chi} = \frac{m_z v (y_1^2 - y_2^2) \left(m_\chi^2 - m_d^2\right)}{2 \left(m_\chi^2 - m_d^2\right)^2 + v^2 \left(4y_1y_2m_\chi m_d + (y_1^2 + y_2^2) \left(m_\chi^2 + m_d^2\right)\right)} \quad (3.11)$$

In the following, I collect the constraints of the parameter space that comes from the dark matter searches through the couplings of the dark matter candidate $\chi$.

### 3.2 Dark matter constraints

As the presence of dark matter is so evident and is absent from the Standard Model, model building toward integrating dark matter is quite popular in the recent years. This led to renewed interest in studying simplified models of dark matter that explores their parameter space which is still left viable from dark matter searches [7, 33–35]. Here, I collect the constraints on the doublet-singlet model based on [36].

From the recent astrophysical maps, the relic density abundance of dark matter in our universe is $\Omega_{dm}h^2 \simeq 0.12$. In the case of thermally created cold dark matter, one can solve the Boltzman equation to calculate the relic abundance and find that is primarily determined by its annihilation cross section [37, 38].

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$$\Omega_{dm}h^2 = 0.3 \left(\frac{x_f}{10}\right) \left(\frac{g_s(m)}{100}\right)^{\frac{1}{2}} \frac{10^{-29} \text{cm}^3\text{s}^{-1}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle} \quad (3.12)$$

where $x = \frac{m}{T}$ is a dimensionless parameter taken at the freeze-out temperature of dark matter $T_f$, that is where the expansion rate of the universe exceeds the annihilation rate of dark matter particles and its abundance is freeze-out to fix value, here $x_f \approx 10 \ldots 20$. The relative velocity of the dark matter particles is $v_{\text{rel}}$ and $g_s$ is the effective number of degrees of freedom.

$$\Omega_{dm}h^2 \simeq \frac{3 \times 10^{-27} \text{cm}^3\text{s}^{-1}}{\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle} \quad (3.13)$$
When the dark matter has only one component, it should create all the relic density. However, dark matter can be made up by several components, too, then one component creates a fraction of the total relic density then it is underabundant and its annihilation cross section can be larger. While the dark matter candidate can create more than the observed relic density then it is called overabundant and its annihilation cross section is too small. In the case when the relic abundance exceeds the observed one, that scenario is excluded.

The direct detection experiments can measure spin-independent (SI) coupling at leading order to the Higgs boson and spin-dependent (SD) couplings to the boson, these are defined by the processes illustrated on figure 3.3. So the absence of direct detection of dark matter constrains these couplings, $c_{h\chi\chi}$ and $c_{Z\chi\chi}$.

$$c_{h\chi\chi} = - \frac{(\sin(2\beta)m_d + m_\chi) y^2 v}{m_d^2 + \frac{1}{2} y^2 v^2 + 2 m_s m_\chi - 3 m_\chi^2}, \quad (3.14)$$

$$c_{Z\chi\chi} = - \frac{m_z y^2 v \cos(2\beta) (m_\chi^2 - m_d^2)}{2 (m_\chi^2 - m_d^2)^2 + y^2 v^2 (2 \sin(2\beta) m_\chi m_d + (m_\chi^2 + m_d^2))}, \quad (3.15)$$

The points of the parameter space where one or both of the two couplings are vanishing, either $c_{h\chi\chi} = 0$ or $c_{Z\chi\chi} = 0$, are called ‘blind spots’ [39]. The blind spots escape the direct detection bounds. On the other hand, if these couplings are small or vanish, the annihilation cross section becomes too small and needs enhancement to avoid overabundance. It can be achieved with a pole in the s-channel, where $m_\chi \approx \frac{m_Z}{2}$ or $\frac{m_H}{2}$ [7].

The following plots from [36] illustrate the dark matter constraints from direct and indirect searches in larger mass region $m_\chi > 100$ GeV. The Yukawa couplings scan over a range from small values that give the same bounds as the representative $y \approx 0.01$ on figure 3.4, through MSSM-like $y \approx 0.2$ on figure 3.5 that is a typical value in the supersymmetric model, up to larger values at the perturbative limit $y \approx 1$ on figure 3.6.

When the Yukawa couplings are small $y \lesssim 0.01$ shown on figure 3.4, the only bound comes from the observed relic density abundance of dark matter, excluding the more doublet-like dark
Figure 3.4: The dark matter constraints on the doublet-singlet model with $y = 1$ Yukawa coupling [36]. The red region is excluded by Fermi-LAT [22] from $\langle\sigma v_{\text{rel}}\rangle_{WW}$. The red line shows the dark matter candidate $\chi$ gives the full dark matter relic density abundance $\Omega_{\text{dm}} h^2 = 0.12$ and the regions where it is over- or underabundant are indicated.

The viable masses for $\chi$ are $m_\chi \gtrsim 280$ GeV for any $\tan \beta$ with $U_{11}^2 \gtrsim 0.5$, where $U_{11}^2$ indicates how much of $\chi$ is coming from the singlet $\chi^0$ as written in equation (3.9). In the Yukawa couplings region preferred by supersymmetric models $y = 0.2$, the mass is allowed from $m_\chi \gtrsim 220$ GeV, again for more singlet-like dark matter with $U_{11}^2 \gtrsim 0.65$. While in the $m_s \approx m_d$ region where the thermal $\chi$ can give the full relic abundance, the direct detection experiments become relevant as LUX [20] excludes the mass up to $m_\chi \gtrsim 1$ TeV. In the largest Yukawa region $y = 1$, even larger part of the parameter space is excluded. The allowed dark matter mass is $m_\chi \gtrsim 275$ GeV, unless it is purely singlet, $U_{11}^2 \gtrsim 0.8$.

In the lower mass region, the allowed regions are around the blind-spots where the vanishing Higgs and $Z$ couplings help escape the direct detection constraints and leave $\chi$ to be more singlet-like. Although here the collider bounds especially the invisible Higgs and $Z$ decays become more important. In the smaller Yukawa coupling range $y \approx 0.1$, 80 GeV $\lesssim m_\chi \lesssim 220$ GeV can be excluded for $U_{11}^2 \lesssim 0.65$ and no further constraints can drawn on the larger Yukawa region $y = 1$.

Overall, dark matter searches favor the regions where the dark matter particle is either higgsino- or bino-like. Its mass is close to $m_s$ or $m_d$ and its couplings to the Higgs and $Z$ bosons are small. In these special cases, solving the characteristic equation (3.7) for the masses and couplings in the neutral sector simplifies. In following section, I present an analytical study around these special points.
Figure 3.5: The dark matter constraints on the doublet-singlet model with MSSM-like Yukawas, \( y = 0.2 \) and different ratios on 3.5a with \( \tan \beta = -2 \), on 3.5b with \( \tan \beta = 2 \), on 3.5c with \( \tan \beta = -20 \) and on 3.5d with \( \tan \beta = 20 \) [36]. The red region is excluded by \( \langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{WW} \) from Fermi-LAT [22]. The red line shows the full relic density abundance \( \Omega_{\text{dm}} h^2 = 0.12 \) and indicates the over- and underabundant regions. The blue region is excluded by spin-independent direct detection \( \sigma_{\text{SI}} \) by LUX [20] and the magenta by spin-dependent detection \( \sigma_{\text{SD}} \) at IceCube [23].
Figure 3.6: The dark matter constraints on the doublet-singlet model with large Yukawa couplings $y = 1$ and different ratios, on 3.6a with $\tan \beta = -2$, on 3.6b with $\tan \beta = 2$, on 3.6c with $\tan \beta = -20$ and on 3.6d with $\tan \beta = 20$ [36]. The exclusion from Fermi-LAT [22] is the red by $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{WW}$ and the green region by $\langle \sigma_{\text{ann}} v_{\text{rel}} \rangle_{t\bar{t}}$. By direct detection the blue region is excluded from spin-independent $\sigma_{\text{SI}}^N \chi$ measurement [20] and magenta by spin-dependent $\sigma_{\text{SD}}^{p\chi}$ measurement [23], the green region is also by spin-dependent $\sigma_{\text{SD}}^{p\chi}$ measurement at PICO [19].
Figure 3.7: The dark matter constraints in the low mass region of the doublet-singlet model with Yukawa couplings favored by supersymmetric models, $y = 0.2$, with two mixing angle $\tan \beta = 2, -20$ [36]. The gray region is excluded by direct detection experiments, the purple region by indirect detection experiments. The blue region is excluded by invisible Higgs decay. It is clearly seen that the underabundant regions are all excluded except the blind spot regions around $m_\chi \approx m_h/2$ and $m_Z/2$.

### 3.3 Regions of small couplings

The direct detection experiments that measure how dark matter couples to the nucleons did not capture any dark matter, yet. That constrains severely the dark matter couplings to the Higgs in equation (3.14) and $Z$ boson in equation (3.15), $c_{h\chi\chi} \lesssim 0.01 \ldots 0.1$ and $c_{Z\chi\chi} \lesssim 0.01 \ldots 0.1$ depending on the dark matter mass [39]. These are the small regions around the blind spots where the two coupling vanish. So this region is experimentally favored and also possible to study analytically.

From the form of the Higgs and $Z$ couplings in equation (3.14) and (3.15), the conditions of vanishing couplings are the following,

$$c_{Z\chi\chi} = 0, \text{ if } |y_1| = |y_2| \text{ or } |m_\chi| = m_d$$

$$c_{h\chi\chi} = 0, \text{ if } m_\chi + \sin(2\beta)m_d = 0$$

where $\sin(2\beta) = \frac{2y_1y_2}{y_1^2 + y_2^2}$ and $m_\chi$ is the smallest eigenvalue of the characteristic equation (3.7). When $m_\chi = \pm m_d$ then the other condition in equation (3.16) is also holds $y_1 = \mp y_2$ or $\tan \beta = \mp 1$, so it is enough to study the environment of $y_1 = \pm y_2$. If the Higgs coupling vanishes, from the characteristic equation (3.7) it follows that the corresponding eigenvalue happens to be one of the initial mass parameters, $m_\chi = m_d$ or $m_s$. In the case of both blind spots, solving the characteristic equation (3.7) simplifies a lot by knowing one of the mass parameters.
Figure 3.8: The mass eigenvalues (that can be negative) and the positive masses for the $y_2 = -y_1$ case. Illustrating both of the two possible hierarchy between the mass parameters $m_s$ and $m_d$. With $m_d = 500$ GeV, on 3.8a $m_d$ is larger with $m_s = 300$ GeV, on 3.8b $m_s$ is larger with $m_s = 800$ GeV.

eigenvalue is either $m_d$ or $m_s$. I perform the expansion around these special points.

### 3.3.1 Small dark matter-$Z$ coupling with $\tan \beta \simeq -1$

In the first case, at the blind spot of $c_{Z\chi\chi} = 0$, the two Yukawa couplings have different sign, but same absolute value, $y_2 = -y_1$. The characteristic equation (3.7) simplifies to the following second order equation,

$$(\lambda - m_d) (m_s - \lambda) (\lambda + m_d) + y_1^2 v^2 = 0 \quad (3.18)$$

In this special case, the spectrum is shown in figure 3.8 and the eigenvalues are the following,

$$m_1 = m_d$$

$$m_{2,3} = \frac{m_s - m_d}{2} \pm \frac{1}{2} \sqrt{(m_s + m_d)^2 + 2y^2 v^2} \quad (3.19)$$

The mass eigenvalues expanded around the $c_{Z\chi\chi} = 0$ blind spot with the expansion parameter $\frac{m_d(y_1 \pm y_2)^2 v^2}{(2m_s + 6m_d + 3y^2 v^2)^2}$ are the following,

$$m_1 = m_d(1 - x_-)$$

$$m_{2,3} = \frac{m_s - m_d}{2} \pm \frac{1}{2} \sqrt{(m_s + m_d)^2 + 2y^2 v^2} + \frac{m_d x_-}{2} \left(1 \mp \frac{m_s - 3m_d}{\sqrt{(m_s + m_d)^2 + 2y^2 v^2}}\right) \quad (3.20)$$

where the corrections are proportional to $x_-,$

$$x_- = \frac{y^2 (1 + \sin(2\beta)) v^2}{4m_d(m_s - m_d) + y^2 v^2} \quad (3.21)$$

25
For $m_s < m_d$ and $y < \frac{2}{\sqrt{m_d}}\sqrt{m_d(m_d - m_s)}$, the dark matter candidate is the one with mass $m_2$ and its coupling are to the Higgs and $Z$ bosons are the following,

$$c_{h\chi\chi} = \frac{8y^2 v}{m_s + m_d} = \frac{8(y_1^2 + y_2^2)v}{m_s + m_d}$$

$$c_{Z\chi\chi} = \frac{y^2 v \cos(2\beta)m_Z}{2(m_s^2 - m_d^2)} = \frac{(y_1^2 - y_2^2)v m_Z}{2(m_s^2 - m_d^2)}$$

(3.22)

where we can see that the coupling to the $Z$ scales accordingly to the blind spot condition, that is with the small $y_1 + y_2$, but for the small Higgs coupling, small overall Yukawa is required compared to the relatively large mass parameters, $y \frac{v}{m_s + m_d} \ll 1$.

When the Yukawa coupling becomes large enough, $y > \frac{2}{v}\sqrt{m_d(m_d + m_s)}$ or $m_d < m_s$, then $m_1 = m_d$ will be the smallest mass and the corresponding eigenstate is the mostly doublet-like dark matter candidate. Its couplings are the following,

$$c_{h\chi\chi} = \frac{(1 + \sin(2\beta))y^2 v m_d}{2m_d(m_d - m_s) - y^2 v^2} = \frac{(y_1 + y_2)^2v m_d}{2m_d(m_d - m_s) - 3(y_1^2 + y_2^2)v^2}$$

$$c_{Z\chi\chi} = \frac{\cos(2\beta)y^2 v m_Z}{4m_d(m_s - m_d) + 2y^2 v^2} = \frac{(y_1^2 - y_2^2)v m_Z}{4m_d(m_s - m_d) + 2(y_1^2 + y_2^2)v^2}$$

(3.23)

Both couplings go to zero now with $y_1 + y_2$, making this region of the parameter space the most favored one, even for larger Yukawa couplings, as they are tuned $|y_1 + y_2| \simeq 0$.

When the two mass parameters are equal $m_s = m_d$, there is a smooth limit between the previous two cases with the couplings,

$$c_{h\chi\chi} = \frac{(1 + \sin(2\beta))m_d}{v} = \frac{(y_1 + y_2)^2m_d}{(y_1^2 + y_2^2)v}$$

$$c_{Z\chi\chi} = \frac{\cos(2\beta)m_Z}{2v^2} = \frac{(y_1^2 - y_2^2)m_Z}{2(y_1^2 + y_2^2)v^2}$$

(3.24)

where again, both coupling goes to zero with $y_1 + y_2$.

### 3.3.2 Small dark matter-$Z$ coupling with $\tan \beta \simeq 1$

When the two Yukawa couplings are equal $y_1 = y_2$, the $Z$ coupling vanishes again, too. The characteristic equation (3.7) simplifies to a second order equation, here too.

$$(\lambda + m_d) \left[ (m_s - \lambda)(\lambda - m_d) + y_1^2 v^2 \right] = 0$$

(3.25)
Figure 3.9: The mass eigenvalues (that can be negative) and the positive masses for the $y_2 = y_1$ case. Illustrating both of the two possible hierarchy between the mass parameters $m_s$ and $m_d$.

With $m_d = 500 \text{ GeV}$, on 3.8a $m_d$ is larger with $m_s = 300 \text{ GeV}$, on 3.8b $m_s$ is larger with $m_s = 800 \text{ GeV}$.

The solution in this special case is the following mass eigenvalues shown in figure 3.9,

\[
m_1 = -m_d \\
m_{2,3} = \frac{m_s + m_d}{2} \pm \frac{1}{2} \sqrt{(m_s - m_d)^2 + 2y^2v^2} \tag{3.26}
\]

Now expanding around the blind spot $c_{Z\chi\chi} = 0$ with the previous expansion parameter $\frac{m_d(y_1 \mp y_2)v^2}{(2m_s^2 + 6m_d^2 + 3y^2v^2)^{1/2}}$, the mass eigenstates are the following,

\[
m_1 = -m_d(1 + x_+) \\
m_{2,3} = \frac{m_s + m_d}{2} \pm \frac{1}{2} \sqrt{(m_s - m_d)^2 + 2y^2v^2} + \frac{m_dx_+}{2} \left( 1 \mp \frac{m_s + 3m_d}{\sqrt{(m_s - m_d)^2 + 2y^2v^2}} \right) \tag{3.27}
\]

where the corrections are proportional to $x_+$,

\[
x_+ = \frac{(1 - \sin(2\beta))y^2v^2}{4m_d(m_s + m_d) + y^2v^2} \tag{3.28}
\]

The lightest fermion that will be the dark matter candidate has mass $m_3$. With no Yukawa couplings, $m_3$ coincides with $m_s$ or $m_d$ and decrease from there as the Yukawa couplings grow. The leading behavior of the Higgs coupling for $m_s \neq m_d$,

\[
e_{hXX} = \frac{y^2v}{|m_d - m_s|} = \frac{(y_1^2 + y_2^2)v}{|m_d - m_s|} \tag{3.29}
\]

That is small only when the Yukawa mass correction $\frac{yv}{\sqrt{2}}$ is small compared to the doublet-singlet mass splitting $|m_s - m_d|$.
The $Z$ coupling for $m_s < m_d$,

$$c_{Z\chi\chi} = \frac{\cos(2\beta)y^2vm_Z}{2(m_d^2 - m_s^2)} = \frac{(y_1^2 - y_2^2)vm_Z}{2(m_d^2 - m_s^2)} \quad (3.30)$$

and for $m_s > m_d$,

$$c_{Z\chi\chi} = \frac{\cos(2\beta)y^2vm_Z}{2m_d(m_s - m_d)} = \frac{(y_1^2 - y_2^2)vm_Z}{2m_d(m_s - m_d)} \quad (3.31)$$

In both case, the $Z$ coupling goes to zero according to the blind spot condition with the small $y_1 - y_2$.

When the two mass parameters are equal $m_s = m_d$ the couplings are the following,

$$c_{h\chi\chi} = -y_1 \cos \beta = -y_1$$

$$c_{Z\chi\chi} = \frac{y^2 \cos(2\beta)m_Z}{16(2m_d - y_1 v)} = \frac{(y_1^2 - y_2^2)m_Z}{2m_d - y_1 v} \quad (3.32)$$

Where the $Z$ coupling goes to zero with $y_1 - y_2$, but the Higgs coupling only small for small Yukawa coupling $y_1$.

These were the blind spot regions of the vanishing dark matter-$Z$ coupling $c_{Z\chi\chi} = 0$. In the following, I analyze the blind spot regions around the vanishing dark matter-Higgs coupling.

### 3.3.3 Small dark matter-Higgs coupling with $\sin(2\beta) \sim -\frac{m_\chi}{m_d}$

To achieve a vanishing dark matter coupling to the Higgs $c_{h\chi\chi} = 0$, either $m_\chi = \pm m_d$ is needed that lead to $|\tan \beta| = 1$ which was discussed in the previous analyzes of the $Z$ coupling blind spot or $m_\chi = m_s$ this second case will be discussed here.

The mass spectrum is shown in figure 3.10 and expanded around the blind spot with the small parameter

$$\left(\frac{(m_s^2 + m_d m_\chi y_2)v^2}{2m_d^2 + 6m_d^2 + 3y_2^2 v^2}\right)^z$$

the masses are the following,

$$m_1 = m_s(1 - z)$$

$$m_{2,3} = \pm \sqrt{m_d^2 + \frac{y^2 v^2}{2}} - z \left(\frac{m_s^2}{\sqrt{m_d^2 + \frac{y^2 v^2}{2}}} \mp m_s\right) \quad (3.33)$$

where the corrections are proportional to $z$,

$$z = \frac{m_d \sin(2\beta) y^2 v^2 + y^2 v^2}{2m_d^2 - 2m_s^2 + y^2 v^2} \quad (3.34)$$
Figure 3.10: The mass eigenvalues (that can be negative) and the positive masses for the $m_\chi = m_s$ and the other two masses are degenerate. Illustrating both of the two possible hierarchy between the mass parameters $m_s$ and $m_d$. With $m_d = 500$ GeV, on 3.10a $m_d$ is larger with $m_s = 300$ GeV, on 3.10b $m_s$ is larger with $m_s = 800$ GeV.

The couplings of the dark matter candidate eigenstate corresponding to $m_1$ is the following,

$$
\begin{align*}
    c_{h\chi\chi} &= 4z \frac{m_s}{v} \\
    c_{Z\chi\chi} &= -\cos(2\beta)y^2 v m_Z \\
    &= \frac{4m_d (m_d - m_s)}{v^2 (m_d - m_s) + 2y^2 v^2}
\end{align*}
$$

(3.35)

The Higgs coupling is small, since it is proportional to the small parameter $z$, while to achieve small $Z$ coupling, it needs additional tuning for small overall Yukawa coupling.

As the phenomenologically important regions are close to a few special blind spots in the parameter space where the dark matter couplings are small, analytical study of these sectors was possible. Now I will look at further constraints on the model considering a field theoretical approach coming from perturbative unitarity conditions.

### 3.4 Perturbative unitarity constraints

As we see in section 2.3, investigating two-particle scattering and applying the perturbative unitarity conditions on their scattering amplitudes can lead to non-trivial constraints on the model in question. So in this section, I study two-to-two scatterings in the doublet-singlet model. In the Standard Model formulation, processes with massive gauge bosons gave the most usable limits, since their longitudinal polarization is proportional with the momenta and may result in amplitudes that growing with the energy.

In the doublet-singlet model, there are four new tree-level scatterings that can be relevant. The ones that involve heavy gauge boson are the $\Psi^-\Psi^+ \rightarrow W^-W^+$ and $\Psi^-\chi_i \rightarrow W^-h$ scatterings. Also, the new fermion scatterings that contain the new parameters can result in limiting those parameters, as the $\Psi^-\Psi^+ \rightarrow \Psi^-\Psi^+$ and $\chi_i\chi_i \rightarrow \chi_i\chi_i$ scatterings.

The electroweak couplings of the new fermions which are used in the scatterings are the
\[ \Psi^+ \rightarrow \gamma^\mu = ie\gamma^\mu \] (3.36)

\[ \Psi^- \rightarrow Z^\mu = \frac{ig}{\cos \theta_w} \left( \sin^2 \theta_w - \frac{1}{2} \right) \gamma^\mu \] (3.37)

\[ \Psi^- \rightarrow W^{+\mu} = \frac{ig}{\sqrt{2}} \gamma^\mu \] (3.38)

\[ \Psi^0 \rightarrow \chi^0 \rightarrow h = \frac{i}{2\sqrt{2}} (y_+ + y_-\gamma_5) \] (3.39)

where \( y_{\pm} = y_1 \pm y_2 \).

For the calculation of the scattering amplitudes, I wrote a Mathematica program, that matches the Lorentz indices and then calculates the amplitude for any fermion helicity. It is included in appendix A.

### 3.4.1 \( \Psi^-\Psi^+ \rightarrow W^-W^+ \) scattering

The most promising process is the charged fermion scattering to longitudinally polarized \( W^\pm \) bosons. In the Standard Model, the related process for example is the \( e^-e^+ \rightarrow W^-W^+ \) scattering which has four exchange channels, see in 2.3.2 on figure 2.4. With the lastly added crucial s-channel Higgs exchange that compensates for the energy dependence coming from the s-channel \( Z, \gamma \) and t-channel fermion exchange, that makes the process unitary. In the doublet-singlet model, however, the charged fermions have no couplings to the Higgs, leaving only three contributing graphs, shown in figure 3.11.
Figure 3.11: Feynman graphs for $\Psi^-\Psi^+ \to W^-W^+$ scattering, that compared to the SM process $e^-e^- \to W^-W^+$, lacks the Higgs exchange graph that makes the SM process unitary. In this case, since the vector-like fermion couplings have no axial couplings with $\gamma_5$, leaving the three graphs together with a constant amplitude.

The three scattering amplitudes are the following,

$$i\mathcal{M}_{Z(s)} (\Psi^-_{s_1}(p_1)\Psi^+_{s_2}(p_2) \to Z(q_s) \to W^{-\mu}(k_3)W^{+\nu}(k_4)) =$$

$$= \frac{ig}{\cos \theta_w} \left( \sin^2 \theta_w - \frac{1}{2} \right) (\bar{v}_{s_2}(p_2)\gamma^\sigma u_{s_1}(p_1)) \frac{-i}{s-m_Z^2} \left( g^\sigma \rho - \frac{q_\rho q_s}{m_Z^2} \right)$$

$$\cdot \left( (q_s + k_4)_\mu g_{\rho\nu} + (-k_4 + k_3)_\rho g_{\mu\nu} + (-k_3 - q_s)_\nu g_{\mu\rho} \right) \epsilon^\mu(k_3)\epsilon^{*\nu}(k_4) \quad (3.40)$$

$$i\mathcal{M}_{\gamma(s)} (\Psi^-_{s_1}(p_1)\Psi^+_{s_2}(p_2) \to \gamma(q_s) \to W^{-\mu}(k_3)W^{+\nu}(k_4)) =$$

$$= (-ig \sin \theta_w)^2 (\bar{v}_{s_2}(p_2)\gamma^\sigma u_{s_1}(p_1)) \frac{-ig^\sigma}{s}$$

$$\cdot \left( (q_s + k_4)_\mu g_{\rho\nu} + (-k_4 + k_3)_\rho g_{\mu\nu} + (-k_3 - q_s)_\nu g_{\mu\rho} \right) \epsilon^\mu(k_3)\epsilon^{*\nu}(k_4) \quad (3.41)$$

$$i\mathcal{M}_{\Psi^0(t)} (\Psi^-_{s_1}(p_1)\Psi^+_{s_2}(p_2) \to \Psi^0(q_t) \to W^{-\mu}(k_3)W^{+\nu}(k_4)) =$$

$$= \left( \frac{ig}{\sqrt{2}} \right)^2 (\bar{v}_{s_2}(p_2)\gamma^\nu \frac{i(q_t + m_d)}{t-m_d^2} \gamma^\mu u_{s_1}(p_1)) \epsilon^\mu(k_3)\epsilon^{*\nu}(k_4) \quad (3.42)$$

In the t-channel fermion exchange, all intermediate states should be summed up, that are the three neutral mass eigenstates. Nevertheless, changing the intermediate states with a unitary transformation and summing over the new transformed states gives the same result. That said, I use the weak eigenstates as intermediate states where only the doublet part $\Psi^0$ has coupling to $W^\pm\Psi^-$ and only one diagram contributes in this way.

I calculated the total amplitude with all fermion helicities, that are defined in equation (2.27). The helicity channels are in the following order in the amplitude matrix,

$$\mathcal{M}_{s_1s_2} : \begin{pmatrix} - & - & + \\ + & + & + \end{pmatrix} \quad (3.43)$$
Figure 3.12: Feynman graphs for the $\Psi^- \chi_i \rightarrow W^- h$ scattering.

The total amplitude matrix then follows,

$$M_{s_1 s_2} = \begin{pmatrix}
\left( \frac{g^2 (1 - \tan^2 \theta_w) \sin \theta}{4} + O \left( \frac{m_W^2}{s} \right) \right) & O \left( \frac{m_W}{\sqrt{s}} \right) \\
O \left( \frac{m_W}{\sqrt{s}} \right) & \left( \frac{-g^2 (1 - \tan^2 \theta_w) \sin \theta}{4} + O \left( \frac{m_W^2}{s} \right) \right)
\end{pmatrix}$$  (3.44)

As the amplitude matrix elements are constants at leading order, integrating out the scattering angle to get the zeroth partial wave amplitude, $a_0$ will result in constants again. That will be proportional to the gauge coupling that satisfies the unitarity condition (2.16).

### 3.4.2 $\Psi^- \chi_i \rightarrow W^- h$ scattering

Another interesting process is the $\Psi^- \chi_i \rightarrow W^- h$ scattering that is taken naively has a t-channel process with an incoming $\chi_0$ and an s-channel one with an incoming $\Psi^0$, illustrated in figure 3.12.

The two related scattering amplitude are the following,

$$iM_{\Psi^0(t)} \left( \Psi^-_{s_1}(p_1) \chi_{s_2}(p_2) \rightarrow \Psi^0(q_t) \rightarrow W^-\mu(k_3) h(k_4) \right) =$$

$$= \left( \frac{ig}{\sqrt{2}} \right) \left( \frac{i}{2\sqrt{2}} \right) \left( \bar{v}_{s_2}(p_2)(y_+ + y_\gamma_5) \frac{i(q_t + m_d)}{l - m_d^2} \gamma_\mu u_{s_1}(p_1) \right) \epsilon^\mu(k_3)$$  (3.45)

$$iM_{W^- (s)} \left( \Psi^-_{s_1}(p_1) \Psi^0_{s_2}(p_2) \rightarrow W^-(q_s) \rightarrow W^-\mu(k_3) h(k_4) \right) =$$

$$= \left( \frac{ig}{\sqrt{2}} \right) \left( igm_W \right) \left( \bar{v}_{s_2}(p_2) \gamma_\mu u_{s_1}(p_1) \right) \frac{-i}{s - m_W^2} \left( g^{\rho\sigma} - \frac{q_\rho q_\sigma}{m_W^2} \right) g_{\rho\sigma} \epsilon^\mu(k_3)$$  (3.46)

The amplitude matrices in the same helicity bases as in equation (3.43),

$$M_{s_1 s_2} \left( \Psi^- \chi_0 \rightarrow W^- h \right) = \begin{pmatrix}
0 & \frac{gq_\mu \sqrt{s}}{2m_W} + O \left( \frac{m_W}{\sqrt{s}} \right) \\
-\frac{gq_\mu \sqrt{s}}{2m_W} + O \left( \frac{m_W}{\sqrt{s}} \right) & 0
\end{pmatrix}$$  (3.47)
Figure 3.13: Feynman graphs for $\Psi^- \Psi^+ \rightarrow \Psi^- \Psi^+$ scattering.

$$
M_{s_1s_2} (\Psi^- \Psi^0 \rightarrow W^- h) = \left( \begin{array}{cc}
-\frac{g^2 \sin \theta_w}{2\sqrt{2}} & 0 \\
0 & \frac{g^2 \sin \theta_w}{2\sqrt{2}}
\end{array} \right) + \mathcal{O} \left( \frac{m_W}{\sqrt{s}} \right)
$$

While the the $\Psi^- \Psi^0 \rightarrow W^- h$ process has a constant amplitude at leading order, the amplitude of $\Psi^- \chi^0 \rightarrow W^- h$ is growing with the energy $\sqrt{s}$. When correctly calculating with the mass eigenstates $\chi_i$, then its couplings contain $\gamma_5$ matrix that makes the two amplitude canceling each other and become constant at leading order for the full process.

### 3.4.3 $\Psi^- \Psi^+ \rightarrow \Psi^- \Psi^+$ scattering

The scattering of the two charged fermions can be a nice option to constrain the fermion couplings. In the case of chiral fermions with masses coming only from the Higgs interactions, it could provide bounds on the Higgs coupling that is correlated with the mass.

There are four relevant Feynman graphs shown in figure 3.13, two s-channel graphs with $Z$ or $\gamma$ exchange, and two t-channel ones with the same $Z$ or $\gamma$ exchange.

$$
i M_{s_1s_2} (\Psi^- s_1(p_1) \Psi^+ s_2(p_2) \rightarrow Z q_s) \rightarrow \Psi^- s_3(p_3) \Psi^+ s_4(p_4)) =
\left( \frac{ig}{\cos \theta_w} \right)^2 \left( \sin^2 \theta_w - \frac{1}{2} \right)^2 (\bar{v}_{s_2}(p_2) \gamma_\alpha u_{s_1}(p_1)) \frac{-i}{s - m_Z^2} \left( g^\sigma \rho - g_5^2 q_\alpha^\rho \right) \frac{m_Z^2}{m_Z^2} \left( \bar{u}_{s_3}(p_3) \gamma_\rho v_{s_4}(p_4) \right)
$$

(3.49)

$$
i M_{s_1s_2} (\Psi^- s_1(p_1) \Psi^+ s_2(p_2) \rightarrow \gamma q_s) \rightarrow \Psi^- s_3(p_3) \Psi^+ s_4(p_4)) =
\left( -ig \sin \theta_w \right)^2 (\bar{v}_{s_2}(p_2) \gamma_\alpha u_{s_1}(p_1)) \frac{-ig^\sigma \rho}{s} \left( \bar{u}_{s_3}(p_3) \gamma_\rho v_{s_4}(p_4) \right)
$$

(3.50)
\[ iM_{Z(t)} \left( \Psi^-(p_1) \Psi^+(p_2) \rightarrow Z(q_t) \rightarrow \Psi^-(p_3) \Psi^+(p_4) \right) = \]
\[ = \left( \frac{ig}{\cos \theta_w} \right)^2 \left( \sin^2 \theta_w - \frac{1}{2} \right)^2 \left( \bar{v}_{a_2} (p_2) \gamma_\alpha v_s (p_4) \right) - \frac{i}{t - m^2_Z} \left( g_\sigma^\rho - \frac{q^\rho q^\sigma}{m^2_Z} \right) \left( \bar{u}_{a_3} (p_3) \gamma_\rho u_s (p_1) \right) \]
\[ = \left( i \frac{g}{2} \cos \theta_w \right)^2 \left( \sin^2 \theta_w \right)^2 \left( \bar{u}_{a_3} (p_3) \gamma_\rho u_s (p_1) \right) \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]  
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]
(3.51)

\[ iM_{\gamma(t)} \left( \Psi^-(p_1) \Psi^+(p_2) \rightarrow \gamma(q_t) \rightarrow \Psi^-(p_3) \Psi^+(p_4) \right) = \]
\[ = \left( -ig \sin \theta_w \right)^2 \left( \bar{v}_{a_2} (p_2) \gamma_\alpha v_s (p_4) \right) - \frac{i g_\sigma^\rho}{t} \left( \bar{u}_{a_3} (p_3) \gamma_\rho u_s (p_1) \right) \]
\[ = \left( -ig \sin \theta_w \right)^2 \left( \bar{v}_{a_2} (p_2) \gamma_\alpha v_s (p_4) \right) - \frac{i g_\sigma^\rho}{t} \left( \bar{u}_{a_3} (p_3) \gamma_\rho u_s (p_1) \right) \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]
(3.52)

Again, I calculated the process in all possible fermion helicities that are defined in equation (2.27). The fermion helicities in the amplitude matrix are in the following order,
\[ M_{s_1 s_2 \rightarrow s_3 s_4} : \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]
\[ \begin{array}{cccc}
- & - & - & - \\
- & - & - & + \\
+ & - & - & + \\
+ & - & + & + \\
\end{array} \]  
(3.53)

The full amplitude matrix then follows,
\[ M_{s_1 s_2 \rightarrow s_3 s_4} = \]
\[ \begin{pmatrix}
\frac{g^2 \sin^2 \frac{\theta_w}{2}}{2 \cos^2 \theta_w} & 0 & 0 & \frac{g^2 \cos^2 \frac{\theta_w}{2}}{2 \cos^2 \theta_w} \\
0 & \frac{g^2 \cos^2 \frac{\theta_w}{2}}{2 \cos^2 \theta_w} & 0 & 0 \\
\frac{g^2 \cos^2 \frac{\theta_w}{2}}{2 \cos^2 \theta_w} & 0 & \frac{g^2 \sin^2 \frac{\theta_w}{2}}{2 \cos^2 \theta_w} & 0 \\
0 & \frac{g^2 \sin^2 \frac{\theta_w}{2}}{2 \cos^2 \theta_w} & 0 & \frac{g^2 \sin^2 \frac{\theta_w}{2}}{2 \cos^2 \theta_w} \\
\end{pmatrix} + O \left( \frac{m^2_W}{s} \right) \quad (3.54) \]

The zeroth partial wave amplitudes are again constants that are proportional to the gauge coupling constant which satisfies the unitarity constraint.

### 3.4.4 $\chi_i \chi_i \rightarrow \chi_i \chi_i$ scattering

The last investigated process is the two neutral fermion scattering that can provide constraints on
the Yukawa couplings. The only contributing Feynman-graph is an s-channel Higgs exchange graph, shown in figure 3.14.

\[ iM_{h(s)} \left( \Psi_0^0 (p_1) \chi^0 (p_2) \rightarrow h(q_s) \rightarrow \Psi_0^0 (p_3) \chi^0 (p_4) \right) = \]
\[ = \left( \frac{i}{2\sqrt{2}} \right)^2 \left( \bar{v}_{a_2} (p_2) (y_+ + y_- \gamma_5) u s_1 (p_1) \right) \frac{i}{s - m^2_h} \left( \bar{u}_{a_3} (p_3) (y_+ + y_- \gamma_5) v s_4 (p_4) \right) \]  
(3.55)

[34]
Figure 3.14: Feynman graphs for $\Psi_0^0\chi_0^0 \to \Psi_0^0\chi_0^0$ scattering.

The helicities are in the order as in equation (3.53).

$$M_{s_1s_2 \to s_3s_4} = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & \frac{y_2^2}{2} + O\left(\frac{m^2}{s}\right) & -\frac{y_1y_2}{2} + O\left(\frac{m^2}{s}\right) & 0 \\
0 & -\frac{y_1y_2}{2} + O\left(\frac{m^2}{s}\right) & \frac{y_1^2}{2} + O\left(\frac{m^2}{s}\right) & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$ \hspace{1cm} (3.56)

The for the four non-vanishing matrix elements, I calculated the zeroth partial wave amplitudes,

$$a_0(\bar{--} \rightarrow ++) = \frac{y_2^2}{32\pi} \hspace{1cm} (3.57)$$

$$a_0(\pm\mp \rightarrow \mp\pm) = -\frac{y_1y_2}{32\pi} \hspace{1cm} (3.58)$$

$$a_0(++ \rightarrow --) = \frac{y_1^2}{32\pi} \hspace{1cm} (3.59)$$

Applying the unitarity bounds $|\text{Re}a_0| \leq \frac{1}{2}$, the constraints on the Yukawa couplings are the following.

$$|y_{1,2}| \leq 4\sqrt{\pi} \approx 7.1 \hspace{1cm} (3.60)$$

In conclusion, after investigating the two-particle scattering processes of the new fermions, I found that no amplitude is growing with the energy as was expected from renormalizability. Also, while the mass parameters of the model remained unconstrained, I obtained a theoretical limit on the new Yukawa couplings, $|y_{1,2}| \leq 7.1$, that would be otherwise unconstrained by experiments and observations.
Chapter 4

Extension of the Standard Model with a singlet scalar

In December 2015, the ATLAS and CMS collaborations announced an excess in the diphoton searches in the 13 TeV LHC run-2 data. The ATLAS [10] data is compatible with a spin-2 with 3.9σ or spin-0 with 3.6σ local significance and large width. The reanalyzed run-1 data in the spring 2016 shows 2σ excess fro spin-0 and also compatible with the spin-2 resonance. The CMS collaboration [11] prefers low width with 3.4σ significance both for spin-0 and spin-2 resonance from the combined run-1 and run-2 analysis. The absence of other channels and the clean diphoton excess hint toward a new scalar or pseudoscalar resonance with mass $m_S = 750$ GeV. After the announcement, there was a huge excitement in model building and a few hundred articles appeared that explain the new observation. New data that can confirm the existence of the hypothetical new particle is expected in the summer of 2016.

The photon-photon final state resemblance to the Higgs discovery and the lack of other channels motivates the assumption, that the new resonance is an electroweak singlet scalar particle. If this new resonance is a pseudoscalar, the results will be similar qualitatively. Since there is no observation in the fermion final state and the mixing with the Higgs is severely constrained, the direct couplings to the fermions and the Higgs are suppressed. So I assume that the new scalar couples to the gauge bosons via five-dimensional effective operators that are induced at loop level by new vector-like fermions. After the electroweak symmetry breaking, the scalar couples to $\gamma\gamma$, $Z\gamma$, $ZZ$ and $W^+W^-$. The production rate of these four channel depends only on two parameters $\kappa_B$ and $\kappa_W$. Only their combination is determined by the collider data from the $S \rightarrow \gamma\gamma$, that is $\kappa_\gamma = \kappa_B + \kappa_W$.

In this chapter, I first introduce the effective theory of the one scalar extended Standard Model [40]. The production of the new resonance is accepted to be either gluon-gluon or photon-photon fusion. The allowed range of the effective couplings can be determined from the experimental data [12, 13, 41–58]. Studying two-particle gauge boson scatterings as expected
from a non-renormalizable effective model, I found their scattering amplitudes grow with the center-of-mass energy $\sqrt{s}$. Requiring perturbative unitarity, I gave the perturbative limits of the model that are rather low. That nearly rules out the scenario with large $S$-width favored by the early ATLAS data and disfavors the production from light-by-light scattering.

At the scale where perturbative unitarity is violated, some new perturbative physics or new strong dynamics should appear to correct the high energy behavior of the processes. Here, I consider that the effective operators are coming from a new heavy fermion loop. From this scenario, a natural assumption on the ratio of the couplings of $S$, $\frac{\kappa_W}{\kappa_B}$, can be given. Starting from the effective theory, taking into account the experimentally determined limits on the couplings, one can set important constraints on the mass and Yukawa coupling of the new heavy fermion that are not limited otherwise in the case of vector-like fermions.

### 4.1 Effective approach

To interpret the surprising collider data, the Standard Model is extended with a real singlet scalar $S$ with the mass $m_S$. Since the observed channel so far is the two-photon final state, it is practical to consider the effective theory, where the scalar $S$ is coupled to the photon and due to the $SU(2)\text{weak} \times U(1)_Y$ gauge invariance, similarly couples to the other gauge fields. In this section, I consider the following five-dimensional effective couplings, that contains the most general gauge invariant terms for a new singlet scalar,

$$L_{\text{eff}} \supset \frac{\alpha_{\text{em}}}{4\pi s_w^2} \frac{\kappa_W}{4m_S} S W_{\mu\nu} W^{a\mu\nu} + \frac{\alpha_{\text{em}}}{4\pi c_w^2} \frac{\kappa_B}{4m_S} S B_{\mu\nu} B^{\mu\nu} + \frac{\alpha_s}{4\pi} \frac{\kappa_g}{4m_S} S G_{\mu\nu}^a G^a_{\mu\nu}$$ (4.1)

This follows the notation of [41], where the SM couplings are explicitly taken out as they would appear in the one-loop integrated renormalizable model. Here, the Weinberg angle is noted as $s_w = \sin \theta_w$ and $c_w = \cos \theta_w$, and $\alpha_{\text{em}}$ is the electromagnetic coupling constant. In theory, the new scalar could also couple to fermions or mix with the Higgs, but those coupling are severely constrained by a large number of experiments and neglected here.

After the electroweak symmetry breaking, the scalar couplings to the heavy weak gauge bosons and to the photon. The couplings to $\gamma$ and $Z$ bosons are

$$\Gamma_{SV_1V_2}^{\mu\nu} = \frac{\kappa_V \alpha_{\text{em}}}{4m_S} \left( \frac{p_{V_1} \cdot p_{V_2} g^{\mu\nu} - p_{V_1}^\mu p_{V_2}^\nu}{m_S} \right),$$ (4.2)

with $V = \gamma$ or $Z$ and $\kappa_V$ couplings are the following combinations of $\kappa_B$ and $\kappa_W$,

$$\kappa_\gamma = \kappa_B + \kappa_W,$$

$$\kappa_Z = \frac{c_w^2}{s_w^2} \kappa_W + \frac{s_w^2}{c_w^2} \kappa_B.$$ (4.3)
While the coupling to the $W^\pm$ boson is the following,

$$\Gamma^\mu_{SW_1W_2} = \frac{\alpha_{em} \kappa_W}{m_S 4\pi s_w^2} \left( p_{W_1} \cdot g_{\mu\nu} p_{W_2} - p^\mu_{W_1} p^\nu_{W_2} \right).$$ \hspace{1cm} (4.4)$$

In the following section, I will investigate the two-to-two gauge boson scatterings to establish the validity range of the effective description.

### 4.1.1 $\gamma\gamma \to S \to \gamma\gamma$ scattering

So far the only experimentally observed decay channel is $S \to \gamma\gamma$, which makes the study of the $\gamma\gamma$-scattering available without much further theoretical assumptions on the couplings of $S$. I consider only the scatterings via $S$ exchange which is growing with the central-of-mass energy $\sqrt{s}$, while the Standard Model contribution is only a constant amplitude and can be neglected. There are three relevant Feynman-graphs shown in figure 4.1.

The corresponding s-, t- and u-channel scattering amplitudes $M_{s,t,u}(\gamma\gamma \to \gamma\gamma)$ are the following,

$$i M_s(\gamma^\mu_{\lambda_1}(k_1)\gamma^\nu_{\lambda_2}(k_2) \to \gamma^\sigma_{\lambda_3}(k_3)\gamma^\rho_{\lambda_4}(k_4)) = \frac{1}{2} \left( \frac{\alpha_{em} \kappa_{\gamma}}{4\pi m_S} \right)^2 \epsilon^\mu_{\lambda_1}(k_1)\epsilon^\nu_{\lambda_2}(k_2) \cdot \left( k_1 \cdot k_2 g_{\mu\nu} - k_{1\mu} k_{2\nu} \right) \frac{i}{s - m_S^2} (k_3 \cdot k_4 g_{\sigma\rho} - k_{3\sigma} k_{4\rho}) \epsilon^*_{\lambda_3}(k_3)\epsilon^*_{\lambda_4}(k_4) \hspace{1cm} (4.5)$$

$$i M_t(\gamma^\mu_{\lambda_1}(k_1)\gamma^\nu_{\lambda_2}(k_2) \to \gamma^\sigma_{\lambda_3}(k_3)\gamma^\rho_{\lambda_4}(k_4)) = \frac{1}{2} \left( \frac{\alpha_{em} \kappa_{\gamma}}{4\pi m_S} \right)^2 \epsilon^\mu_{\lambda_1}(k_1)\epsilon^*_{\lambda_2}(k_2) \cdot \left( k_1 \cdot k_2 g_{\mu\sigma} - k_{1\mu} k_{2\sigma} \right) \frac{i}{t - m_S^2} (k_3 \cdot k_4 g_{\nu\rho} - k_{3\nu} k_{4\rho}) \epsilon^*_{\lambda_3}(k_3)\epsilon^*_{\lambda_4}(k_4) \hspace{1cm} (4.6)$$
I chose a fixed helicity base for the photons, where the in- and outgoing polarization vectors \( u^\lambda \) and \( u^{\lambda} \) are defined as:

\[
\epsilon_{\text{in,}1}(k_{1,2}) = (0, 1, 0, 0) \quad \epsilon^{*}_{\text{out,}1}(k_{3,4}) = (0, \cos \theta, 0, -\sin \theta)
\]

\[
\epsilon_{\text{in,}2}(k_{1,2}) = (0, 0, 1, 0) \quad \epsilon^{*}_{\text{out,}2}(k_{3,4}) = (0, 0, 1, 0)
\]

Then I calculated the non-vanishing partial wave coefficients, \( a_0 = \int_{-1}^1 d(\cos \theta) M, \) that are included in appendix A.
growing with $s$,

$$a_0(11 \to 11) = \frac{1}{192\pi} \left( \frac{\alpha_{em}\kappa\gamma}{4\pi} \right)^2 \frac{s}{m_S^2} + O \left( \left( \frac{m_S^2}{s} \right)^0 \right)$$

(4.12)

$$a_0(11 \to 22) = a_0(22 \to 11) = \frac{1}{128\pi} \left( \frac{\alpha_{em}\kappa\gamma}{4\pi} \right)^2 \frac{s}{m_S^2} + O \left( \left( \frac{m_S^2}{s} \right)^0 \right)$$

(4.13)

$$a_0(12 \to 12) = a_0(21 \to 21) = \frac{1}{768\pi} \left( \frac{\alpha_{em}\kappa\gamma}{4\pi} \right)^2 \frac{s}{m_S^2} + O \left( \left( \frac{m_S^2}{s} \right)^0 \right)$$

(4.14)

$$a_0(21 \to 12) = a_0(12 \to 21) = -a_0(12 \to 12)$$

(4.15)

$$a_0(22 \to 22) = O \left( \left( \frac{m_S^2}{s} \right)^0 \right)$$

(4.16)

I required perturbative unitarity from the partial wave amplitudes $|\text{Re}a_0| \leq \frac{1}{2}$ from equation (2.16). The strongest limits on the energy $\sqrt{s}$ come from those matrix elements where only the $s$-channel gives contribution, $M_{11 \to 22} = M_{22 \to 11} \propto \frac{s^2}{s-m_S}$,

$$\sqrt{s} \lesssim \frac{32\pi^2 m_S^2}{\alpha_{em}\kappa\gamma} = 1.7 \times 10^7 \text{ GeV}$$

(4.17)

I derived the bounds on $\kappa\gamma$ based on the analysis of [12] in appendix B,

$$\kappa\gamma \in [23.7, 143.1] \quad (1012.0)$$

(4.18)

where the bounds come from considering minimal total width, meaning the only production channels are the gluon-gluon and photon-photon ones, $\Gamma_{\text{tot}} = \Gamma_{S \to gg} + \Gamma_{S \to \gamma\gamma}$. The smaller couplings come from where the $S$ production is dominated by gluon fusion, while the larger values corresponds to production dominated by photon fusion. The value given in the parenthesis corresponds to the large $S$ width scenario, the best fit by ATLAS. Translated to the energy $\sqrt{s}$, the constrains on the validity of the model is a few hundred TeV.

$$\sqrt{s} \lesssim 118.8 \ldots 716.3 \text{ TeV} \quad (16.8 \text{ TeV})$$

(4.19)

The calculation of the gluon-gluon scattering goes similarly, however, the $Sgg$ couplings is less constrained from the experiment and result in weaker limits [12]. Stronger bounds are expected from the massive gauge boson scatterings, as they have longitudinal polarization growing with the momentum $e^\mu(k) \sim \frac{k^\mu}{m_{W,Z}}$. Though their couplings to $S$ are also less constrained due to the lack of experimental observation.
Figure 4.2: Feynman graphs for the longitudinally polarized $Z_L Z_L$ scattering via $s$-, $t$- and $u$-channel $S$ exchange.

### 4.1.2 $Z_L Z_L \rightarrow S \rightarrow Z_L Z_L$ scattering

In the case of massive gauge boson scatterings, the transverse polarization results are similar to the photon-photon scattering, while stronger bounds come when considering the longitudinal polarization. Calculating the $Z \gamma$ scattering could result in stronger limit than the $\gamma \gamma$ scattering, but not as strong as considering only heavy gauge boson scattering. In the high energy limit, where $\sqrt{s} \gg m_{Z,W}$, the longitudinal polarization is proportional to the momenta of the gauge bosons, $\epsilon^\mu_L(k) \approx \frac{k^\mu}{m_{Z,W}}$. Similarly to the $\gamma \gamma$ scattering, the Standard Model contribution to the amplitude can be neglected, as the scatterings via $S$ exchange grows with $s^3$.

There are again three relevant Feynman-graphs shown in figure 4.2. The corresponding scattering amplitudes $\mathcal{M}_{s,t,u}(Z_L Z_L \rightarrow Z_L Z_L)$ are the following.

\[
i \mathcal{M}_s(Z_L^\mu(k_1)Z_L^\nu(k_2) \rightarrow Z_L^\sigma(k_3)Z_L^\rho(k_4)) = \frac{1}{2} \left( \frac{\alpha_{em} \kappa_Z}{4 \pi m_S} \right)^2 \frac{k_1^\mu \ k_2^\nu}{m_Z m_Z} \cdot \frac{i}{s - m_S^2} (k_1 \cdot k_2 g_{\mu \nu} - k_1^\mu k_2^\nu) \frac{k_3^\sigma \ k_4^\rho}{m_Z m_Z} \quad (4.20)
\]

\[
i \mathcal{M}_t(Z_L^\mu(k_1)Z_L^\nu(k_2) \rightarrow Z_L^\sigma(k_3)Z_L^\rho(k_4)) = \frac{1}{2} \left( \frac{\alpha_{em} \kappa_Z}{4 \pi m_S} \right)^2 \frac{k_1^\mu \ k_2^\nu}{m_Z m_Z} \cdot \frac{i}{t - m_S^2} (k_3 \cdot k_4 g_{\sigma \rho} - k_3^\sigma k_4^\rho) \frac{k_1^\mu \ k_2^\nu}{m_Z m_Z} \quad (4.21)
\]

\[
i \mathcal{M}_u(Z_L^\mu(k_1)Z_L^\nu(k_2) \rightarrow Z_L^\sigma(k_3)Z_L^\rho(k_4)) = \frac{1}{2} \left( \frac{\alpha_{em} \kappa_Z}{4 \pi m_S} \right)^2 \frac{k_1^\mu \ k_2^\nu}{m_Z m_Z} \cdot \frac{i}{u - m_S^2} (k_2 \cdot k_3 g_{\mu \sigma} - k_2^\mu k_3^\sigma) \frac{k_3^\rho \ k_4^\sigma}{m_Z m_Z} \quad (4.22)
\]
The kinematics of the scattering in the center-of-mass frame with \( k = \sqrt{\frac{s}{4} - m_Z^2} \) and \( \theta \) scattering angle,
\[
\begin{align*}
  k_1 &= \left( \frac{\sqrt{s}}{2}, 0, 0, k \right) & k_3 &= \left( \frac{\sqrt{s}}{2}, k \sin \theta, 0, k \cos \theta \right) \\
  k_2 &= \left( \frac{\sqrt{s}}{2}, 0, 0, -k \right) & k_4 &= \left( \frac{\sqrt{s}}{2}, -k \sin \theta, 0, -k \cos \theta \right)
\end{align*}
\]  
(4.23)

The full \( Z_L Z_L \rightarrow Z_L Z_L \) scattering amplitude becomes
\[
i\mathcal{M}(Z_L(k_1)Z_L(k_2) \rightarrow Z_L(k_3)Z_L(k_4)) = \frac{i \alpha_{em}^2 \kappa_Z^2}{512 \pi^2 m_Z^2 m_Z^4} \left( \frac{s^2 (s - 4m_Z^2)^2}{s - m_S^2} + \frac{t^2 (t - 4m_Z^2)^2}{t - m_S^2} + \frac{u^2 (u - 4m_Z^2)^2}{u - m_S^2} \right)
\]  
(4.24)

Again, calculating the zeroth partial wave amplitude \( a_0 \) that grows with \( s^3 \),
\[
a_0 = \left( \frac{\alpha_{em} \kappa_Z}{4 \pi m_Z^2} \right)^2 \frac{s^3}{m_S^2} \left( \frac{1}{1024 \pi} + \frac{5}{1536 \pi} \frac{m_S^2}{s} + \mathcal{O} \left( \left( \frac{m_S^2}{s} \right)^2 , \left( \frac{m_Z^2}{s} \right) \right) \right)
\]  
(4.25)

Imposing the partial wave unitarity condition on \( a_0 \) from equation (2.16), at leading order it results in the following constraint on the energy \( \sqrt{s} \).
\[
\sqrt{s} \lesssim 4\sqrt{\pi} \sqrt{\frac{2m_Z^2 m_S}{\alpha_{em} \kappa_Z}} = \frac{7.35 \text{ TeV}}{\sqrt{\kappa_Z}}
\]  
(4.26)

To quantify the above constraint, I need more information on the couplings of \( S, \kappa_W \) and \( \kappa_B \), since the experimentally available is only constraining their sum, \( |\kappa_\gamma| = |\kappa_B + \kappa_B| \). I assume that they are positive, I have a lower and upper bound.
\[
0 \leq \kappa_{W,B} \leq \kappa_B + \kappa_W = 23.7 \ldots 143.1 \quad (1012.0)
\]  
(4.27)

where again, the small couplings corresponds to gluon fusion dominated \( S \) production and the larger couplings to the photon fusion being the dominant production channel, while the value given in parenthesis is coming from the large \( S \) width scenario preferred by ATLAS.

Mathematically translating the experimental limits of equation (4.27) to the effective \( SZZ \) coupling \( \kappa_Z \), it can have the following values,
\[
\kappa_Z = \frac{s^2_w}{s_w^2} \kappa_W + \frac{s^2_w}{c_w^2} \kappa_B \in [7.1, 479.2] \quad (3388.1)
\]  
(4.28)

Using the mathematical bound on \( \kappa_Z \) alongside with the energy limit coming from the \( Z_L Z_L \) scattering in equation (4.26), gives strong limits on the validity of the effective description. I specify the constraints in two limiting case, where either \( \kappa_W \) or \( \kappa_B \) vanishes.
In the first case, when $S$ does not couple to the $W^\pm$, $\kappa_W = 0$, the $SZZ$ coupling is small,

$$\kappa_Z = \frac{s_w^2}{c_w^2} \kappa_B \in [71.1, 428.4] \quad (302.3)$$

In this scenario, the $Z_LZ_L$ scattering gives the strongest energy bound among the gauge boson scatterings, since the better $W^+_LW^-_L$ scattering does not get a contribution from $S$ exchange, as their coupling is absent.

$$\sqrt{s} \lesssim 2.1 \ldots 3.8 \text{ TeV} \quad (1.1 \text{ TeV})$$

In the other limit, when the new scalar does not couple to the hypercharge generator $B_\mu$ and $\kappa_B = 0$, the $SZZ$ coupling is much larger,

$$\kappa = \frac{c_w^2}{s_w^2} \kappa_W \in [79.5, 479.2] \quad (3388.1)$$

Resulting in a more tight energy bound than in the previous case,

$$\sqrt{s} \lesssim 0.9 \ldots 1.7 \text{ TeV} \quad (0.5 \text{ TeV})$$

The $W^+W^-$ scattering will provide even stronger bounds, however, the $SW^+W^-$ coupling can vanish, i.e. $\kappa_W = 0$, and in this case, the $ZZ$ scattering offers the tightest limit on the validity of the effective model.

### 4.1.3 $W^-_LW^+_L \rightarrow S \rightarrow W^-_LW^+_L$ scattering

As in the case of the $Z$ bosons, I only calculate the scattering of the longitudinally polarized $W^\pm$ bosons, where I expect the strongest limits. Now, the u-channel is absent and there are only two relevant Feynman-graphs shown in figure 4.3, through s- and t-channel $S$ exchange.
Then calculating the zeroth partial wave amplitude
\[ a_{SW} \]

The kinematics is the same as in the ZZ scattering in equation (4.23), with \( k = \sqrt{\frac{s}{4} - m_W^2} \).

The full \( W_L^- W_L^+ \) scattering amplitude then follows,

\[
iM_t(W_L^- (k_1) W_L^+ (k_2) \rightarrow W_L^- (k_3) W_L^+ (k_4)) = \frac{i \alpha_{em}^2 \kappa_W^2}{512 \pi^2 s_W^2 m_W^2 m_S^2} \left( \frac{s^2(s - 4m_W^2)^2}{s - m_S^2} + \frac{t^2(t - m_W^2)^2}{t - m_S^2} \right)
\]  

(4.35)

Then calculating the zeroth partial wave amplitude \( a_0 \),

\[
a_0 = \left( \frac{\alpha_{em} \kappa_W}{4 \pi s_W^2 m_W^2} \right)^2 s^3 \left( \frac{3}{2048 \pi} + \frac{1}{384 \pi} \frac{m_S^2}{s} \right) + O \left( \left( \frac{m_S^2}{s} \right)^2, \left( \frac{m_W^2}{s} \right) \right)
\]  

(4.36)

After imposing the partial wave unitarity condition from equation (2.16) on \( a_0 \), at leading order the constraint on the energy \( \sqrt{s} \) becomes

\[
\sqrt{s} \lesssim 4\sqrt{\pi} \sqrt{\frac{2m_W^2 m_S s_W^2}{\sqrt{3} \alpha_{em} \kappa_W}} = \frac{3.86 \text{ TeV}}{\sqrt{\kappa_W}}
\]  

(4.37)

As mentioned in the previous section, in the limit of vanishing \( S B_\mu B_\nu \) coupling, \( \kappa_B = 0 \), the \( W_L^- W_L^+ \) scattering gives the strongest bound on the energy. In this case, the full \( S \gamma \gamma \) coupling comes from the \( W^\pm \kappa_w = \kappa_W \). With the same assumptions as in equation (4.27), the \( S W^+ W^- \) coupling is in the following range,

\[
\kappa_W = \kappa_\gamma \in [23.7, 143.1] \text{  (1012.0)}
\]  

(4.38)

With this values for \( \kappa_W \) the energy constraint from the \( W_L^- W_L^+ \) scattering given in equation

44
Figure 4.4: Resolving the vertex of the new scalar and gauge bosons with a new heavy fermion loop.

(4.37) is the following,

$$\sqrt{s} \lesssim 0.7 \ldots 1.3 \text{ TeV} \quad (0.4 \text{ TeV})$$

(4.39)

Again, the value given in parenthesis is coming from the large $S$ width fit that gives extremely low energy bound, even smaller than the resonance mass $m_S = 750$ TeV. These energy constraints are coming from unitarity conditions, signaling that our assumptions are not correct and new mechanisms or particles should appear that correct the non-unitary processes.

I arrived at these bounds from assuming that one of the couplings vanishes. In a more physically motivated scenario, since the couplings $\kappa_B$ and $\kappa_W$ are expected to be induced at one-loop level by particles with ordinary weak and hypercharges, that is not the case. A general estimate is that $\kappa_W \simeq \kappa_B$ or at least they are at the same order of magnitude. For example in the case of a standard vector-like weak doublet fermion, where $\kappa_W = \kappa_B$, as I show in section 4.2.2.

### 4.2 Renormalizable extension

The effective couplings that were introduced in the previous section, are induced at loop level in renormalizable theory either by new fermions or (pseudo)scalars running in the loop. The main difference between the two scenario is the sign of the resulting coupling, since the fermion loop has an extra minus sign compared to the scalar one. Here, I consider the case, where a new heavy vector-like fermion $T$, with mass $m_T$, is added to the Standard Model beside the singlet scalar $S$ with a Yukawa-type coupling between the two. To avoid the unobserved decay of $S \rightarrow \bar{T}T$ the fermion mass should be $m_T \gtrsim 375$ GeV.

$$\mathcal{L}_T \supset \bar{T}i\gamma^\mu D_\mu T - \lambda_{ST}\bar{T}TS$$

(4.40)

To relate the fermion couplings directly to $\kappa_B$ and $\kappa_W$, I considered the $S \rightarrow B^\mu B^\nu$ and $S \rightarrow W^{\mu\nu}W^{\nu\omega}$ at energies higher than the electroweak symmetry breaking scale. The relevant couplings dictated by the representation of $T$ are shown in equations (4.41) and (4.42) below.
\[ S(k_1 + k_2) \rightarrow B^\mu(k_1) \]
\[ B^\nu(k_2) \]

Figure 4.5: $\Gamma_{SBB}^{\mu\nu}$ effective interaction of $SB^\mu B^\nu$ from the lagrangian (4.1).

\[ T \rightarrow B^\mu = ig'Y_T \gamma^\mu = ig \tan \theta_w Y_T \gamma^\mu \quad (4.41) \]

\[ T \rightarrow W_3^\mu = \frac{i}{2} g' \gamma^\mu \quad (4.42) \]

where I use $Y = Q - T_3$ and the relation of the electroweak couplings $\frac{g'}{g} = \tan \theta_w$.

In the next section, I show how to resolve the effective vertex of $SB_\mu B_\nu$ with the new fermion running in the loop.

### 4.2.1 Resolving the $S \rightarrow B^\mu B^\nu$ interaction

I go through the resolution of the effective coupling $SB_\mu B_\nu$ that is illustrated in figure 4.5 and is the following from the effective lagrangian (4.1),

\[ i\Gamma_{SBB}^{\mu\nu} = \frac{i\alpha_{em} k_B}{4\pi c S m_S} (k_1 \cdot k_2 g^{\mu\nu} - k_1^\mu k_2^\nu) \quad (4.43) \]

In the renormalizable theory that is further extended with the heavy fermion $T$, the related Feynman-graph is illustrated in figure 4.6. This generates the effective interaction in equation (4.43) between the scalar $S$ and the hypercharge generators that has the following amplitude,

\[ i\mathcal{M}_{S \rightarrow BB}^{\mu\nu} = -i\lambda_{ST} \int \frac{d^4k}{(2\pi)^4} \text{Tr} \left[ \frac{i(k - m_T)}{k^2 - m_T^2 + i\varepsilon} (ig'Y_T \gamma^\nu) \right. \]
\[ \left. \frac{i(k + k_1 + k_2 + m_T)}{(k + k_1 + k_2)^2 - m_T^2 + i\varepsilon} (ig'Y_T \gamma^\mu) \frac{i(k + k_1 + k_2 + k + m_T)}{(k + k_1 + k_2)^2 - m_T^2 + i\varepsilon} \right] \quad (4.44) \]

To simplify the calculation of the momentum integral, one can use the trick to project out the
Lorentz indices with the transverse projector operator $P_{\mu\nu}$. I use the normalization $P_{\mu\nu}P^{\mu\nu} = 2$,

$$P_{\mu\nu} = g_{\mu\nu} - \frac{k_1\nu k_2\mu}{k_1 \cdot k_2} \quad (4.45)$$

For the calculation, the kinematics of the process is $k_2^2 = 0 = k_2^2$ and $m_S^2 = (k_1 + k_2)^2 = 2k_1 \cdot k_2$. Then, I can calculate the trace in equation (4.44) after projecting out the Lorentz indices.

$$T^{\mu\nu} = \text{Tr}[((k + m_T)\gamma^\nu(k + \bar{k}_2 + m_T)\gamma^\mu(k + \bar{k}_1 + \bar{k}_2 + m_T))] \quad (4.46)$$

$$P_{\mu\nu}T^{\mu\nu} = 4m_T \left(-m_S^2 + 3m_T^2 - \frac{8}{m_S^2}(k \cdot k_1)(k \cdot k_2) - 2(k \cdot k_2) + k^2\right) \quad (4.47)$$

Now I can integrate out the momentum in equation (4.44) for the projected amplitude,

$$iP_{\mu\nu}{\cal M}_{\mu\nu}^{S \to BB} = i\frac{\lambda_{ST}(g'Y_T)^2m_T}{2\pi^2}(1 + (1 - \tau)f(\tau)) = i\frac{\lambda_{ST}(g'Y_T)^2}{2\pi^2}\frac{m_S^2}{4m_T}\tilde{f}(\tau) \quad (4.48)$$

where $\tau = \frac{4m_T^2}{m_S^2}$, and the function $f(\tau)$,

$$f(\tau) = \begin{cases} \arcsin\frac{1}{\sqrt{\tau}} & \text{if } \tau > 1 \\ -\frac{1}{4}\left(\log\frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi\right) & \text{if } \tau < 1 \end{cases} \quad (4.49)$$

and I use the notation $\tilde{f}(\tau) = \tau(1 + (1 - \tau)f(\tau))$. The relevant region of $f(\tau)$ is the $\tau > 1$, where $1 > \tilde{f}(\tau) > \frac{2}{3}$, and $\frac{2}{3}$ is a good approximation for $m_T > m_S$, as illustrated on figure 4.7.

Projecting out the effective coupling in equation (4.43), too, we can see why it was convenient to separate the Standard Model contribution in the effective couplings, $\alpha_{em} = \frac{g^2\alpha^2}{4\pi}$, and that the relevant scale is the mass of the particle running in the loop $m_T$ rather than the scalar mass $m_S$.

$$iP_{\mu\nu}{\Gamma}_{\mu\nu}^{S \to BB} = i\frac{\alpha_{em}k_Bm_S}{4\pi c_w} \quad (4.50)$$
Figure 4.7: The function $\tilde{f}(\tau) = \tau (1 + (1 - \tau) f(\tau))$ in the region $\tau \geq 1$. $\tau = 4$ corresponds to $m_T = m_S$.

$$\kappa_B = 2m_S \frac{\lambda_{ST} Y_T^2}{m_T} \tilde{f}(\tau)$$ (4.51)

The form of the effective coupling $\kappa_B$ in equation (4.51) applies when only one vector-like fermion is running in the loop. When there are more than one new fermions $T_f$ that couples to the scalar $S$, then we need to sum over them to get the effective coupling. If each of the new fermions appear in $N_{C,f}$ color and $N_{F,f}$ flavor, then the generalization is

$$\frac{\kappa_B}{2m_S} = \frac{\lambda_{ST}}{m_T} \left( \sum_f N_{F,f} N_{C,f} Y_{T_f}^2 \right) \tilde{f}(\tau)$$ (4.52)

Similarly, taking one of the three weak gauge bosons, e.g $W^{\alpha 3}$, from relating the $S \to W^{3\mu} W^{3\nu}$ effective coupling $\kappa_W$ to the amplitude of the integrated fermion-loop, $\kappa_W$ can be expressed as follows,

$$\frac{\kappa_W}{2m_S} = \frac{\lambda_{ST}}{m_T} \left( \sum_f N_{F,f} N_{C,f} T_{3T_f}^2 \right) \tilde{f}(\tau)$$ (4.53)

$$\frac{\kappa_W}{\kappa_B} = \frac{\sum_f N_{F,f} N_{C,f} T_{3T_f}^2}{\sum_f N_{F,f} N_{C,f} Y_{T_f}^2}$$ (4.54)

Now, in addition to the experimental value of $|\kappa_{\gamma}| = |\kappa_B + \kappa_W|$, the ratio of the two effective coupling $\frac{\kappa_W}{\kappa_B}$, that depends only on the electroweak quantum numbers of the new fermion, is known, too.
4.2.2 \(SU(2)_{\text{weak}}\) doublet vector-like \(T\)

As a special case, consider a theory with only one additional vector-like \(SU(2)_{\text{weak}}\) doublet fermion \(T = \begin{pmatrix} T^0 \\ T^- \end{pmatrix}\). The quantum numbers of \(T\) are \(Y_T = \frac{1}{2}\) and \(T_3^T = \pm \frac{1}{2}\). So the ratio of the two effective couplings with \(\kappa_W\) from equation (4.53) and \(\kappa_B\) from equation (4.52) becomes 1.

\[
\frac{\kappa_W}{\kappa_B} = \frac{T_{3T}^2}{Y_T^2} = 1 \quad (4.55)
\]

Considering this as an input for the effective description, the energy limit from the \(W_L^- W_L^+\) scattering of equation (4.37) is the following,

\[
\kappa_W = \kappa_B = \frac{\kappa_\gamma}{2} \in [11.9, 71.6] \quad (506.0) \quad (4.56)
\]

\[
\sqrt{s} \lesssim 0.9 \ldots 1.7 \text{ TeV} \quad (0.5 \text{ TeV}) \quad (4.57)
\]

These bounds are similar to the \(\kappa_B = 0\) case in equation (4.38). As for the fermion mass \(m_T \gtrsim 375\) GeV, the large scalar width scenario preferred by ATLAS, with the corresponding limit given in the parenthesis, leaves no room for the effective interactions. Also, the lower bounds from the photon fusion dominated part are not favored, leaving the gluon fusion production the most likely scenario, if we suppose one heavy fermion doublet.

Turning around the context, taking the couplings from the experiment from equation (4.53), I constrain the ratio of the fermion mass and Yukawa coupling.

\[
\frac{\lambda_{ST}}{m_T} = \frac{\kappa_W}{2m_S} \frac{1}{f(\tau)} \frac{\tau^{\geq 1}}{3\kappa_W} \frac{4m_S}{4m_S} \in [0.008, 0.05] \quad (0.34) \quad (4.58)
\]

\[
m_T \approx \lambda_{ST} \cdot (21 \ldots 126 \text{ GeV}) \quad (\lambda_{ST} \cdot (3 \text{ GeV})) \quad (4.59)
\]

The perturbative limit on the Yukawa coupling is \(\lambda_{ST} \lesssim 4\sqrt{\pi}\), then the fermion mass limit translates to

\[
m_T \approx 148 \ldots 896 \text{ GeV} \quad (21 \text{ GeV}) \quad (4.60)
\]

These low bounds are coming from only one color singlet, weak doublet vector-like fermion \(T\), when its Yukawa coupling to \(S\) is close to its perturbative limit. These constraints can be loosened by considering colorful fermions or having multiple doublet. Nevertheless, this scenario favors the gluon dominated \(S\) production with minimal \(S\) width and rules out completely the large width scenario.
Chapter 5

Conclusion

The Standard Model successfully describes all particle physics phenomena observed in high energy experiments. Although it cannot be the final fundamental theory of nature. Evidences of physics beyond Standard Model appear at cosmological scales that are not observable in the current collider experiments. To learn about physics at higher energies, we can use theoretical arguments to study the influence of high energy physics on low energy observables.

In this thesis I introduced a theoretical tool that can be used to study the effects of higher energy phenomena in the low energy regions that are available. From the unitarity of scatterings which means that all possibilities add up to one, we can derive constraints on the scattering amplitudes at every order in the gauge coupling constants. The most basic scatterings are the two-particle scatterings that can be parametrized by only two parameters, the center-of-mass energy and the scattering angle. After expanding the two-particle scattering amplitude in partial waves, the coefficients only depend on the center-of-mass energy. The perturbative unitarity limits are applied on the partial wave amplitudes which then translate into constraints on the couplings appearing in the amplitude, or on the energy. When an amplitude grows with the energy, then it will violate unitarity at some point. This is the cut-off of the theory, where it loses predictive power. If we want to create a theory that is valid up to arbitrary high energies, it must respect perturbative unitarity in all scattering amplitude at all energies.

Studying the scattering processes using perturbative unitarity constraints, we can draw limits on the parameters of the theory or establish the validity scale of the theory where new physics should appear to correct the unitarity violating processes. Also, depending on which scatterings violate unitarity, we can forecast the new physics that unitarizes the process. That makes perturbative unitarity a useful tool studying new theories.

In this thesis, I studied two models. The first one was a simplified model of dark matter, where the Standard Model was extended with a pair of weak vector-like doublets and a singlet vector-like fermion with renormalizable couplings. There are four new parameters in the model, two are the dimensionful mass term of the vector-like fermions and two are the dimensionless
Yukawa couplings between the singlet, one doublet and the Higgs boson. After introducing the constraints of the model from various dark matter searches, I gave a short analytic study on the dark matter candidates coupling in the regions that are favored by experiments and can be solved analytically. Then I calculated different two-to-two scatterings that were promising in providing constraints on the model. I derived the perturbative limits on the Yukawa couplings, however, the mass parameters cannot be constrained in this way.

The second study was motivated by the newly announced resonance in the diphoton channels of the ATLAS and CMS detectors at CERN. From the data, I introduced an effective model that extends the Standard Model with a new singlet scalar that couples to the gauge boson via five-dimensional operators. Again, I calculated gauge boson two-particle scatterings which were growing with the center-of-mass energy. After applying the perturbative unitarity constraints, I obtained that the model has low validity scale. In some scenarios, even lower than the mass of the newly introduced scalar, signaling that those scenarios are not viable after theoretical consideration. At the end, I presented a renormalizable model that included a new heavy fermion in addition to the new scalar. I was able to translate the energy constraints to the mass and Yukawa couplings of the new fermion.

What we can see from the two studies is that perturbative unitarity is a useful tool in complementing the experimental constraints on the parameter space and establish the limits of effective models.
Appendix A

Scattering amplitudes with Mathematica

This appendix contains a sample of my Mathematica notebook that calculates the scattering amplitudes for the doublet-singlet model analysis in chapter 3.

High energy scattering amplitudes in 2D1SWeyl model

This notebook calculates the high energy behavior of the scattering amplitude in the model with two doublet and a singlet vector-like fermions.

The processes are the following: \( \Psi^- \Psi^+ \rightarrow W^- W^+ \), \( \Psi^- \Psi^+ \rightarrow \Psi^- \Psi^+ \), (\( \eta k \eta k \rightarrow \eta k \eta k \)).

Processes and notation.

The notation for the amplitudes will be the following: each process has the letter of their subsection, the channel (s, t, u) and the intermedeate state (Z, A for \( \gamma \), P for \( \Psi \), h, ...).

As we are interested in the worst high energy behavior, for the vector boson polarization we use \( e^\mu (k) \approx \frac{k^\mu}{m_W} \).

A. \( \Psi^- \Psi^+ \rightarrow W^- W^+ \)

In the s - channel there are two processes, one through Z boson and the other through a photon. In the t - channel, since we sum over all possible intermedeate states, we can use the weak eigenstates, which means only one process through \( \Psi^0 \).
AsZ\([s_1,s_2]\):

\[i_A (\Psi^- (p_1, s_1) \Psi^+ (p_2, s_2) \to Z (q_s) \to W^- (k_3) W^+ (k_4)) = \frac{ig}{\cos \theta_w} \left( \sin^2 \theta_w - \frac{1}{2} \right) (-ig \cos \theta_w)\]

\[\times (\bar{v}_{s_2} (p_2) \gamma_\mu u_{s_1} (p_1)) \frac{-ig \sigma^\rho}{s - m_z^2} \left[ (q_s + k_4)_\mu g_{\rho \nu} + (-k_4 + k_3)_\rho g_{\nu \mu} + (-k_3 - q_s)_\rho g_{\mu \rho} \right]\]

\[\times \epsilon^\mu (k_3) \epsilon^{\nu*} (k_4)\]

AsA\([s_1,s_2]\):

\[i_A (\Psi^- (p_1, s_1) \Psi^+ (p_2, s_2) \to A (q_s) \to W^- (k_3) W^+ (k_4)) \]

\[= (-ig \sin \theta_w)^2 (\bar{v}_{s_2} (p_2) \gamma_\mu u_{s_1} (p_1)) \frac{-ig \sigma^\rho}{s} \left[ (q_s + k_4)_\mu g_{\rho \nu} + (-k_4 + k_3)_\rho g_{\nu \mu} + (-k_3 - q_s)_\rho g_{\mu \rho} \right]\]

\[\times \epsilon^\mu (k_3) \epsilon^{\nu*} (k_4)\]

AtP\([s_1,s_2]\):

\[i_{A_0} (\Psi^- (p_1, s_1) \Psi^+ (p_2, s_2) \to \Psi^0 (q_t) \to W^- (k_3) W^+ (k_4)) \]

\[= \left( \frac{ig}{\sqrt{2}} \right)^2 (\bar{v}_{s_2} (p_2) \gamma_\nu \frac{i (q_t^\alpha \gamma_\alpha + m_d)}{t - m_d^2} \gamma_\mu u_{s_1} (p_1)) \epsilon^\mu (k_3) \epsilon^{\nu*} (k_4)\]

**B. \(\Psi^- \Psi^+ \to \Psi^- \Psi^+\)**

In the s-channel, as previously, there are two processes, one through Z boson and the other through a photon. In the t-channel, here are also two processes, similarly one through Z boson and the other through a photon.

BsZ\([s_1,s_2,s_3,s_4]\):

\[i_B (\Psi^- (p_1, s_1) \Psi^+ (p_2, s_2) \to Z (q_s) \to \Psi^- (p_3, s_3) \Psi^+ (p_4, s_4)) \]

\[= \left( \frac{ig}{\cos \theta_w} \right)^2 \left( \sin^2 \theta_w - \frac{1}{2} \right)^2 (\bar{v}_{s_2} (p_2) \gamma_\mu u_{s_1} (p_1)) \frac{-ig \sigma^\rho}{s - m_z^2} \left[ \bar{u}_{s_3} (p_3) \gamma_\mu v_{s_4} (p_4) \right]\]

BsA\([s_1,s_2,s_3,s_4]\):
iB_A \left( \Psi^- (p_1, s_1) \Psi^+ (p_2, s_2) \rightarrow A (q_8) \rightarrow \Psi^- (p_3, s_3) \Psi^+ (p_4, s_4) \right) \\
= \left( -\text{igsin} \theta_w \right)^2 \left( \bar{v}_s (p_2) \gamma_\sigma u_s (p_1) \right) -i\frac{\text{ig} \gamma_\rho}{s} \left( \bar{u}_s (p_3) \gamma_\rho u_s (p_4) \right)

BtZ[s_1, s_2, s_3, s_4]:

iB_Z \left( \Psi^- (p_1, s_1) \Psi^+ (p_2, s_2) \rightarrow Z (q_6) \rightarrow \Psi^- (p_3, s_3) \Psi^+ (p_4, s_4) \right) \\
= \left( -\text{ig} \left( \text{cos} \theta_w \right)^2 - \frac{1}{2} \right)^2 \left( \bar{v}_s (p_2) \gamma_\sigma v_s (p_4) \right) -i \left( \frac{g^{\sigma \rho} - q^\sigma q^\rho}{m_z^2} \right) \left( \bar{u}_s (p_3) \gamma_\rho u_s (p_1) \right)

BtA[s_1, s_2, s_3, s_4]:

iB_A \left( \Psi^- (p_1, s_1) \Psi^+ (p_2, s_2) \rightarrow A (q_6) \rightarrow \Psi^- (p_3, s_3) \Psi^+ (p_4, s_4) \right) \\
= \left( -\text{igsin} \theta_w \right)^2 \left( \bar{v}_s (p_2) \gamma_\sigma v_s (p_4) \right) -i \frac{\text{ig} \gamma_\rho}{t} \left( \bar{u}_s (p_3) \gamma_\rho u_s (p_1) \right)

C. Try $\Psi^0 \chi^0 \rightarrow \Psi^0 \chi^0$

Only one process through the higgs.

CsH[s_1, s_2, s_3, s_4]

iC_h \left( \Psi^0 (p_1, s_1) \chi^0 (p_2, l_2) \rightarrow h (q_6) \rightarrow \Psi^0 (p_3, s_3) \chi^0 (p_4, l_4) \right) \\
= \left( \frac{i}{2\sqrt{2}} \right)^2 \left( \bar{v}_l (p_2) (y_+ + y_- \gamma_5) u_s (p_1) \right) \frac{i}{s - m_h^2} \left( \bar{u}_s (p_3) (y_+ + y_- \gamma_5) v_l (p_4) \right)

Notation: $y_\pm = y_1 \pm y_2$

D. $\Psi^+ \chi^0 \rightarrow W^+ h$ and $\Psi^+ \Psi^0 \rightarrow W^+ h$

DtP[s_1, s_2]

iD_{\Psi^0} \left( \Psi^+ (p_1, s_1) \chi^0 (p_2, s_2) \rightarrow \Psi^0 (q_6) \rightarrow W^+ (k_3) \gamma_\mu (k_4) \right) \\
= \left( \frac{ig}{\sqrt{2}} \right) \left( \frac{i}{2\sqrt{2}} \right) \left( \bar{v}_s (p_2) (y_+ + y_- \gamma_5) \frac{i (\gamma_\rho q^\rho + m_d)}{t - m_d^2} \gamma_\mu u_s (p_1) \right) \epsilon^\mu (k_3)

DsW[s_1, s_2]

54
\[
\begin{align*}
\text{id}_W (\Psi^+ (p_1, s_1) \Psi^0 (p_2, s_2) & \to W (q_s) \to W^+ (k_3) \ h (k_4)) \\
& = \left( \frac{ig}{\sqrt{2}} \right) (\text{igm}_w) (\bar{u}_{s_2} (p_2) \gamma_\mu u_{s_1} (p_1)) \frac{-i \left( g^{\mu \alpha} - \frac{q_\alpha q_\mu}{m_w^2} \right) g_{\alpha \nu} e^\nu (k_3)}{s - m_w^2}
\end{align*}
\]

**Dealing w/ Lorentz indices & defining the matrix elements.**

**Quit[ ]:**

1: ‘gammarep’ takes the \( \gamma \) matrices from the expressions, replaces with slashed momenta.

\[\text{SetAttributes}[\eta, \text{Orderless}]\]

\[\text{gammarep} = \{\text{Dot}[x\_\_, \eta[a\_\_, \ y\_\_], p\_[a\_]] :\to \text{Dot}[x, (\text{slash}[p])], y]\}, \text{Dot}[x\_\_, \gamma[a\_\_, \ y\_\_] \eta[a\_\_, \ b\_]] :\to \text{Dot}[x, \gamma[b], y]\};\]

2: ‘indexmatch’ should finally make the minkowski scalar products of two similar indices.

\[\text{indexmatch} = \{\text{Times}[\eta[a\_\_, \ b\_\_, \ \eta[b\_\_, \ c\_\_], x\_\_] :\to \text{Times}[\eta[a, c], x], \eta[a\_\_, \ a\_] :\to 4, \eta[a\_\_, \ b\_]^2 :\to 4, \]

\[\text{Times}[\eta[a\_\_, \ b\_\_, \ x\_\_, \ y\_\_] :\to \text{Times}[x[b], y], \text{Times}[\eta[a\_\_, \ b\_\_, \ x\_[b\_\_, \ y\_\_]] :\to \text{Times}[x[a], y], \]

\[\text{Times}[x\_[a\_\_, \ y\_[a\_\_, \ z\_\_]] :\to \text{Times}[\text{mink}[x, y], z], \text{Times}[x\_[a\_\_], y\_\_] :\to \text{Times}[\text{mink}[x, x], y]\};\]

3: ‘gammasum’ deals with the four-spinor case, where \( (\bar{u} \gamma_\mu u) (\bar{v} \gamma^\mu v) \) can appear.

\[\text{gammasum} = \{\text{Times}[\text{Dot}[x\_\_, \gamma[a\_\_, \ y\_\_], \text{Dot}[z\_\_, \gamma[a\_\_, \ r\_\_] :\to \]

\[\text{Sum}[\text{Times}[\text{Dot}[x, \text{gamma}[[i]]], y], \text{Dot}[z, \text{gamma}[[i]], r]], \{i, 4]\}];\]

\[A. \ \Psi^- \Psi^+ \to W^- W^+\]

\[\text{AsZ}[s_1\_\_, s_2\_\_] := -g^2 \text{Sin}[\text{thw}]^2 (1 - 1/2) (1/(s - m_\_\_z^2)) (\text{(Expand}[s_2.\gamma[s].s_1]) (\eta[s, r] - \text{qs}[s] \text{qs}[r]/m_\_\_z^2) \]

\[((\text{qs}[m] + k_3[m])_\eta[r, n] + (-k_4[r] + k_3[r])_\eta[m, n] + (-k_3[n] - \text{qs}[n])_\eta[m, r]) \]

\[k_3[m] k_4[n]/m_\_\_z^2 //\text{gammarep} //\text{.indexmatch}\]

\[\text{AsA}[s_1\_\_, s_2\_\_] := g^2 \text{Sin}[\text{thw}]^2 (1/s) (\text{(Expand}[s_2.\gamma[s].s_1]) \eta[s, r] \]

\[((\text{qs}[m] + k_4[m])_\eta[r, n] + (-k_4[r] + k_3[r])_\eta[m, n] + (-k_3[n] - \text{qs}[n])_\eta[m, r]) \]

\[k_3[m] k_4[n]/m_\_\_z^2 //\text{gammarep} //\text{.indexmatch}\]

\[\text{AtPvar}[s_1\_\_, s_2\_\_] := -(g^2/2) \text{Sin}[\text{thw}]^2 (1/(s - m_\_\_z^2)) \]

\[((\text{Expand}[s_2.\gamma[n].((\text{slash}[q]) + \text{mdI4}).\gamma[m].s_1]) k_3[m] k_4[n]/m_\_\_z^2) //\text{gammarep} //\text{.indexmatch}\]

\[\text{AtPvar}[s_1, s_2]\]
\[ g^2 \text{slash}[k_4] \cdot (\text{I}[4\text{md} + \text{slash}[q_1]) \cdot \text{slash}[k_3] \cdot s_1 \over 2 \text{mw}^2 (\text{md}^2 + t) \]

AtP[s_{1a}, s_{2a}]: = \frac{g^2 \text{slash}[k_4] \cdot (\text{I}[4\text{md} + \text{slash}[q_1]) \cdot \text{slash}[k_3] \cdot s_1}{2 \text{mw}^2 (\text{md}^2 + t)}

B. \: \Psi^- \Psi^+ \rightarrow \Psi^- \Psi^+

C. \: \Psi^0 \chi^0 \rightarrow \Psi^0 \chi^0

CsH[s_{1a}, l_{2a}, s_{3a}, l_{4a}]: = - (1/8)(1/(s - \text{m}^2))(l_2 \cdot (\eta_4 + \eta_5).s_1)(s_3 \cdot (\eta_4 + \eta_5).l_4)

C'. \: \chi^0 \Psi^0 \rightarrow \chi^0 \Psi^0

Same.

D. \: \Psi^+ \chi^0 \rightarrow W^+h \text{ and } \Psi^+ \Psi^0 \rightarrow W^+h

DtP[s_{1a}, s_{2a}]: = - (g/4)(1/(t - \text{md}^2))(1/\text{mw})(s_2 \cdot (\eta_4 + \eta_5).((\text{slash}[q_t] + \text{md}l_4) \cdot \text{slash}[k_3] \cdot s_1)

DsW[s_{1a}, s_{2a}]: = (g^2/\sqrt{2})(1/(s - \text{m}^2))

\[
(\text{Expand}[s_2 \cdot \eta[r].s_1]) \cdot (\eta[r, s] - q[s]q[s]/\text{m}^2) \eta[s, m]k_3[m]\text{.gammarep}\text{.indexmatch})
\]

Put in the kinematics & everything else.

What do I need for the calculations?

The \(\gamma\)-matrices: G0-G3 & \(\gamma\) gamma, the 4x4 identity: I4, the metrics, Minkowski scalar product, “slashed notation”: \(p_\mu \gamma^\mu = \text{slash}[p]\).

Then some kinematics for the 2\(\rightarrow\)2 scattering.

G0 = \{0, 0, 1, 0\}, \{0, 0, 0, 1\}, \{1, 0, 0, 0\}, \{0, 1, 0, 0\};

G1 = \{0, 0, 0, 1\}, \{0, 0, 1, 0\}, \{0, -1, 0, 0\}, \{-1, 0, 0, 0\};

G2 = \{0, 0, 0, -I\}, \{0, 0, I, 0\}, \{0, I, 0, 0\}, \{-I, 0, 0, 0\};

G3 = \{0, 0, 1, 0\}, \{0, 0, 0, -1\}, \{-1, 0, 0, 0\}, \{0, 1, 0, 0\};

G5 = \{-1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\};

I4 = \{1, 0, 0, 0\}, \{0, 1, 0, 0\}, \{0, 0, 1, 0\}, \{0, 0, 0, 1\};

56
\[\text{gamma} = \{\text{G0, G1, G2, G3}\};\]
\[\text{metr} = \{\{1, 0, 0, 0\}, \{0, -1, 0, 0\}, \{0, 0, -1, 0\}, \{0, 0, 0, -1\}\};\]
\[\text{mink}[p, k] := \text{metr}.k;\]
\[\text{slash}[p] := \text{metr}.\text{gamma};\]

**A.** \[\Psi^-\Psi^+ \rightarrow W^-W^+\]

**B.** \[\Psi^-\Psi^+ \rightarrow \Psi^-\Psi^+\]

**C.** \[\Psi^0\chi^0 \rightarrow \Psi^0\chi^0\]

**C’.** \[\chi^0\Psi^0 \rightarrow \chi^0\Psi^0\]

**D.** \[\Psi^+\chi^0 \rightarrow W^+h\]

\[p_1 = \{(s + m_d^2 - m_s^2)/(2\sqrt{s}), 0, 0, p\};\]
\[p_2 = \{(s + m_s^2 - m_d^2)/(2\sqrt{s}), 0, 0, -p\};\]
\[k_3 = \{(s + m_w^2 - m_h^2)/(2\sqrt{s}), k\sin[\theta], 0, k\cos[\theta]\};\]
\[k_4 = \{(s + m_h^2 - m_w^2)/(2\sqrt{s}), -k\sin[\theta], 0, -k\cos[\theta]\};\]
\[q_s = p_1 + p_2;\]
\[q_t = p_1 - k_3;\]
\[p = \sqrt{-m_d^2 + (s + m_d^2 - m_s^2)^2/4s};\]
\[k = \sqrt{-m_w^2 + (s + m_w^2 - m_h^2)^2/4s};\]
\[t = q_t.\text{metr}.q_t;\]

The in - and out - going fermion spinors. With zero momentum, then boosted to \(p\). (\(h\) is for easier summation over the spins.)


\[u1p = \{1, 0, 1, 0\};\]
\[ u1n = \{0, 1, 0, 1\}; \]
\[ v2p = \{0, 1, 0, 1\}; \]
\[ v2n = \{-1, 0, -1, 0\}; \]
\[ uf[p, -1] := \text{Boost}[p].u1n; \]
\[ uf[p, 1] := \text{Boost}[p].u1p; \]
\[ vf[p, -1] := \text{Boost}[p].v2n; \]
\[ vf[p, 1] := \text{Boost}[p].v2p; \]
\[ \text{vbar}[p, -1] := G0.vf[p, -1]; \]
\[ \text{vbar}[p, 1] := G0.vf[p, 1]; \]
\[ h = \{-1, 1\}; \]

\[ \text{DtPtab} = \text{Table}[\text{FullSimplify}[\text{DtP}[uf[p1, h[[i]]], \text{vbar}[p2, h[[j]]]]], \]
\[ \text{Assumptions} \rightarrow mw > 0 & \& mh > 0 & \& md > 0 & \& ms > 0 & \& s > 0, \{i, 2\}, \{j, 2\}] \]

\[ \left\{ \begin{array}{l}
0, -\frac{1}{4mw} \sqrt{\frac{1}{-ms^2 + (md + \sqrt{s})^2}} \sqrt{\frac{1}{-md^2 + (ms + \sqrt{s})^2}} (md + ms + \sqrt{s}) \left( \sqrt{md^4 + (ms^2 - s)^2 - 2md^2 (ms^2 + s)yn + ((md - ms)^2 - s)} yp \right) \\
\frac{1}{4mw} \sqrt{\frac{1}{-ms^2 + (md + \sqrt{s})^2}} \sqrt{\frac{1}{-md^2 + (ms + \sqrt{s})^2}} (md + ms + \sqrt{s}) \left( -\sqrt{md^4 + (ms^2 - s)^2 - 2md^2 (ms^2 + s)yn + ((md - ms)^2 - s)} yp \right), 0 \end{array} \right\} \]

Let us see how it behaves for high energies!

As \( s \to \infty \), the small parameter is \( w^2 = \frac{ms^2}{s} \to 0 \).

\( s = mw^2/2w^2; \)

\[ \text{FullSimplify}[\text{Series}[\text{DtPtab}.\{yp \to y1 + y2, yn \to y1 - y2\}, \{w, 0, 1\}], \]
\[ \text{Assumptions} \rightarrow ms > 0 & \& md > 0 & \& mw > 0 & \& mh > 0 & \& w > 0 \]

\[ \left\{ \begin{array}{l}
0, \frac{g}{2w} - \frac{g (-2mdmsy1 + (md^2 + ms^2) y2) w}{4mw^2} + O[w]^2 \\
\frac{g}{2w} + \frac{g (md^2 + ms^2) y1 - 2mdmsy2) w}{4mw^2} + O[w]^2, 0 \end{array} \right\} \]
This graph is growing with $\sqrt{s}$ and naively the following s-channel graph cannot compensate it. However, when looking at mass-eigenstates instead of $\chi^0/\Psi^0$, the $\Psi^-\eta(W$ coupling keeps the $\gamma_5$ as well, and growing with $\sqrt{s}$, compensating exactly the above t-channel graph. Calculations are in P+P0toWh_inAssymSD.nb notebook.

$$D^*, \Psi^+\Psi^0 \rightarrow W^+/h$$

\[ p_1 = \{(s + m_d^2 - m_s^2)/(2\sqrt{s}), 0, 0, p\}; \]
\[ p_2 = \{(s + m_s^2 - m_d^2)/(2\sqrt{s}), 0, 0, -p\}; \]
\[ k_3 = \{(s + m_w^2 - m_h^2)/(2\sqrt{s}), k\text{Sin}[\theta], 0, k\text{Cos}[\theta]\}; \]
\[ k_4 = \{(s + m_h^2 - m_w^2)/(2\sqrt{s}), -k\text{Sin}[\theta], 0, -k\text{Cos}[\theta]\}; \]
\[ q_s = p_1 + p_2; \]
\[ q_t = p_1 - k_3; \]
\[ p = \sqrt{-(m_d^2 + (s + m_d^2 - m_s^2)^2)/(4s)}; \]
\[ k = \sqrt{-(m_w^2 + (s + m_w^2 - m_h^2)^2)/(4s)}; \]
\[ t = q_t\text{metr}.q_t; \]

The in- and out-going fermion spins. With zero momentum, then boosted to p. (h is for easier summation over the spins.)

\[
\]
\[
\]
\[
\text{u1p} = \{1, 0, 1, 0\};
\]
\[
\text{u1n} = \{0, 1, 0, 1\};
\]
\[
\text{v2p} = \{0, 1, 0, -1\};
\]
\[
\text{v2n} = \{-1, 0, 1, 0\};
\]
\[
\text{uf}[p_.,-1] := \text{Boost}[p].\text{u1n};
\]
\[
\text{uf}[p_.,1] := \text{Boost}[p].\text{u1p};
\]
\[
\text{vf}[p_.,-1] := \text{Boost}[p].\text{v2n};
\]
\[
\text{vf}[p_.,1] := \text{Boost}[p].\text{v2p};
\]
vfbar[p\_\_1] := G0.vf[p, -1];

vh\_bar[p\_\_2] := G0.vf[p, 1];

h = \{-1, 1\};

DsWtab = Table[FullSimplify[{DsW[uf[p1, h[i]], vfhbar[p2, h[j]]]} /. ms -> md],
    Assumptions -> mw > 0 && mh > 0 && md > 0 && s > 0, \{i, 2\}, \{j, 2\}]

\[
\begin{align*}
\left\{ \left\{ \frac{g^2 \sqrt{mh^4 + (mw^2 - s)^2 - 2mh^2 (mw^2 + s) \sin[\theta]}}{2 \sqrt{2} \ (mw^2 - s)} \right\}, \\
\frac{g^2 md \sqrt{mh^4 + (mw^2 - s)^2 - 2mh^2 (mw^2 + s) \cos[\theta]}}{s} \right\}, \\
\left\{ \frac{-g^2 \sqrt{mh^4 + (mw^2 - s)^2 - 2mh^2 (mw^2 + s) \sin[\theta]}}{2 \sqrt{2} \ (mw^2 - s)} \right\}
\end{align*}
\]

Let us see how it behaves for high energies!

As s \to \infty, the small parameter is \( w^2 = \frac{m_w^2}{s} \to 0. \)

\( s = mw^2/w^2; \)

FullSimplify[Series[DsWtab, \{w, 0, 1\}], Assumptions -> md > 0 && mw > 0 && mh > 0 && w > 0]

\[
\left\{ \left\{ -\frac{g^2 \sin[\theta]}{2 \sqrt{2}} + O[w]^2 \right\}, \left\{ -\frac{g^2 md \cos[\theta] w}{\sqrt{2} mw} + O[w]^2 \right\} \right\}, \left\{ \left\{ -\frac{g^2 md \cos[\theta] w}{\sqrt{2} mw} + O[w]^2 \right\}, \left\{ \frac{g^2 \sin[\theta]}{2 \sqrt{2}} + O[w]^2 \right\} \right\}
\]

**Partial wave bounds**

For the bound:

\( a_0 = \frac{1}{32\pi} \int_{-1}^{1} |M| d(\cos\theta) \leq \frac{1}{2}, \) when M does not depend on the angle; \( M \leq 8\pi \approx 25.1 \)

\( N[8\Pi] \)

25.1327

C. \( \Psi^0 \chi^0 \to \Psi^0 \chi^0 \)

\( \left\{ \frac{y_1^2}{2}, \frac{y_1 y_2}{2}, \frac{y_2^2}{2} \right\} \leq 8\pi \to \{ y_1 < 4\sqrt{\pi}, y_1 y_2 \leq 16\pi, y_2 \leq 4\sqrt{\pi} \} \)

\( \left\{ \frac{y_1^2}{2}, \frac{y_1 y_2}{2}, \frac{y_2^2}{2} \right\} \leq 25.1 \to \{ y_1 < 7.1, y_1 y_2 \leq 50.3, y_2 \leq 7.1 \} \)
Translate this into masses: \( v = 246 \text{GeV} \rightarrow m_y = \frac{\sqrt{2} \pi}{\sqrt{2}} \leq 1.23 \text{TeV} \).

\( v = 246; \)

\( \sqrt{2} \pi v \)

\( N[\sqrt{2} 8\text{Pi}]v/\sqrt{2} \)

2\( \sqrt{2} \pi v \)

\( N[\sqrt{2} 8\text{Pi}]v/\sqrt{2} \)

1233.26

\( N[8\text{Pi}] \)

\( N[8\text{Pi}v] \)

25.1327

6182.65
Appendix B

Experimental bounds on $\kappa_\gamma$ and notation conversion

This appendix contains the experimental bounds on the couplings in the effective model for the extension of the Standard Model with a singlet scalar that I study in chapter 4 that has the following lagrangian,

$$\mathcal{L}_{\text{eff}} \supset \frac{\alpha_{\text{em}}}{4\pi s_w^2} \frac{\kappa_W}{4m_S} S W^\mu W^\nu_{\mu\nu} + \frac{\alpha_{\text{em}}}{4\pi c_w^2} \frac{\kappa_B}{4m_S} S B_{\mu\nu} B^{\mu\nu} + \frac{\alpha_s}{4\pi} \frac{\kappa_g}{4m_S} S G^a_{\mu\nu} G^{a\mu\nu}$$

(B.1)

The scalar coupling to the photons can be expressed as $\kappa_\gamma = \kappa_W + \kappa_B$.

From the experiment, interpreting the excess as the process $pp \rightarrow S \rightarrow \gamma\gamma$, one can constrain $\Gamma_{S\rightarrow\gamma\gamma} = \Gamma(S \rightarrow \gamma\gamma)$ after making assumptions on the production of $S$ and then translate it to the coupling $\kappa_\gamma$. I used the analysis of [12], their illustration is shown in figure B.1. The decay width is the following,

$$\Gamma_{S\rightarrow\gamma\gamma} = \frac{1}{64\pi} \left( \frac{\alpha_{\text{em}} \kappa_\gamma}{4\pi} \right)^2$$

(B.2)

The yellow region shows what can be fitted from the diphoton rate assuming $gg \rightarrow S \rightarrow \gamma\gamma$. The upper green boundary where the total width is $\Gamma_{\text{tot}} \approx 0.06$, the best width fit from ATLAS with large $S$ width.

$$\frac{\Gamma_{S\rightarrow\gamma\gamma}}{m_S} \approx 2 \times 10^{-3}$$

(B.3)

The lower blue boundary assumes minimal total width, $\Gamma_{\text{tot}} = \Gamma_{S\rightarrow gg} + \Gamma_{S\rightarrow\gamma\gamma}$. On one end, the gluon fusion dominates the production of $S$, $\Gamma_{S\rightarrow gg} \gg \Gamma_{S\rightarrow\gamma\gamma}$.

$$\frac{\Gamma_{S\rightarrow\gamma\gamma}}{m_S} \approx 1.1 \times 10^{-6}$$

(B.4)

On the other end of the blue region of minimal total decay width, the photon fusion dominates
in the production, $\Gamma_{S\rightarrow \gamma\gamma} \gg \Gamma_{S\rightarrow gg}$,

$$\frac{\Gamma_{S\rightarrow \gamma\gamma}}{m_S} \approx 4 \times 10^{-5} \quad (B.5)$$

These give the range of the $S\gamma\gamma$ coupling

$$\kappa_{\gamma} \in [23.7, 143.1] \quad (1012.0) \quad (B.6)$$

where the lower end is from the gluon fusion dominated production, while the upper bound is from the photon fusion dominated region. In parenthesis is given the ATLAS best fit of large $S$ width.

These $\kappa$ values are in agreement with other estimations in the the literature. As several different notations are used for the couplings, here I give a brief review of the conversion between them.

In [42], the authors use the effective lagrangian as follows with the new resonance denoted as $\phi$, including a derivative coupling to the Higgs as well,

$$\mathcal{L}_{\phi,\text{eff}} = \phi \left( \frac{1}{f_g} (G_{\mu\nu})^2 + \frac{1}{f_B} (B_{\mu\nu})^2 + \frac{1}{f_W} (W_{\mu\nu})^2 + \frac{1}{f_H} |D_\mu H|^2 (+\ldots) \right) \quad (B.7)$$
The relation between the above dimensionful couplings and the dimensionless $\kappa_B$ and $\kappa_W$ couplings can be read off,

$$\frac{1}{f_B} = \frac{\alpha_{\text{em}} \kappa_B}{4\pi c_w^2 4m_S} \quad \text{(B.8)}$$
$$\frac{1}{f_W} = \frac{\alpha_{\text{em}} \kappa_W}{4\pi s_w^2 4m_S} \quad \text{(B.9)}$$

In [13], the authors only consider the scalar coupling to the photons and gluons with the following effective lagrangian,

$$\mathcal{L}_{S,\text{eff}} = \frac{e^2}{4v} c_{s\gamma\gamma} S A_{\mu\nu} A_{\mu\nu} + \frac{g_s^2}{4v} c_{sgg} S G_{a\mu\nu} G_{a\mu\nu} \quad \text{(B.10)}$$

The dimensionless photon coupling can be related to $\kappa_\gamma = \kappa_B + \kappa_W$,

$$c_{s\gamma\gamma} = \frac{v}{m_S} \frac{\kappa_\gamma}{16\pi^2} \quad \text{(B.11)}$$

where the authors take out the Higgs VEV $v$ as the relevant scale instead of the new mass $m_S$. 
Bibliography


65


Summary

The Standard Model (SM) of particle physics is one of the most successful theories in the past decades. It describes all the observed phenomena in high energy experiments. However, it is not a fundamental theory, since there are observations that the SM cannot explain, such as the presence of dark matter. On the other hand, ultraviolet complete theories, for example supersymmetric models, predict a large number of new particles that are not observed and their huge parameter space makes it difficult to compare them with ongoing experiments. However, to understand the new phenomena, it is important to know their effects at the available energies.

One alternative is the effective description, where the particle content is integrated out above a given cut-off scale and appear in the form of non-renormalizable operators with dimensionful couplings. One limitation of effective theories is that they lose predictive power at energies above their cut-off scale. Another way out is introducing simplified models that extend the SM with the relevant sector from a larger theory.

In this thesis, I use perturbative unitarity as a theoretical tool to investigate extensions of the SM. The unitarity of the scattering matrix ensures that the probabilities of the events add up to one. In effective models, there are processes with amplitudes that grow with the energy and violate unitarity above a scale, that is the cut-off of the theory. In simplified models, from unitarity conditions one can establish bounds on the couplings that appear in the scatterings.

After a short presentation on how unitarity considerations helped formulating the electroweak theory, I introduce two models and study with perturbative unitarity to complement the experimental constraints. The first is a simplified model for dark matter that extends the SM with a vector-like pair of weak doublet fermions and a singlet. Due to the Yukawa coupling between the new fermions, there is a mixing in the new neutral sector, making analytical studies rather difficult. Dark matter searches constrain the parameter space and it is possible to expand around the experimentally preferred regions, where I carry out an analytical study, focusing on the properties of the dark matter candidate. Then, I investigate two-particle scatterings in the model, requiring unitarity, and give a perturbative limit on the Yukawa couplings, $|y_{1,2}| \leq 7.1$.

The other model is an effective theory that extends the SM by a singlet scalar, which may explain the recently announced excess in the diphoton final state by the ATLAS and CMS experiments. Here I study two-to-two gauge boson scattering amplitudes via the new scalar particle which is growing with the energy. Applying the unitarity condition, I can establish the validity range of the model, where new mechanism is expected to compensate the processes. That also helps discriminate between scenarios that are experimentally viable such as the large scalar width that is preferred by ATLAS, but as I show, it is in conflict with unitarity. Finally, I present a further extension with a new heavy fermion that can induce the effective operators at loop level and express the constraints on the new fermion mass and Yukawa coupling.
Összefoglalás

A részecskefizika Standard Modellje (SM) az utóbbi idők egyik legsikeresebb modellje, amely leírja a nagyenergiás kísérletekben megfigyelhető összes jelenséget. Mindemellett ez nem egy fundamentális elmélet, mert egyes megfigyeléseket a SM nem tud megmagyarázni, mint például a sötét anyag jelenlétét. Másfelől az ultraibolya teljes elméletek, mint a szuperszimmetrikus modellek, nagy mennyiségű új részecskét jósolnak, amelyeket eddig nem figyeltünk meg, valamint a túl nagy paraméterterületnek megnehezíti a kísérleti megfigyeléseket való összevetést. Ennek ellenére az új jelenségek megértésében fontos, hogy tudjuk milyen hatásaik vannak az általunk megfigyelhető energiákon.

Alternatívát nyúttanak az effektív leírások, ahol egy adott lebonyolított energia felett kiintegráljuk a további részecskék, ennek hatása dimenziós csatolásokat tartalmazó effektív operátorkban jelenik meg. Márszélti bevezethetünk egyszerűsített modelleket, amelyek egy konkrét jelenség szempontjából releváns szektorral egészítik ki a SM-t.

A dolgozatban a perturbatív unitaritást használom az SM kiterjesztéseinek vizsgálatára. Effektív modellekben megjelennek olyan folyamatok, amelyek az energiával nőnek és sértik az unitaritást egy adott energiaskála felett, ez az elmélet érvényességi határa. Az egyszerűsített modellekben a szórási folyamatokban is megjelenő új paraméterekre lehet korlátozni az unitaritás segítségével.

A dolgozatomat az unitaritás feltételek áttekintésével kezdem, majd alkalmazom két modellt, kiegészítve a kísérletekből jövő korlátokat. Az első egy egyszerűsített modell, amelyben egy vektor-szerű gyenge dublett fermion pár és egy szinglet fermion egészíti ki a SM-t.

Az új fermionok közti Yukawa csatolás miatt keveredik és jön létre az új semleges szektorban, ami megnehezíti az analitikus tanulmányozást. A paraméterteret korlátozzák a sötét anyag megfigyelése, és a kísérletek által preferált tartományok körül kifejthetőek a paraméterek, amelyekre analitikus leírást adok a sötét anyag-jelölt részecske tulajdonságaira koncentrálva. Ezután két-részecske szórásokat vizsgálok, amelyekre kísért a perturbatív unitaritási feltételeket, korlátokat adok a Yukawa csatolás erősségeire, \( |y_{1,2}| \leq 7.1 \).