Modelling of Geoid, calculation of gravity mass with inversion method.

DOCTORAL DISSERTATION

Made:

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1. **Introduction, objectives of the work**

My previous dissertation examined gravity anomalies. In the 1997 dissertation I examined the theory of Professor György Barta in three-dimensional case. I tried to approach the geoid as a result of two rotationally symmetric weight distribution. In my 2003 dissertation I dealt with the interpretation of local gravity anomalies as hot spots and subduction zones and took over the major gravity anomaly models published until then. Calculations were carried out about hot spots and subduction zones caused gravity anomalies (the direct problem solving). On this basis, I came to the conclusion with my consultant to present my thesis: can I define the structure of hot spots about the inversion of the above gravity anomalies.

In recent years, increased the number of publications, which has a significant advantage in the development of computer technology in addition to complicated mathematical models. These are like mantle flow models, as well as multi-parameter inversion calculations. The plums modelling is performed as a function of different ways of the desired physical parameters examined. The mantle flow models to define the parameters of the updraft of different viscosity parameters and modelling of flows. The seismic tomography provides information about the Earth's interior through the velocity inhomogeneity.

The gravitational anomalies examined by me are consequences of density inhomogeneity in the crust and mantle. The density surplus in the crust, the magma chambers and the magma density decrease in the mantle are the consequences of magma uplift. Of course, the physical parameters are not independent of each other, but because the relationship is very complicated so we can get new or different information from different methods.

My aim is the continuation of modelling of the gravity anomalies and geoid, which was started in my dissertation with increasing the details of the models, or by using other methods, for example, to determine the key parameters by inversion, furthermore the collection of surface data and satellites data which are needed for the calculations. My aim the calculation of the gravity effects of above described and modelled upper mantle and the topography.

2. **Methods**

The data sources used

The National Oceanic and Atmospheric Administration (NOAA) in the USA freely available and downloadable database is the ETOPO1 1 minute resolution elevation model (Amante, Eakins, 2009). Gravity data were available from the National Geophysical Data Center (NGDC), published by the US "Gravity Edition 1994 CD" (Hittleman et al., 1994). From the data CD, the following date which are called free air gravity anomaly were used, which were
determined in March 1993 the gravitational measurements and the altimetry data of sea surface of GEOS3 / SEASAT / GEOSAT by Rapp and Basic (1992), 72-72 and 0 - 360 degrees across the Earth 7.5 minute resolution.

**Correction calculation methodology**

The available Rapp92 had to corrected with the topographical, bathymetric correction, and the Bouguer gravity anomaly data. The adjustments to the sign which shall be considered to be dependent upon the location of the mass relative to the reference plane, which is caused by the gravitational effects if over a positive effect if it is negative below it. In the case of islands in the reference plane in the literature and I, was in the sea level, while in the case of the continental areas on the surface.

The principle laid down by Vening Meinesz the isostatic effect of an island volcano for the discharge of volcanic basalt higher density material such as the environment, it causes additional load, this load of the lithosphere (as a flexible disc) takes the sag. A wide expanse of negative gravity anomaly corresponding to the Vening Meinesz's regional isostatic model comes from load of the crust (Heiskanen, Vening Meinesz, 1958).

In the current study is important hot spots can be linked to the projections of the geoid altitude and the topography is greater than +1 (Hawaii of 3.5 - 6 m / km, Iceland 1.5 m / km, Reunion 2.2m / km), which forecast a relatively large depth and a dynamic compensation current is present.

Isostatic effect as defined by Vening Meinesz not modelled because of the large proportion of the geoid topography, so the isostatic equilibrium does not occur, and assume a dynamic effect which is that the thermal column expand at the boundary of the mantle and the crust and occurs a mass miss.

So, this isostatic, cratonic roots and thermal column sprawl geometrical parameters wanted to receive as a result of the inversion calculations.

Using the above, I got a residual gravity anomaly, also known as the Bouguer anomaly data, which is expected to inversion. Topography and bathymetry data came from ETOPO1 as it has previously been described. Density data required for calculation bought form the literature. Polygonal prismatic gravitational effect to the relative impact of sea level of the topography was calculated form the 500m contour levels of ETOPO1, determined Plouff (1976). The bathymetric correction was calculated similar way to the topographical corrections, which was described above.

After the above adjustments and the removal of the Bouguer residual anomaly I received the residual gravity anomaly, which I carried out the inversion.
Inversion of the gravity anomalies of hot spot

During the inversion I assumed the anomalies caused by a mass which is a vertical cylinder. The required equations of this form gravity effect are in the published literature (Nabighian, 1962; Nagy, 1965).

The model

The cylinder can be also a good choice because of its well-published and inversion of mathematical equations to be described. In case of a cylinder the defining four parameters necessary that the density and the physical dimension (radius, height and depth). In the calculations of physical parameters of the selected models were determined.

The inversion

Nonlinear inversion obtained if the density or density contrast known and suppose we wish to determine the parameters of the selected geometric model.

The inversion was carried out by Bayesian method with the use in case of Laplace and Gaussian distribution (Box and Tiao, 1973; Tarantola, 1987). The essential of the Bayesian inversion is to create the a posteriori model by combining the a priori model and the measurement data. It is expected to describe the reality better than the a priori model.

The solution of the minimum problem I used simplex and simulated annealing methods (Walsh, 1975, Kirkpatrick et al., 1983).

In my calculations, three parameter: the radius of the cylinder model, roof and base depth wanted to determine, however, in addition to pre-recorded variable density.

3. Results and theses

Thesis 1. - In relation to the free air gravity anomalies

In the dissertation studied six cases of free air gravity anomalies compared stated that the case when the islands developed on the Pacific plate, shape of the anomaly is similar and generally concentric around the island. The anomaly compared to the centre of the island in the event of a shift can be observed if the plum and the Pacific plate has moved relative to one another since the formation of the island, these are Easter Island and Reunion Island. The free air gravity anomaly in the continental plate cases compared to the oceanic plates are similar in shape, but there are a definite shift in the place of the hot spots, which is published in the literature I experienced it in the case of Ethiopia.
Thesis 2. - In relation to the residual gravity anomalies
The residual gravity anomaly cases in the oceanic plates are circular symmetrical and well-modelled, when the hot spots positioned far away from the edges of the oceanic plate. In the case of Easter Island where are many microplate and mantle upwelling is located relatively close to the plate edge, in the residual gravity anomaly the effects of the acting bodies are recognisable, but they were not nearly circular symmetrical. The parallel volcanos identified as local maximum on the residual gravity anomaly (Hawaii main island, Reunion Island, Iceland), in spite of the fact that local measurements have been only partially taken into account and a high-wavelength part is kept, appeared and were invertible, which was correlated with bibliographic data. In the case of Ethiopia and Yellowstone anomalies they were not nearly circular symmetrical.

Thesis 3. – About inversion
In nonlinear case, the Bayesian inversion solution for the chosen vertical cylindrically symmetric fixed density of three varying-parameter (radiation, bottom depth, top depth) model experience is for simplex and simulated annealing processes the following, the simplex method is more sensitive to steps, in case of few and high deviation a priori information gives better results, more density sensitivity, it requires less CPU time than simulated annealing process. I concluded that between the Gaussian and Laplacian distributions the Laplace distribution is more stable, if the Gaussian distribution has result, then with a relatively wide scale start parameter was resulted approximately the same results with small deviation. The change of the density was not almost affected for the radius, whereas it affected the mass position so as the density decreases the mass "stretched" and "creeping" up. I found it is better to tend the parameters underestimate and the inversion solution was first established with the simplex method and Laplace distribution. While earlier gravity anomalies modelled on other celestial bodies considerably easier morphology, assuming the model proved to be usable terrestrial environments. (Kis et al., 2007.; Kis et al., 2010; Kis et al., 2011; Kis et al., 2012)

Thesis 4. – In relation with isostasy
The result of Vening Meinesz's isostasy published literature I got a good approximation in case the main island of Hawaii and Reunion Island. The Iceland and the Ethiopian residual gravity anomaly is well correlated with the literature on these areas where flexural isostasy calculations used. I managed to separate the Vening Meinesz's isostasy wavelength and amplitude to the plum sprawl on the boundary of Earth mantle lithosphere. (Toronyi, 2016)
Thesis 5.- With the plum spreading on the boundary of the lithosphere
In two specific cases (the main Hawaiian Islands and Reunion Island) were identified the parameters of a cylindrically symmetric model which is in the Earth mantle, with inverse and direct method, as a possible result of the meeting of a plum with a margin of the lithosphere. (Toronyi, 2016)

4. Conclusions
1. In connection with plum spreading on the boundary of the lithosphere
Based on the Thesis 5 it is worthy if additional calculations made on island volcanos on the mid-oceanic plate, where hot spots are found.

5. Short reference list
6. Publication about thesis

7. Other publication