Thesis statements

Integrable form factor program and its applications

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Objectives of the work

In my thesis, I studied the integrable form factor bootstrap program and some of its applications. The importance of the knowledge of the form factors, i.e. matrix elements of local operators between asymptotic states, lies in their appearance in different physical quantities. The Wightman theorem states that all information about a quantum field theory is encoded into the knowledge of the correlation functions of local operators. Taking such a correlator and inserting a complete set of multiparticle states between the operators results in its spectral representation, involving the form factor functions. They play an essential role in the theory of finite temperature mean values of operators, or in the description of non-integrable perturbations of integrable models. Recently, there has been an increasing interest in the form factor bootstrap program, and in its slightly generalised versions, due to their emergence in the domain of the three-point structure constants in the AdS/CFT correspondence.

A \(1 + 1\) dimensional integrable field theory is defined by its scattering matrix which satisfies certain consistency requirements, the so-called bootstrap axioms. An impurity, localised in space, may break the integrability of the system. In interacting theories, an integrable impurity is either a purely transmitting defect or a purely reflecting boundary. Furthermore, the defect transmission- and the boundary reflection matrices must satisfy the corresponding bootstrap axioms. The scattering-, the transmission- and the reflection matrices contain all the on-shell information about a model.
However, the complete solution of a model includes the determination of all the correlation functions of local operators. The form factors are the fundamental building blocks in the spectral representation of correlation functions, hence their knowledge is essential. In integrable models, the form factors satisfy certain consistency conditions, the form factor bootstrap axioms. These axioms are complicated functional relations connecting the matrix elements between different states. Although the bootstrap program for the scattering-, the transmission- and the reflection matrices is well known since long time, the defect- and boundary form factor programs are much less analysed. In my research, I studied these latter ones. The solutions of form factor bootstrap axioms were determined in [1, 2] for both the defect and boundary versions of the scaling Lee-Yang model. The axiomatic approach for the boundary changing operators was introduced in [3] which was also elaborated in the cases of the free boson and the scaling Lee-Yang models.

During the last years, the maximally supersymmetric Yang-Mills theory was intensively studied, due to the discovery of the AdS/CFT duality. The Yang-Mills gauge theory is a close relative of quantum chronodynamics, obeying several similar properties. The AdS/CFT duality states that this gauge theory is dual to the theory of superstrings propagating on the direct product of a five dimensional Anti-de Sitter space and a five dimensional sphere such that, the strongly interacting - thus non-perturbative - regime of the gauge theory corresponds to the perturbative regime of string theory. Later on, it turned out that this correspondence is integrable: choosing the uniform
light cone gauge on the string theory side, the model is mapped to a $1+1$ dimensional integrable, non-relativistic, massive scattering theory. During the last decade, the finite volume spectral problem got in the focus of research, achieving important advancement towards the complete solution of the problem. However, being the Yang-Mills theory conformal, its complete solution, besides the spectral information, would also include the determination of the three-point functions. Recently, the analysis of these three-point functions got more and more attention. A special type, the so-called Heavy-Heavy-Light three-point function establishes an interesting connection to the theory of form factors: according to the conjecture of Bajnok, Janik and Wereszczynski, these three-point functions correspond to diagonal form factors in finite volume [6]. In my thesis, I studied this connection in the perturbative regime of the Yang-Mills theory. The leading order study of short operators showed that the conjecture of Bajnok, Janik and Wereszczynski indeed holds at weak coupling, and the infinite volume connected form factors were also determined [4].
Thesis statements and conclusions

Form factors of defect operators in the scaling Lee-Yang model

The form factor bootstrap program for defect operators, as well as the generic parametrisation of the form factors have been known [7]. The parametrisation contains an operator-dependent polynomial factor which, due to the form factor axioms, satisfies certain recurrence relations. The explicit solutions for these polynomials determine the form factors of the operator. For low particle number, these polynomials can be calculated easily, however, by increasing the particle number, the problem quickly becomes cumbersome. I gave an explicit form for the polynomials for the primary fields of the model [1]. The resulting solutions were checked numerically: comparing the spectral decomposition of the two-point functions to the ones calculated from the high energy conformal field theory supports the validity of the solutions.

Form factors of boundary operators in the scaling Lee-Yang model

Similarly to the defect case, the bootstrap approach of form factor of boundary operators and their generic parametrisation, containing a polynomial factor, have been known since long time [8]. The axioms result in recursive relations for the polynomials which can be determined explicitly for low particle number, however, the problem becomes complicated as the number of particles increases. In the boundary version of the scaling Lee-Yang model,
the recursion polynomials, at leading order in the rapidity, coincide with the ones in the bulk case, therefore the boundary solutions can be calculated as corrections to the bulk ones. In my thesis, I gave the explicit form of these polynomials for all the primary fields and for both integrable boundary conditions [2].

**Bootstrap axioms for the boundary changing operators**

Although the form factor axioms for bulk-, boundary- and defect operators are known since long time [7, 8, 10], the case of boundary changing operators is much less studied. In the paper [3], by extending the previous results [11], the bootstrap approach was formulated for these boundary changing operators. The consistency of the system of axioms was also checked. The completeness of the axioms is demonstrated by proving that, in the scaling Lee-Yang model, the set of solutions of the bootstrap program is isomorphic to the space of local operators of the theory. A systematic method of solving the axioms as well as their generic parametrisation are also presented.

**Form factors of boundary changing operators in the free boson theory**

In the free boson theory, the form factor bootstrap axioms take a relatively simple form that were solved explicitly in [3]. The theory obeys a Lagrangian description which allows us to perform independent calculations. Defining the system in finite volume and mapping the problem to the already solved equations for open-closed string vertex
In the case of the scaling Lee-Yang model, as the first step, an important building block of the generic parametrisation, the minimal one particle form factors of boundary changing operators were determined. The parametrisation further contains a polynomial term for which recursive equations can be derived from the axioms. The polynomials appearing in these recurrence relations are closely related to the ones for the identity boundary, therefore the solutions for the identity boundary [2] can be generalised to the boundary changing case [3]. The resulting solutions were numerically checked: the truncated spectral series of the two-point functions at two particle level agrees with the ones computed from the ultraviolet conformal field theory, which supports the results.
Weak coupling check of the conjecture of Bajnok, Janik and Wereszczynski

Analysing the so-called Heavy-Heavy-Light three-point functions of the AdS/CFT integrable model, Bajnok, Janik and Wereszczynski formulated a conjecture for their form for any coupling [6]. They supported their conjecture by several examples at the strong coupling limit, while the weak coupling study was presented in [4]. In this limit the problem is to calculate the three-point functions of the $\mathcal{N} = 4$ supersymmetric Yang-Mills theory at tree level. This can be mapped to the computation of the diagonal matrix elements of local spin operators between on-shell Bethe states of the XXX$_{1/2}$ Heisenberg spin chain in finite volume. This can be achieved by means of the Algebraic Bethe Ansatz technique and using the solution of the Quantum Inverse Scattering Problem. A generic argumentation was presented to show that the volume dependence of the diagonal matrix elements of spin operators is captured by minors of the Gaudin determinant, which supports the conjecture of Bajnok, Janik and Wereszczynski. The multipliers of the minors are the connected diagonal form factors in infinite volume, which can be read off from the explicit results. By analysing these form factors for short operators with low particle numbers, a conjecture was formulated for generic number of excitations.
References

Publications containing the thesis statements:


Publications independent of the Ph.D thesis:


Further references:


