Ph.D. Dissertation

Balázs Szabó

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Shear zones in dry granular materials

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Balázs Szabó

Institute for Solid State Physics and Optics
Wigner Research Centre for Physics
Hungarian Academy of Sciences

Supervisor: Tamás Börzsönyi, Ph.D.

EÖTVÖS LORÁND UNIVERSITY
Graduate School of Physics
Head of the School: Prof. Dr. László Palla, D.Sc.

Materials Science and Solid State Physics Program
Program Leader: Prof. Dr. János Lendvai, D.Sc.

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1. Introduction to Granular Materials

Granular materials can be found everywhere in the nature due to erosion and other geological processes. The basic mechanical properties of the granular samples are still not completely understood because they can depend sensitively on the packing density, particle shape, size distribution, grain surface properties, interparticle material, velocity, external forces, and the memory of the actual sample can also play an important role. Therefore, instead of a universal description of the behavior of sheared granular media, specific models or equations have been developed for different problems.

1.1 General properties of the granular materials

Granular materials consist of large number of macroscopic grains. These particles are driven by forces known from classical physics: friction with each other and the container walls, hard core repulsion or elastic interaction between the grains and external forces like gravity are the most common forces occurring in granular experiments. However, due to the large number of bodies a new experiment or simulation often leads to an unexpected behavior of such a granular system. In the following I will give a few examples of phenomena which demonstrate the peculiar features of granular samples. The three most relevant general features related to the subject of this dissertation are presented in separate sections. In Sec. 1.2 the Reynolds dilation is introduced, Sec. 1.3 is denoted to the Janssen-effect and the different flow regimes are described in Sec. 1.4.

In granular flows particles often collide, and these collisions are not perfectly elastic. Therefore all the granular systems are dissipative as kinetic energy is converted to thermal energy by the large number of grain collisions. There are different ways to keep the sample flowing: energy can be pumped into the system by gravity, external mechanical forces, pumping air or other interparticle fluid or even by applying an external electromagnetic field. A nice overview of the frequently used experimental
geometries can be found in the work of the GDR MiDi group [1]. Gravity is very important for example in inclined plane flows or characterizing the physics of sand piles. External mechanical forces can be vibrations of the walls, which is a well controlled way for observing pattern formation (see for example the review paper by Aranson and Tsimring [2] or the paper by Umbanhowar et al. [3]), or a non-periodic movement of a container wall (e.g. shear flows).

The microscopic structure of a granular sample is described by 6 degrees of freedom per particle in general. The high total degree of freedom of the particles allows the sample to remember its previous states. Therefore the response of a granular system to an external effect can strongly depend on the history of the actual sample. A special attention is needed to generate the initial condition for an experiment or a simulation, otherwise the reproducibility of the investigation may not be satisfied. When reaching a stationary flow field is possible in the actual experiment, the system can forget its initial state, and the microstructure is purely determined by the actual flow field. This is referred to as critical state, especially in the engineering literature.

The typical size distribution of the grains can play an important role in the observed dynamics. For monodisperse spheres crystallization occurs frequently. Cyclic shearing of such a sample leads to a crystallized state resulting in a denser packing of the grains [4, 5], but continuous shear can also lead to a crystallized state [6, 7]. Applying shear and vibrations simultaneously allows us to control precisely the shear induced crystallization [8, 9]. The crystals can start to grow at the container walls (especially if they are smooth), but nucleation starts also in the bulk of the sample [10].

If the granular particles have a wide size distribution, segregation occurs very often in granular flows independently of the geometry of the setup. A well known example for segregation is the Brazil-nut effect, where due to shaking the large grains move to the top of the sample. When a rock avalanche starts to flow on an inclined plane, smaller objects go below the bigger ones, which is called kinetic sieving. But not only size differences can lead to segregation. Grains with lower density will also rise during shaking. Previous studies analyzed shear induced segregation driven by size [11] or density [12] difference. A comprehensive review paper by Ottino and Khakhar describes various aspects of segregation in granular materials [13].
When the volume of a granular sample is kept fixed, the observed behavior strongly depends on the packing fraction, which is the volume occupied by the individual particles divided by the total volume of the sample. Below a certain packing density particles can flow freely, but above the critical packing fraction the system becomes rigid, and particles can not move anymore. This jammed state is not special for granular materials, it can be observed for foams, colloidal or molecular systems [14], or it can even occur in our everyday life in the form of traffic jams [15] for example. The phase transition to a jammed state in general is not only determined by the packing density of the sample, as also the external load on the system or the temperature can influence the critical packing fraction [16]. It is numerically proven that for granular particles a critical packing density exists at zero temperature (no continuous flow or vibrations used) and zero applied stress, where the particles start to jam, and this critical point can be captured by monitoring many different physical parameters [17]. In experiments power law behavior has been observed near the critical point e.g. for the coordination number and for the pressure [18]. For granular materials particles can locally get stuck without experiencing a phase transition or a precise control of the packing density, which is referred to as clogging. This clogging happens when arches form during the flow which are stable enough to hold the material above it, typically when the grains have to flow through a hole smaller than about five particle diameters [19]. This arch formation depends strongly on the particle shape [20].

In addition to the properties of the grains, the interparticle fluid can also be varied as wet and dry sand have completely different physical properties. In many cases the interparticle material is air, but often the particles are immersed in a liquid or liquid bridges are generated [21]. Typically for slow motions, where the maximal speed of the grains is much less than their speed in free fall, air has a little influence on the observed dynamics of the grains and therefore it can be neglected.

The most important force in a granular system is the hard core repulsion. Friction between the particles is always present in the experiments, however, in many of the numerical studies frictionless grains are implemented as friction represents an extra complication in the simulations and for many cases it is not essential for the studied mechanism. The observed macroscopic friction of the granular material is influenced by the inelastic collision of the particles, the microscopic friction and the surface roughness
of the grains. When the particle size of a granular sample is above 0.1 mm, van der Waals forces are not relevant. Electrostatic forces can also occur in the experiments for fast flows originating from the friction between the particles and the wall of the container, but with a good choice of the investigated material and the container wall this effect can be suppressed.

In the experiments described in this dissertation, crystallization or segregation did not occur as all the studied materials had a polydispersity of about 10%, which is sufficiently large for suppressing shear induced positional ordering (crystallization) but not too large for avoiding segregation. As in our measurements the material was allowed to dilate during shear, the packing fraction stayed far below the jamming limit. Only dry granular samples were investigated in this work, and the maximal speed of the grains was much less than their speed in free fall (about 10 m/s), which allows us to neglect the effect of air between the particles. In a review paper by Jaeger and Nagel the authors underline that analogies with other physical systems often help to better understand the physics of granular materials [22]. As we will see this turns to be true for several of our experiments.

1.2 The Reynolds dilatation

Granular materials often dilate due to shear, if the volume of the sample is allowed to change. Particles can move next to each other in shear flows only if some additional free space is available, therefore typically the so-called Reynolds dilation occurs [23, 24]. This process is illustrated on Fig. 1.1.

Recent MRI experiments provided information about the Reynolds dilation of the material under shear [25], and showed that the density decreases with the accumulated strain and saturates when the local strain becomes unity. The fully dilated stationary state was obtained when the accumulated strain reached the order of one.

1.3 The Janssen-effect

A general property of the granular samples is that force chains can develop inside the material both in static and dynamic conditions. Similarly, if granular particles are filled
in a vertical tube, the pressure detected on the bottom wall of the container will not increase linearly with the filling height as for a liquid, but it saturates (see Fig. 1.2) due to the arches which transmit forces to the side walls.

For large filling heights the weight of the material is mostly loaded to the side wall by the friction between the grains. This phenomenon has a serious consequence on
industrial applications as sometimes enormous forces can occur on the side walls of the silos, which makes the designing of such a silo more complicated.

1.4 The different flow regimes

For granular materials consisting of hard particles we distinguish three different regimes according to the relevance of the inertial collisions. The so-called inertial number defined as \( I = \dot{\gamma}d\sqrt{\rho/P} \) describes the relevance of the dynamical forces in the system. Here \( \dot{\gamma} \) is the shear rate, \( d \) denotes the particle diameter, \( \rho \) is the density of particles and \( P \) is the pressure. The inertial number can be considered as the ratio of the inertial time scale of particle relaxations and the time scale of the macroscopic shear. Its significance was pointed out by da Cruz et al. [26].

For slow granular flows the inertial effects are negligible, and particles are always in contact. We refer to this as quasistatic regime. Practically for this regime \( I < 10^{-3} \). When collisions are dominating in the flow, we are in the collisional regime, which is defined as \( I > 10^{-1} \). For the intermittent values of the inertial number, both grain contacts and collisions transfer significant forces and the observed dynamics is more complex. In the literature this is called as dense flow regime.
2. Previous studies and objectives

In this chapter I summarize the most relevant previous results for each of the three main topics of this dissertation. At the end of the sections the research objectives are also listed.

2.1 Shape of the shear zones

Deforming a granular sample the shear is often localized to a certain region called shear zone. Shear banding can be observed very often in granular materials from laboratory size to geological scales, and is an important factor in various industrial and geological processes. The localization of the shear can occur in a wide range of geometries. Often the shear zone develops close to a container wall, which influences the velocity profile. In this case the width of the shear zone is determined by the roughness of the surfaces nearby and the particle size.

In certain cases (e.g. in the so-called split-bottom shear cells), the shear zone is located far enough from all the surrounding walls, and it’s width is determined by parameters like the filling height of the material. Therefore the description of the material in such cases is even more complex than in the first scenario. Fenistein and van Hecke named this latter case as wide shear zones, while the first type is referred as narrow shear zones [27]. For dry granular materials several features of the stationary flow have been explored in the last two decades for both narrow and wide shear zones.

2.1.1 The stationary shape

After the studied system reaches the critical state (see Sec. 1.1), one can make reproducible experiments easily. A traditional geometry to study the response of a fluid to shear stress is the Couette cell, where an inner cylinder is rotating, while the bottom and the outer cylinder is stationary. This setup can be also used for granular materials
with slight modifications. Here narrow shear bands are formed, as reported by Losert et al. [28]. The mean velocity was found to have a maximum near the inner cylinder, and decay rapidly with distance from the moving surface. When dealing with nearly uniform grains Mueth used magnetic resonance imaging (MRI) and X-ray Computed Tomography (CT) to get quantitative information about the shear profile inside the granular material in the stationary state [29]. Veje et al. analyzed the two dimensional version of this setup and concluded that a second order phase transition occurs at a critical packing fraction [30].

The first experiments on wide shear zones were performed in a different cylindrical configuration, where the bottom plate consists of a rotating circular plate and a stationary outer ring. Fenistein et al. determined both the position and the width $w$ of the shear band for many different kind of particles [27, 31]. They found that the width of the shear zone at the surface depends on the filling height $H$ as a power law, and the exponent is between 0.5 and 1, as it is shown on Fig. 2.1. An another important result is that the surface velocity profile can be fitted well by an error function.

![Fig. 2.1: (Left) The surface width of the shear zone as a function of the filling height $H$. Four different sizes of glass beads were used, and the lines indicate the slope of $1/2$, $2/3$ and 1. (Right) The zone width as a function of the distance from the bottom $H_b$, for three different filling heights. Figures were taken from Fenistein et al., 2004 [31].](image)

To explain why these zones are wider than expected, one possible approach is to include non-local aspects of the rheology. Nichol et al. proposed that there cannot be a local relation between applied stress and observed strain rate [32]. The continuum model by Henann and Kamrin was based on a grain size dependent non-local rheology, which was first applied on emulsions [33]. They tested their continuum model on a couple of different geometries: flow on an inclined plane driven by gravity, Taylor-Couette flow and flow in the cylindrical split-bottom shear cell. They concluded that
incorporating this additional length scale enables to capture the stationary shape of the shear zones accurately. In the so-called fluctuating narrow band model first published by Unger et al. the non-locality was included in a way that not only the lattice sites at the location of the shear zone are updated but also the neighboring sites [34, 35]. A detailed description of this model can be found in Sec. 2.3.1.

Depken et al. had another approach to the problem of the wide shear zones by assuming a non-constant (angle dependent) effective friction coefficient throughout the zone [36]. Without this feature the shape of the zone observed in the experiments was not reproduced in their calculations for the split-bottom geometries. They also gave a possible explanation for the angle dependence: the competition between the organizational tendencies of the flow and the gravitational pull. The shear flow leads to a dilation of the material with respect to the random close packed state, where the number of grain contacts is increased in the compressional direction and decreased in expanding directions, while gravitational forces locally might favor a larger number of vertical contacts. A more careful analysis proved that the stress and strain-rate tensors are co-linear, but the friction coefficient is set by the local geometry [37].

Describing wide shear zones is a hard test for the numerical models as a lot of them are unable to capture the width of the shear zones in the split-bottom geometries. For example the spot model first published by Kamrin and Bazant [38] successfully describes the flow profile in a wide range of geometries, but according to the paper of Woldhuis et al. [39], it can not capture the larger scale of the wide shear zones as only one length scale is included in the model.

Luding made experiments and simulations in both two and three dimensions [40, 41] using the circular shear cell. He found that the width of the shear zone at the surface depends nearly linearly on the filling height (See Fig. 2.2), while its width in the bulk is found to change faster close to the bottom (See Fig. 2.2). Unfortunately, the layer depth vs. zone width curve \( w(z) \) was not fitted by any function. As for the numerical simulations, a slight dependence of the zone width on the shear velocity was observed indicating that the quasistatic conditions were not fully satisfied.

Two numerical works studied the shape of the shear zones in the linear split-bottom shear cell accurately. Ries et al. performed molecular dynamics simulations to reveal the details of the shear zone [42]. The filling height vs. zone width relation \( w(H) \) was
observed to have an exponent of about 2/3, see Fig. 2.3. The zone width as a function of the depth $w(z)$ was fitted by a quarter of a circle as it is shown on Fig. 2.3. They also tested different fitting functions for the zone width and concluded that the error function fitted better the velocity profile when larger deformations were applied. The authors analyzed their system with and without gravity (i.e. applying a constant force on the top layer) and got almost the same results, meaning that the shape of the zone has a geometric origin. Jagla used an elasto-plastic model originally developed for directed polymers [43]. He fitted the shape of the shear band $w(z)$ with a Gaussian function, which is slightly different from the quarter of a circle close to the bottom. In both papers no significant differences were observed between the cell with opened surface and the cell with double height and a splitted top.

After the paper by Fenistein et al. from 2003 [27], a logical question was to study the effect of the layer thickness in the cylindrical split-bottom shear cell. If the inner cylinder is present, for high enough filling heights we most likely get back to the case of narrow shear zones, where the highest velocity is measured close to the inner wall, and the velocity profile is not affected by the change in the filling height. But if we remove the inner cylinder, a more optimal shear band was found, which is not reaching the surface (see Fig. 2.4). The relevant system parameter is the ratio of the filling height $H$ and the radius of the rotating plate $R_s$. In the first numerical study by Unger et al. a simple variational model was developed and a transition was found between the two above mentioned regime at around $H/R_s \approx 0.72$ [34].
Fig. 2.3: (Left) The surface width of the shear zone as a function of the filling height $H_{\text{fill}}$. The lines indicate the slope of exponent $2/3$. (Right) The zone width as a function of the layer height $z/H_{\text{total}}$. Different symbols denote the system size. All the curves lie on a square root function. Figures were taken from Ries et al., 2007 [42].

works addressed to measure this parameter a somewhat smaller value was obtained, about $H/R_s \approx 0.6$ by Cheng et al. using MRI technique [44], and $H/R_s \approx 0.65$ Fenisstein et al. by optical imaging [45]. Then the original variational model was refined to reproduce better the experimental results and it also gave predictions to induce further experimental work [35]. A hydrodynamic model developed by Jop, involving an inertial number dependent friction coefficient also reproduced this transition at about $H/R_s \approx 0.7$ [46]. The non-local model by Henann and Kamrin gave a very similar value for this parameter $H/R_s \approx 0.72$ [33]. An experimental work by Nichol and van Hecke applied larger filling heights to create axial shear, and they analyzed the fluidized region over the shear zone [32].

Recent discrete element method simulations of simple plane shear show that shear localization near smooth walls is qualitatively different for different wall velocities [47]. The roughness of the container wall strongly influences the velocity profile for narrow shear zones under slow oscillatory shear [48]. With changing the surface roughness of the wall and the filling height in the cylindrical split-bottom shear cell, one can find more than one sheared region in the system at the same time [49].

Stationary shear zones were discussed in a number of review papers and monographs as well. Jaeger and Nagel summarized the unusual solid-like and fluid-like behaviors of dry granular materials in their review paper in 1996 [50], mainly focusing on the static properties, convection maps and pattern formation. The review paper by the French
GDR MiDi group collected the most widely used geometries studying dense flows and sorted the recent results in 2004 [1]. In the same year a very comprehensive book was published edited by Hinrichsen and Wolf [51]. The review paper by Aranson and Tsmir- ing collected the recent results of pattern formation including vibrated monolayers and multilayers, gravity-driven flows and segregation [2]. Dijksman and van Hecke summarized the latest results obtained in the split-bottom geometries in 2010 [52]. Shear band formation in foams, emulsions, suspensions and granular media was reviewed by Schall and van Hecke in the same year [53].

### 2.1.2 Initial transient effects

If we prepare a system with a random configuration or reverse the shear direction after reaching the critical state, we can provide a reproducible initial condition and study the initial transient. As for wide shear zones, discrete element method simulations by Ries et al. found that starting the system from a random configuration initially the shear zone is wider, then it shrinks to its final width as the stationary state is reached [42]. They gave a possible explanation that at the beginning of the process the resistance to shear is lower, later it converges to a larger value exponentially with a characteristic deformation scale of about $\gamma = 0.2$. This value is measured locally and reached faster.
in the center part of the shear zone. In the early stage of the process where the critical zone (the region where the critical state is reached) is smaller than the sheared region, the shear rate is slightly enhanced outside the critical region, resulting a wider zone. After the whole shear zone becomes critical this additional widening effect ceases and the zone width reduces to its stationary value.

![Fig. 2.5](image)

**Fig. 2.5:** The surface zone width as a function of the slider displacement $\lambda$. Initially the zone width is wider, then it converges to its final value. Figure was taken from Ries et al., 2007 [42].

Toiya et al. studied narrow shear zones in a Taylor-Couette cell, and focused on the transients and the case of oscillatory shear [54]. After reaching the critical state they reversed the direction of shear, and they measured a larger shear zone width, a higher density and a lower resistance to shear, which parameters later converged back to their original value. Particles far from the shear zone move faster during this transient than in the final state. The typical inner cylinder displacement in particle diameter units for the fast compaction process was about $0.5 \, d$, while for the shear stress $5 - 10 \, d$ was needed to reach the critical state again. During oscillatory shear the shear stress (torque) remained below the steady state stress for small amplitudes. Increasing the amplitude of the oscillations, first the shear stress becomes larger compared to the stationary value in one direction, but for large enough amplitudes it will converge to the stress in the critical state. These results mean that small amplitude oscillatory shear leads to a compaction of the material and strengthening of the sample.

Utter et al. used a two dimensional Couette cell filled with bidisperse photoelastic disks where the force network and the rotation of the particles was visualized [55]. The force chains had a preferred direction which is neither parallel nor perpendicular to
the shear flow, as it is shown on Fig. 2.6. The force network over the sample remained essentially the same after stopping the shear. After restarting the shear in the original direction, the steady state was reached immediately. But if the shear was applied in the opposite direction, a structural rearrangement in the force network was detected. The new force network had a preferred direction nearly perpendicular to the original one (see Fig. 2.6), which was built up during an inner cylinder displacement of 3 $d$, while the velocity profile set back in 30 $d$ of displacement.

![Fig. 2.6: The visualized force chain network when the system is sheared in the original (a) and in the opposite (b) direction. The light gray arrows show the rotation of the inner circle. The white solid lines illustrate the average orientation of the strong force chains. Figures were captured from Utter et al., 2004 [55].](image)

Recently, contact dynamics simulations were performed by Azéma et al. using non-spherical particles [56]. The grains were generated by gluing 4 overlapping spheres together, where the convexity was controlled by the overlap. A density decrease due to shear was observed for these special types of particles, and the dimensionless local strain $\gamma$ needed to reach the stationary state was found to be around 0.5.

To shortly summarize the previous results about the geometry of shear zones, the stationary position and width of the wide shear zones is relatively well explored. It is known that for wide shear zones the relation between the filling height and the surface width can be fitted by a power law with an exponent of about 2/3. In the cylindrical geometry the shear zone approaches the center for larger filling heights. Above a critical value of the ratio of the filling height and the radius of the rotating plate $H/R_p$, the original radial shear band becomes less pronounced, and axial shear is formed. As for the initial transient effects, a dilation occurs not only for random initial condition (Reynolds dilatancy), but also when in the critical state of the system the direction of
shear is reversed. The relaxation of the force network is much faster than the relaxation of the zone width.

### 2.1.3 Objectives

One open question is the stationary width of the shear zone in the whole cross section, which we aim to determine accurately from experiments and compare the results with the numerical prediction. A more general issue we focus on is the evolution of the shear zone width during the initial transient. We start the system from two different initial conditions: random configuration and stationary state sheared in the opposite direction. A comparison of the straight and the cylindrical shear cell would be useful, especially because for the experimental investigations the cylindrical one is preferred, and in numerical simulations the straight one is more popular.

### 2.2 Alignment of elongated particles in granular flows

Materials consisting of elongated or flattened particles are a part of our daily life. Macroscopic grains like pills and seeds are important for the industrial processes. But other shape-anisotropic particles have a much smaller length scale: we use liquid crystal molecules in our displays, viruses and bacteria can have an elongated shape, nanorods and nanowires are a very active area of research, and even clay consist of flattened microscopic particles. The relevant forces strongly depend on the particle size: e.g. van der Waals forces, capillary forces or hard-core repulsion can play an important role. A common feature of all these shape-anisotropic particles is that they can align due to shear flow.

#### 2.2.1 Ordering of the grains

If we have a small amount of elongated small objects in a viscous fluid, the particles will rotate with respect to the shear flow, as discussed by Jeffery [57]. As we increase the volume fraction of the particles in the fluid, the interactions of the grains become more
pronounced. In this section I summarize the results for granular materials consisting of elongated particles, where interparticle forces are dominating.

Behringer et al. studied a quasi two dimensional hopper flow geometry filled with grass seeds [58]. In their measurements they identified smaller and bigger aligned clusters due to shear flow. Similar cluster formation was reported by Misra [59]. Behringer et al. highlighted a fundamental difference between the interaction of spherical and non-spherical particles. Only for spherical grains, the contact point is located always on the line connecting the two centers of mass. For that reason, torque origins purely from the friction between the grains. For a more complex particle shape with elongation or sharp edges more sophisticated methods of computer simulations are necessary to be implemented, and the required computing time increases rapidly with the complexity of such a simulation code.

In a later work by Ehrentraut and Chrzanowska flow of rice grains on an inclined plane was investigated [60]. In this geometry for homogeneous flows the velocity is in average parallel to the inclined plane, and it only depends on the distance from the bottom plate. It means that the shear is applied perpendicularly to the inclined plane. They found that the rice particles align due to the flow, but the grains are not perfectly parallel to the flow direction (see Fig. 2.7).

![flow direction](image)

**Fig. 2.7:** The bottom layer of flowing rice. The grains are oriented, but in average they are tilted to the direction of the flow. Figure was taken from Ehrentraut and Chrzanowska, 2003 [60].

The authors provided an orientation distribution of elongated particles (see Fig. 2.8), which has rather poor statistics. A very extensive analysis of the Savage-Hutter equations was also included, which is able to capture some basic features of anisotropic granular media.
In numerical simulations a number of different particle shapes were examined to test how the shape affects the static and dynamic properties. Anki Reddy et al. studied two dimensional cells filled with inelastic dumbbells (two overlapping circles) and measured the orientational order parameter as well as the orientational distribution function as a function of the packing fraction and the elongation of the particles [61]. For more elongated particles the ordering is stronger. In a later work they focused on the granular temperature, which is nearly independent of the elongation [62]. Campbell worked with three dimensional ellipsoids [63]. Guo et al. compared elongated particles made of glued spheres and perfect cylinders with various aspect ratios [64]. All four papers concluded that these particles have a similar preferred orientation as Ehrentraut and Chrzanowska: the major axis encloses a small angle with the flow direction, and for more elongated particles this angle becomes smaller.

Azéma and Radjaï considered two dimensional stadium shaped particles built of two circles and a rectangle, and characterized the force chains [65, 66]. In further papers Azéma et al. also studied simple shear flow of two dimensional regular polygons from 3 to 60 sides [67], three dimensional inclined plane flows of polyhedral grains [68, 69] as well as non-convex particles made of 4 overlapping spheres [56]. Donev et al. determined numerically the random closed packing fraction as a function of the elongation [70].

Fig. 2.8: The orientation distribution of the rice grains in the shear zone. The line corresponds to a fitted theoretical curve. Figure was captured from Ehrentraut and Chrzanowska, 2003 [60].
They found that for a certain aspect ratio (the ratio between the particle length $L$ and the diameter $d$) of about $L/d \approx 1.5$ the packing fraction has a maximum. In a later paper by Donev et al. they published advanced numerical methods for simulating effectively both ellipses and ellipsoids [71]. Azéma et al. determined that for larger values of the non-convexity parameters the packing fraction decreases but at the same time the shear strength increases. In two dimensions Cleary varied the particle shape – both the aspect ratio and the blockiness (describing how rectangular the shape is) – and concluded that the shear strength increases faster with the aspect ratio than with the blockiness [72]. Simulations of frictionless ellipses indicated that the threshold of the jamming transition is affected by the particle elongation [73]. Biaxial tests with elongated polygons by Nouguier-Lehon et al. also revealed shear induced alignment, moreover, a continuous global rotation of the particles was observed [74, 75]. Peña et al. found that the shear strength is higher for elongated grains because of the stronger interlocking between the polygons.

The flow and orientation of elongated particles is important in other areas of physics, and even in other natural sciences. In geology the alignment of embedded clasts indicates the original direction of volcanic flows [76, 77, 78]. For suspensions, orientation is well characterized by e.g. Yamamoto et al. [79].

As van Hecke reflected in his review paper, the exact shape of the elongated grains can strongly influence the pattern formation in vibrated systems [80]. Recent advances on the packing and dynamics of granular materials consisting of non-spherical particles are summarized in a review paper by Börzsönyi and Stannarius [81].

### 2.2.2 Objectives

Our goal is to characterize the main properties of the shear induced alignment of shape-anisotropic granular materials. In our experiments the interaction of the gravity and the shear flow sets the local packing density. We aim to determine the orientation of the particles as well as the rotation of individual grains. As mentioned earlier in Sec. 2.1.2, the initial transient effects can help to understand the physics of granular media. The elongation of the grains and the shear velocity should be varied in order to connect our
work with the previous papers. We also aim to connect the evolution of the ordering with other parameters e.g. the packing fraction of the material.

2.3 Position of the shear zone in layered materials

The properties of shear zones were studied extensively in the last decades. However, most of this work focused on homogeneous samples. In the nature such simple systems are rare, typically layers of different materials are lying on each other, built up by geological and biological processes like volcanic activities, fragmentation due to erosion, transport by air or water, sedimentation and so on. The physical properties can change rapidly from layer to layer, which makes it more difficult to build tunnels, mines or to drill wells. The kinetics and dynamics of these layered systems are not well understood. A deeper analysis could help the geologists to understand what circumstances are needed to suppress or induce flows in inhomogeneous samples, and what determines the position of the geological faults.

2.3.1 Previous results

One of the pioneering works studying shear zones in layered granular systems was published by Unger [82]. In this numerical study two independent methods were applied, as the result of a simple variational model developed by Unger et al. [34] was validated by three dimensional discrete element method simulations. The details of this variational model are described together with its numerical extension, the “fluctuating narrow band model” developed by Török et al. [35]. Both models are based on the cross section of the shear cell. In the variational model a continuum medium is supposed, while in the fluctuating narrow band model a (regular or randomized) mesh is placed on the cross section, which has a lattice constant $a$ as a fitting parameter of that model. This constant is close to the particle size in the system, therefore the edges of the mesh can be thought of as particle contacts. The randomized mesh fits better to the discretized nature of granular materials. In this cross section the shear zone is modeled as an infinitely thin sliding path which separates the two rigid blocks sliding next to each other. The variational model only captures the position of the shear zone, while
the fluctuating narrow band model can provide smooth shear profiles by generating as an ensemble of many individual thin sliding paths, enabling us to determine the width of the sheared region.

Taking into account all the possible yielding trajectories which are consistent with the actual boundary conditions, the sample will be deformed along the weakest path. For a given shear stress \( \tau \) at the particle level each contact has a different resistance, which is determined by the length of the edge \( l \), the pressure \( p \) between the particles (we neglect here, that the stress tensor is not isotropic), the effective friction coefficient for that material \( \mu^{\text{eff}} \) and for the fluctuating narrow band model an additional random number \( X \) between 0 and 1 with uniform distribution:

\[
\tau = l \, p \, \mu^{\text{eff}} \, X \quad (2.1)
\]

For cylindrical shear cells an additional parameter should be included. As the cross section represents a radial cut of the original system, all elemental paths are weighted by the radial coordinate to include the effect of the azimuthal direction. This variational problem can be described as:

\[
\int \mu \, p \, ds = \min. \quad (2.2)
\]

In simulations it is straightforward to set a constant pressure in the whole system. This step is useful to find the basic features of both models. An other solution would be to incorporate a hydrostatic pressure through the sample, but it can be used only if the Janssen effect is negligible (see Sec. 1.3). Using the first assumption, the following simple integral can be formulated:

\[
\int \mu \, ds = \min. \quad (2.3)
\]

The variational model provides us only one path, which corresponds to the position of the shear zone. But in the fluctuating narrow band model many different paths are generated. Each minimal path is registered as a shear path. After each path is saved, the random numbers for the elements of the yielding path and for its neighborhood are updated. After collecting thousands of yielding paths, the shear zone profile is defined as the ensemble average of all the paths, which becomes smooth due to the averaging.
Let us consider a system consisting of two components with two different effective internal friction and a straight interface between the two materials. If the shear zone travels through the material crossing this interface, the above described minimization leads to the following formula:

$$\frac{\sin \alpha}{\sin \beta} = \frac{\mu_h^{\text{eff}}}{\mu_l^{\text{eff}}}$$

(2.4)

where $\alpha$ and $\beta$ correspond to the angles of incidence and $\mu_h^{\text{eff}}$ and $\mu_l^{\text{eff}}$ denote to the effective friction coefficients.

The mathematical formalism of this above described variational problem (Eqs. 2.3 and 2.4) is identical to the formalism describing light propagation in geometric optics. According to the Fermat’s principle the trajectory of a ray of light is such a path that the corresponding traveling time is extremal (typically minimal):

$$\int n \, ds = \text{extr.}$$

(2.5)

For a system consisting of two layers this leads to the well-known Snell’s law, describing light refraction at the layer boundary:

$$\frac{\sin \alpha}{\sin \beta} = \frac{n_h}{n_l},$$

(2.6)

where $\alpha$ and $\beta$ are the angles of incidence and $n_h$ and $n_l$ are the optical densities of the materials.

In the discrete element method simulations a long cylindrical container was prepared with two cuts on the top and the bottom (see Fig. 2.9). As it is seen on the figure, two kinds of grains were placed in the system. The side walls were shifted parallel to each other and the red particles had the intermediate velocity in between the two walls. The shear zone was not linear, but it changed its direction at the interface of the two materials. The same effect was observed in the fluctuating narrow band model.

Numeral simulations were performed scanning a wide range of the interface orientations and the angles of incidence were detected. According to the Eq. 2.4, the ratio of the sine of these angles were calculated for each simulation. This ratio was found to be equal to the ratio of the effective friction coefficients within numerical error, as it is shown on Fig. 2.10.
Fig. 2.9: A snapshot of the discrete element method simulation on layered materials. The green particles have a higher friction coefficient than the yellow ones. Shear is applied by moving the left and right side of the walls in the opposite direction. Particles with an intermediate velocity are indicated with red. Figure was taken from Unger, 2007 [82].

This result proved that an analogy is expected between granular materials and geometric optics, moreover, the granular version of the Snell’s law should be valid for these kind of systems. This surprising analogy attracted the attention of the Nature editorial board, and they wrote a short summary in the Editor’s Choice section [83].

Fig. 2.10: The relative index of refraction as a function of the interface orientation, measured in simulations. Each point corresponds to a simulation while lines show the theoretically predicted values. Figure was captured from Unger, 2007 [82].

In order to prove the numerical expectations two independent experimental works studied the refraction of shear zones in layered granular materials. Knudsen and Bergli
studied a geometry similar to the simulations of Unger [84]. To visualize the deformation inside the sample they colored half of each material. After shearing the material was removed layer by layer to explore the displacement profile through the sample. The cylindrical container was placed vertically to suppress the effect of gravity. In this setup they supposed to have a shear zone as it is illustrated on Fig. 2.11. An experimental image shows that the shear zone changes its direction according to the expectations (see Fig. 2.11).

Fig. 2.11: (Left) The numerical prediction for shear zone refraction in the cylindrical geometry. (Right) A single experimental image, which fits to the expectations. Here gray and red samples have a higher friction (sand), while black and yellow ones are easier to shear (glass beads). The shear is located where the color is changing. Figures were taken from Knudsen and Bergli, 2009 [84].

The other experimental work by Börzsönyi et al. used a different geometry which is more suitable to test Snell’s law [85]. In the split-bottom shear cell one end of the shear zone is determined by the setup but the other one can be located everywhere on the surface. The same excavation method was used as in the other paper. As in this setup the pressure is not homogeneous due to gravity, the zone has slightly curved shape (see Fig. 2.12).

Here the zone tries to escape not only from the high friction, but also from the high pressure parts of the sample. Close to the bottom slit the shear zone is propagated almost vertically to leave the high pressure part of the setup, but later it tends to the low friction part of the cell. At the interface of the two materials the zone changes its direction, and it goes vertically to reach the top surface. In this paper a much better
Fig. 2.12: Shear zone refraction in a different experimental geometry. The blue line indicates the layer boundary, and the color intensity is proportional to the local strain rate of the sample. In this setup not only the high friction, but also the high pressure parts are avoided. Figure was captured from Börzsönyi et al., 2009 [85].

statistics of the angles of incidence was published. These investigations formed the basis of my Diploma thesis.

Both experimental works proved that the numerically predicted shear zone refraction exists, and the granular version of the Snell’s law describes that special type of systems accurately.

2.3.2 Objectives

Our main goal is to expand the analogy between optics and granular media. An interesting open question is to discover the limiting case of total reflection for granular materials, as none of the earlier works analyzed this phenomenon. The variational model can be tested by comparing the analytical solution of the integral (2.3) and the measured zone position. We aim to map additional similarities between these two distinct fields of physics and find the limitations of these analogy between them by designing new geometries and measuring the position of the shear zones.
3. Experimental techniques and materials

3.1 Experimental configurations

To obtain a wide shear zone, there are several different geometries described in the literature. The common feature of all these experimental configurations are that the container wall is cut to two parts, and these two parts are moved with respect to each other to generate a shear flow. In our experiments we studied the two most frequently used geometries: a simple straight and a cylindrical cell. The straight cell has the advantage of a translational symmetry, therefore the data analysis and the interpretation becomes easier, but the total achievable deformation in this system during an experiment is limited by the length of the shear cell. In the cylindrical geometry we only have a rotational symmetry, which makes the interpretation a little more complicated, but a serious advantage is that for this case the shear displacement is not limited. The other important aspect in selecting the most suitable geometrical configuration is related to the data collection method (MRI, X-ray CT or simple optical detection), where the equipment can also limit the size and shape of the experimental cell.

3.1.1 Linear split-bottom shear cell

The simplest configuration to generate wide shear zones consists of two long L-shaped sliders, as shown on Fig. 3.1. This geometry has the advantage of translational symmetry, which makes the data evaluation and understanding relatively simple. The deformation of the material is obtained by slowly moving the L-shaped sliders with respect to each other. The grains near to a wall are moving together with the boundary, and the sheared region in the center is indicated with red.

The coordinate system and the parameters describing the geometry of the shear zone are illustrated on Figs. 3.1 and 3.2. The slider is moved in the $x$ direction and the zone width $w$ is defined in the $y-z$ plane as an average along the $x$ axis. The
(y = 0, z = 0) point is located at the meeting point of the two sliders, which are colored with blue on the figure. H is the filling height of the system, therefore the material is placed between 0 ≤ z ≤ H. As in previous studies, we denote the surface zone width by w(H) which can be easily measured by optical methods. As for granular materials many distance-like quantities are scaling with the particle diameter d, I will provide H, w and the axis coordinates in dimensionless particle-diameter-units.
geometries for which the desired effect is clearly visible. Therefore the exact geometries of those measurements are detailed in the Results section (see Sec. 5.3).

3.1.2 Cylindrical split-bottom shear cell

The majority of the measurements were performed in the cylindrical split-bottom shear cell (see Fig. 3.3), which can be considered as a modified Couette cell. In this geometry a circular plate at the bottom of the container is rotated with a constant angular velocity $\omega$, while the rest of the bottom plate and the outer wall are fixed. If the granular layer is sufficiently thin (as discussed in Sec. 2.1.1), the middle part of the granular sample is rotating with the plate. The outer part of the sample is not moving during the rotation of the bottom plate. The region in between these undisturbed parts is the actual shear zone indicated with red on the figure, which can be easily sheared for very long times. In some experiments an additional inner cylinder has been included which is also far from the sheared region and does not affect the shear band formation.

![Fig. 3.3: Illustration of the cylindrical experimental setup. The granular material is sheared as the circular bottom plate is rotating. The strain is localized in the region marked with red.](image)

Two experimental setups were constructed with different sizes. For the optical measurements (see Sec. 3.2.1), a cell with a diameter of 27 cm and a rotating plate with a diameter of 15 cm was built, as it is shown on Fig. 3.4. In the CT machine (described in Sec. 3.2.4) a larger cell was used with an outer diameter of 57 cm and a rotating plate diameter of 39 cm.
Despite the dissimilarities between the two geometries, we expected no significant differences in the resulting shear zones. In the straight cell the displacement of one of the sliders was realized by turning a long screw with a screwdriver (or by hand in the MRI measurements). In the cylindrical split-bottom cell the circular plate was slowly rotated by either a DC motor (for optical imaging) or by hand (for the X-ray measurements). Note that the straight cell was not filled up to the ends of the sliders, otherwise near the ends of the sliders the width and the position of the sheared region would be influenced by the ending walls.

3.2 Imaging techniques, materials

We used various experimental methods to study quasistatic shear flows of granular materials. The first one is optical data collection, a fast and relatively cheap technique. Using high resolution and high speed digital cameras allowed us to map the surface of the opaque granular samples in details. Visualizing the grain displacement inside the sample is more difficult, but there are a couple of methods described in the literature. For non-transparent samples, one option is the so-called excavation method, where successive layers of particles starting from the top layer are removed, and e.g. the strain in the bulk can be detected by observing the position of colored particles. There are two important disadvantages of this technique. It takes considerable time, moreover, once it is applied, the sample is destroyed and the measurement can not be continued.

Advanced three dimensional imaging devices are developed quickly and they became available not only for medical purposes. These expensive machines enable us to capture
the position and orientation of the particles below the surface, without modifying the actual configuration. Our group had access to a nuclear Magnetic Resonance Imaging (MRI) and an X-ray Computed Tomography (CT) machine. On one hand, these devices collect data slower than the optical cameras, and the spatial resolution of the created images are typically lower. But on the other hand, they produce reliable information about the recorded volume in a non-invasive way. Taking advantage of the complementary nature of the above described experimental methods, we aimed to get the most complete set of information of sheared granular materials.

### 3.2.1 Optical imaging

Two types of cameras were used to visualize grain motion at the surface of the sample. In the straight split-bottom cell deformation was applied in small steps and still images were taken in between the steps. This was achieved with a 3 megapixel Tuscen USB camera shown on the left panel of Fig. 3.5, which can image a relatively large region in the flow direction, and having sufficient resolution in the lateral coordinate as well. This technique was used for measuring the width of the shear zone for homogeneous materials and for detecting the position of the zone for layered materials. On the right panel of Fig. 3.5 the high speed camera (Mikrotron EoSens CL MC1362) is shown. The sensor of this camera has higher sensitivity, therefore it was suitable for studying the width of the shear zone as well as the orientation of elongated particles in the cylindrical shear cell, where grains are continuously moving due to the rotation of the motor driven circular plate. This camera has lower resolution (1.2 megapixels) with 8-bit grayscale.

![The cameras used in the optical experiments. (Left) 3 megapixel Tuscen camera, used in the measurements with the straight shear cell. (Right) Mikrotron EoSens CL MC1362 high speed camera, used in the experiments with the cylindrical geometry.](image-url)
color depth, but the maximal achievable frame rate with full resolution is over 500 frames per second. This Mikrotron camera is connected to the computer through a framegrabber card which has an on-board memory of 1 gigabyte. The number of recorded pixels is limited by the memory on the framegrabber card, but when a lower resolution was applied, the number of maximal recorded images increased. When we collected data to determine the width of the shear zone, we recorded a 300 × 1024 pixel area with 60 frames per second. In the experiments about the alignment of elongated granular samples a 1024 × 1024 pixel area was imaged with 18 to 100 frames per second.

Several types of granulates were investigated in the optical measurements. For the experiments on the shear zone width we wished to compare the behavior of materials with different grain shape but similar grain size, therefore we used 2 types of grains in the linear cell, while 3 other samples were studied in the cylindrical shear cell, which are shown on Fig. 3.6(a) and (b). The alignment of elongated particles was determined

(a) Glass beads with diameter \( d = 250 \pm 25 \mu m \) (left) and sand with equivalent diameter \( d = 250 \pm 20 \mu m \) (right), used in the linear geometry for studying the width of the shear zone.

(b) Silica gel beads with diameter \( d = 1.8 \pm 0.2 \, \text{mm} \) (left), corundum with equivalent diameter \( d = 1.8 \pm 0.2 \, \text{mm} \) (middle), and glass rods with diameter \( d = 1.9 \pm 0.3 \, \text{mm} \) and length \( L = 7 \pm 1 \, \text{mm} \) (right), used in the cylindrical geometry for measuring the width of the shear zone.

(c) Three different types of rice grains with a length, diameter and aspect ratio of \( L = 5.0 \, \text{mm}, \, d = 2.0 \, \text{mm}, \, L/d = 2.5 \) (left), \( L = 6.5 \, \text{mm}, \, d = 1.9 \, \text{mm}, \, L/d = 3.4 \) (middle), and \( L = 7.4 \, \text{mm}, \, d = 1.6 \, \text{mm}, \, L/d = 4.5 \) (right), used in the cylindrical geometry for studying the shear induced alignment.

(d) Two types of cylindrical glass rods with \( L = 2.5 \, \text{mm}, \, d = 1.8 \, \text{mm}, \, L/d = 1.4 \) (left) and \( L = 6.6 \, \text{mm}, \, d = 1.9 \, \text{mm}, \, L/d = 3.5 \) (right), used in the same experiments as panel (c).

Fig. 3.6: Materials studied in the optical method. The bar represents 0.5 mm on panel (a) while it has a length of 5 mm for the other three panels.

by using 3 types of rice grains and 2 types of cylindrical glass rods, which are presented
on Fig. 3.6(c) and (d), respectively. The dimensions of the particles are listed in the figure captions. Note that the glass rods with an aspect ratio of $L/d = 3.5$ were used in both types of experiments.

The lamps for illuminating the sample in the optical experiments were supplied by DC power to avoid change in the detected intensities on subsequent frames.

### 3.2.2 The excavation technique

For studying the position of the shear zones in layered granular samples we wanted to visualize the displacement of the material not only at the surface, but also in the whole cross section of the straight shear cell. This was achieved by the excavation technique using a sample where half of the particles for each material was colored. The sample was prepared such a way that the border between the colored and uncolored grains was a sharp plane, perpendicular to the flow direction (see Fig. 3.7).

![Fig. 3.7: The side view (left) and the upper view (right) of the measuring cell for two different measurements. On the right image four types of materials are visible: uncolored corundum (top left corner), corundum colored to dark gray (bottom left corner), glass beads painted to blue (top right corner) and unpainted glass beads (bottom right corner). The particles and their sizes are described on Fig. 3.8.](image)

After shearing the sample the displacement of the material in the bulk can be reconstructed by carefully removing the material layer by layer, and identifying the distorted border line between the colored and uncolored part of the sample in each layer, as it was documented in the paper of Fenistein et al. [45]. For each layer an additional calibration image was taken with a millimeter scale since magnification changes with distance from the camera. For gently removing the grains we used a vacuum cleaner with an
additional extension tube to reduce the flow rate. The whole process took several hours as the distance of the layers was 1 to 2 mm.

We repeated the measurements several times with various configurations. In order to provide the same particle properties for each experiment, we used the same sample for each run. For this we had to separate the colored an uncolored grains after each experiment, which was done by sieving since the four samples consisted of grains with different particle sizes. On the left side of Fig. 3.7, the uncolored (white) corundum had a size of \( d = 0.23 \pm 0.02 \text{ mm} \), while the colored (dark gray) particles had a grain size of \( d = 0.33 \pm 0.02 \text{ mm} \). On the right side, uncolored (light gray) glass beads were \( d = 0.48 \pm 0.02 \text{ mm} \) big, and the colored (blue) ones had a diameter of \( d = 0.56 \pm 0.02 \text{ mm} \). The samples are shown on Fig. 3.8. Using different sized particles did not affect the position of the shear zone significantly.

The samples are shown on Fig. 3.8. Using different sized particles did not affect the position of the shear zone significantly.

![Materials studied in the excavation method.](image)

(a) Corundum, \( d = 230 \pm 20 \mu m \), uncolored.
(b) Corundum, \( d = 335 \pm 20 \mu m \), colored to black.
(c) Glass spheres, \( d = 475 \pm 25 \mu m \), uncolored.
(d) Glass spheres, \( d = 565 \pm 35 \mu m \), colored to blue.

**Fig. 3.8**: Materials studied in the excavation method. For both corundum and glass beads two materials were used with slightly different grain size, which were separated after each run by sieving. The size of the images is 2.5 mm \( \times \) 1.9 mm.

The four materials were placed such a way that all the borders between the different materials were clearly visible. Special attention was paid to prepare the border between the colored and uncolored particles as perpendicular to the flow direction as possible since later we calculated the displacement of the material by comparing the distorted shape of this border line.

### 3.2.3 Magnetic Resonance Imaging

The visualization of the internal deformation of the material by nuclear Magnetic Resonance Imaging (MRI) technique was realized using a Bruker BioSpec 47/20 MRI scanner operating at 200 MHz proton resonance frequency with a magnetic field of
4.7 $T$ at the Leibniz Institute for Neurobiology in Magdeburg, Germany (see Fig. 3.9). The device has an internal coil diameter of 7 cm. According to the geometry of this device, a straight shear cell was used with dimensions optimal for that specific MRI device.

![Fig. 3.9: The Bruker BioSpec 47/20 MRI device located at the Leibniz Institute for Neurobiology in Magdeburg, Germany.](image)

The MRI machine provides a set of horizontal grayscale images. The resolution of the MRI device in the horizontal direction was 0.156 mm/pixel, and the slices were at a vertical distance of 0.8 mm from each other. The recorded volume was $512 \times 282 \times 50$ voxels, which corresponds to a physical volume of $8 \text{ cm} \times 4.4 \text{ cm} \times 4 \text{ cm}$. The longest dimension was parallel to the flow. During the measurement the slider was moved by about 0.25 cm between the subsequent scans, and 20 – 30 recordings were made during one measurement. The total displacement of the slider was around 5 – 7.5 cm. One MRI scan took from 8 to 15 minutes depending on the actual sample.

We recorded the evolution of the shear zone for homogeneous samples. The material used in the MRI device had to contain as much hydrogen atoms as possible to give a good signal. Rape seeds turned out to be very suitable for this experiment, as they contain a large amount of oil and their husk is relatively hard. These grains had an average diameter of $1.8 \pm 0.2 \text{ mm}$ and they are shown on Fig. 3.11. The experimental cell was built from plastic and wood since any metal part would disturb the measurement process.

In the other measurements, for studying the complex shape of the shear zones in layered samples, a different method was used. In this case our granular samples (corundum and glass beads) did not give an MRI signal, therefore tracer particles were added
Fig. 3.10: Rape-seeds with an average diameter of $1.8 \pm 0.2$ mm, applied in the MRI measurements for investigating the shape of the shear zone for homogeneous materials. The bar has a length of 5 mm.

to visualize the displacement of the grains. The tracer particles should be sufficiently large for giving a strong signal on the images, but also not too large for the best spatial resolution. The number of the tracer grains should be optimized: too few grains would lead to regions with undetectable deformation, while too many added particles could have an influence on the flow field or they could be able to segregate. The best signal was obtained by using poppy seeds of diameter $d = 0.75 \pm 0.04$ mm. Since the total strain is relatively small in these experiments, during the course of one experiment there was no noticeable segregation of the poppy seeds.

Fig. 3.11: Sample images taken with the MRI machine. On the left side the measurement cell was filled with rape-seeds, on the right side poppy seeds were added to corundum and glass beads to visualize the motion for the cross section.

The poppy seeds also contain a considerable amount of oil, but they are not as round as the rape-seed. We chose mixing the materials with poppy seeds because their size is closer to the grain size of the corundum and glass beads, which are presented on Fig. 3.8 in Sec. 3.2.2. Note that in the MRI measurements the two colored samples were not needed as the painting was not captured by the machine.
3.2.4 X-ray Computed Tomography

The complete three dimensional configuration of the particles was examined with an
other type of tomographic device, namely with X-ray Computed Tomography (CT).
We used a Siemens Artis zeego machine, which is located at the INKA lab of the
Otto von Guericke University in Magdeburg, Germany. This device is a rotational c-
arm based X-ray device equipped with a flat-panel detector. The machine recorded
about 300 transmitted images in different angles of view, and calculated the original
three dimensional absorption distribution within the imaged region. The reconstructed
volume was $512 \times 512 \times 386$ voxels, which is equal to a volume of $25.2 \times 25.2 \times 19.0 \, \text{cm}$
when the spatial resolution was set to 0.495 voxel/mm. The intensity resolution of a
single reconstructed voxel was 16 bits. The cylindrical shear cell had an outer diameter
of 57 cm and the rotating plate had a diameter of 39 cm. The device is shown on
Fig. 3.12.

Fig. 3.12: The X-ray CT apparatus located in Magdeburg, Germany. This is a Siemens
Artis zeego machine with a rotating c-arm and a flat panel detector.

The CT device detects the X-ray absorption of the sample, which is larger for atoms
with larger atomic mass number. The selected materials were all organic with high
carbon concentration. Using metal in the experimental cell was avoided as it absorbs
X-ray more efficiently and therefore the contrast between the wooden particles and the
air is radically reduced, moreover, metal pieces which were visible on a fraction of the
transmitting images could modify the detected absorption map dramatically. Nearly
spherical peas as well as wooden pegs with three different aspect ratio were studied, which are shown on Fig. 3.13.

\[ L = 10 \text{ mm}, \ d = 5 \text{ mm}, \ L/d = 2.0; \ L = 20 \text{ mm}, \ d = 6 \text{ mm}, \ L/d = 3.3 \text{ and } L = 25 \text{ mm}, \ d = 5 \text{ mm}, \ L/d = 5.0 \]

On the tomograms particles are indicated with white. On each recording thousands of particles are imaged in three dimensions independent of their position or orientation. The walls of the container are sometimes visible on the images with a different intensity. The recorded volume was smaller than the size of the experimental shear cell. Therefore near the edges of a recording artifacts were showing up in the directions where additional particles were visible on some of the transmitting images. One horizontal cross section of a tomogram is presented on Fig. 3.14.

Fig. 3.13: Materials used in the CT measurements. From left to right: nearly spherical peas with an average diameter of $7.7 \pm 0.5$ mm, three types of wooden pegs with a length, diameter and aspect ratio of $L = 10$ mm, $d = 5$ mm, $L/d = 2.0$; $L = 20$ mm, $d = 6$ mm, $L/d = 3.3$ and $L = 25$ mm, $d = 5$ mm, $L/d = 5.0$. The bar represents 10 mm.

Fig. 3.14: A horizontal cross section of a tomogram. The cylindrical cell was filled with wooden pegs with the highest elongation shown on Fig. 3.13. The sheared part of the sample is located between the black and white lines.
Taking one CT scan took less than 2 minutes. Compared to the MRI the recorded volume of the CT tomograms is larger and the measurement time for one scan is much less. However, the spacial resolution of the X-ray CT is significantly worse in the horizontal directions. For experiments with the cylindrical shear cell using the X-ray CT was more suitable, and for studies with the straight shear cell the MRI device was more optimal.

3.3 Methods for detecting grain motion

The two and three dimensional images obtained by the above described techniques can be analyzed with various numerical methods. The goal is to determine the flow field, grain motion and for the case of elongated particles the orientation of the grains. In this section I give a general description of the two techniques we used. Later in Sec. 4 I give a more detailed description of the numerical code developed by myself for the accurate analysis of the images. For the general description of the methods, first I will describe the main elements of Particle Image Velocimetry (PIV), which is based on rectangular image pieces containing dozens of particles and provides their average velocity profiles. This information is needed to determine the local deformation of the sample as a function of the position and time. Then I will focus on a different technique called Particle Tracking Velocimetry (PTV) or particle tracking, which gives detailed information about individual grains. Using elongated particles, the last method is essential to study the orientation of the particles.

Both methods are widely used and implemented in many commercial and open source software, but we developed our own code in “C” programming language to ensure the maximal flexibility and efficiency required for our purposes.

3.3.1 Particle Image Velocimetry

The Particle Image Velocimetry (PIV) has its origins at the late 70s, and it was originally developed for visualizing the flow of a transparent fluid by putting small floating tracer particles in it and lighten the system by a stroboscopic light source. Due to the technical development since then this analyzing method became widely available.
Nowadays, digital cameras allow us to control the shutter time precisely, making the stroboscopic light pointless. For granular samples tracer particles are not necessary as the grains itself can be followed on the recorded images. Even if the experimental details changed in the last decades, the numerical method remained essentially the same.

The traditional two dimensional PIV method uses square shaped cuts (small boxes or “interrogation” areas) on two consequent images. In one cut a few dozens of particles are visible. We fix the position of the box on the first frame, and choose areas with the same size of different positions on the second frame. The algorithm searches for the box on the second frame which has the strongest correlation with the image area on the first image. For a visual explanation see Fig. 3.15.

![Frame 1 and Frame 2](image)

**Fig. 3.15:** Illustration of the PIV method. An image area is selected on the first image. On the second frame we look for a box for which the correlation with the original selection is maximal. On frame 2 this will be area (b). The resulting displacement is given by the position difference between the box on the first frame and the box on the second image with the maximal correlation.

The displacement is then calculated by the positional difference between the original box on the first frame and the box with the strongest correlation on the second frame. When the particles are not moving with respect to each other, the maximal correlation should be 1. However, the noise of the image sensor always reduces the maximal correlation in the experiments, but with a good camera this effect is negligible. When the grains experience shear and they move relative to each other, the maximal correlation is reduced significantly.

Sometimes not only the displacement, but also the rotation of the selected image areas is determined directly, depending on the actual problem. For our measurements we
did not need this information. Traditionally, the correlation is calculated for whole pixel
displacements, and the accurate displacement (fraction of a pixel) for the maximum
correlation is recalculated by linear interpolation.

In this work some modifications were applied to fit the algorithm to our actual
problem. The size of the boxes was reduced to 1 pixel in the lateral (or in the cylindrical
geometry the radial) coordinate, and at the same time it was extended in the direction
of the flow to keep the number of visible particles around 10. The other problem is to
find the actual displacement with sub-pixel resolution. As described above, the simplest
way is to interpolate the correlation function itself and look for the minimum of the
interpolated curve. The result will depend on the choice of interpolation (linear, cosine,
cubic etc.). We found that we can get a more accurate result, if we interpolate the image
intensity to a sub-pixel lattice and then calculate the correlation function with higher
resolution. Of course, this change in the algorithm rapidly increased the computational
time. In the optical experiments I implemented a more accurate cubic interpolation,
while for the evaluation of the MRI data linear interpolation was also suitable due
to the higher contrast between the particles and the background on the images. The
last important difference with the conventional method is that the lateral (or radial)
displacement was ignored. Using the straight cell we set one axis of the imaging device
(camera or MRI machine) parallel to the flow direction. In the cylindrical geometry
boxes were only placed on lines which are crossing the center of rotation and parallel
with the axis of the images.

We applied the PIV algorithm to detect the velocity profiles as a function of the
lateral (or radial) coordinate. This information was used to measure the local strain
rate, and the width of the shear zone, see 5.1.1. Note that from the CT images a more
reliable displacement profile can be extracted by using particle tracking data, therefore
those measurements were not evaluated by the PIV method.

3.3.2 Particle Tracking

The conventional Particle Tracking (PT) algorithm can be separated to two problems:
one is the identification of the particles, and the other is pairing the detected grains
on subsequent images. In a typical application the particles are well separated from
each other, and they are clearly visible on the images. In the CT measurements we can see all the particles in three dimensions, on these scans we were able to separate the particles from each other during the data process. For this the watershed algorithm was applied since it can be fine-tuned according to the properties of the actual material. Typically most of the grains were detected and only less than 1% of the particles were lost.

In our optical measurements, and in general for dense granular flows the grains are continuously in contact, and we only see a projection of the sample with the camera. Therefore the particles are often partially hidden by other grains, making the particle detection more complicated. In Sec. 4 I will introduce the steps of the Particle Tracking code, which I developed for processing the optical images and extract particle data.
4. Developing a specific Particle Tracking code

Here I describe the particle tracking algorithm, developed by myself (in “C” programming language) for the accurate detection of particle motion and particle rotation (for elongated grains) on the two dimensional optical images. Building this specific code was crucial to ensure the maximal accuracy and efficiency needed for these investigations. There are freely available Particle Tracking codes on the web, which are rather optimized for well separated particles. In our measurements however, particles are in contact, as we study dense granular systems. Some software were able to analyze our recordings and to detect the particles correctly, but only with limited accuracy, since they were not able to threat the case of partially hidden or joined grains and shadows.

First of all, the problem of grains in contact was solved by mixing about 8% black tracer particles into the white ones, and our software analyzed only the dark pixel clusters of the images. Our program carefully selected the almost fully visible particles on the pictures, and we did not try to correct or estimate the partially hidden black grains nor separate the joined ones. This way only a fraction of the tracer particles was tracked but the inaccuracy of their positions and orientations was small. To compensate for the small amount of actually detected grains per image, large number of images were collected to reduce statistical noise.

A selected $60 \times 30$ pixel area of a recorded image is shown on Fig. 4.1. For quantifying the stationary flow of a given material we recorded 80 image sequences, each consisting of 1007 frames with a resolution of $1024 \times 1024$ pixel and grayscale intensity depth of 8 bits. These measurements were repeated for 3 different rotational speed $\Omega$ of the bottom cylindrical plate. In addition to the stationary measurements, we collected data during the initiation of the flow (initial transient) with one material. In these measurements
about 3000 image sequences each consisting of 300 frames with the same resolution were recorded. Altogether, about 1820 GB of data was analyzed with our particle tracking software, which also encouraged us to develop a new code.

Fig. 4.1: An enlarged $60 \times 30$ pixel area of the original images for rice (left) and glass rods (right). In the following figures I will describe the steps of the algorithm for detecting the elongated particles.

First we have to distinguish the black particles from the white ones. We applied a binarization on the grayscale images with a threshold determined for each measurement separately. The actual threshold depended on the light intensity, the shutter time and the sensitivity of the digital camera. When painted rice grains were sheared in the experiments, the grains slowly eroded and the black particles became slightly lighter, and at the same time the white grains became darker. Therefore an adjustment of the applied binarization threshold was needed during the data collection. The binarized image selections of Fig. 4.1 are shown on Fig. 4.2.

Fig. 4.2: The binarized images of Fig. 4.1 for rice (left) and glass rods (right). Shadows can have an intensity low enough to appear on the binarized images.

Not all the clusters below the threshold value are real tracer particles. To eliminate the artifacts, we calculated the area of all the clusters consisting of black pixels. All the clusters having a size close to the size of one real particle are marked. The ratio of the allowed maximal and minimal size of an object was about 2. The other clusters can be shadows, partially hidden grains or more grains in touch, or even a combination of these. Identifying the clusters with a wrong size would need too much computing.
power, for that reason we did not analyze them in the following steps. Clusters with the right size are indicated on Fig. 4.3.

![Fig. 4.3](image)

**Fig. 4.3**: We measure the size of the clusters on the binarized images seen on Fig. 4.2 both for rice (left) and glass rods (right). This cluster size is the first filter during the data process, which helps to eliminate the partially visible or joined grains and the shadows.

Even if the actual object has an accepted size, it can still be an artifact. For example, if we see a huge shadow next to a partially hidden grain, this filter is not sufficient to distinguish it from a real particle. For that reason we determined the major and minor axes of the particles. When the grain is not perfectly horizontal, its apparent aspect ratio decreases. We did not want to exclude such grains, thus the allowed maximal and minimal ratio of the major and minor axes were far from each other, and their ratio was about 2 for all the studied particles.

The major and minor axes were calculated by the following method. We drew lines crossing the center of mass in different directions. Then we calculated the average distance between the pixels of the actual object and the line. For the major axis this distance is minimal, while for the minor axis it is maximal. The angle between the major and minor axes are always close to $90^\circ$. For both particle shapes studied in this work, the length of the major axis was proportional to the largest average distance (which provided the direction of the minor axis), and the length of the minor axis was calculated similarly. Therefore the aspect ratio can be determined directly. The resulting major and minor axes are indicated on Fig. 4.4.

The extension of the particles in the vertical direction is not directly measurable from the images, but in average their horizontal projection becomes somewhat shorter. This information was only used for calculating the order parameter (introduced in Sec. 5.2.2), which quantifies the strength of the orientational order.

Even if a cluster has an appropriate size and aspect ratio, the direction of the axes can be wrong due to a small shadow. To reach a higher accuracy in the orientational
Fig. 4.4: The major and minor axes of the particle for rice (left) and glass rods (right). If the ratio of the two extensions are far from the expected one, the particle is registered as wrong.

angle, all the grains having such a shadow should be separated. Therefore we fitted the shape of all the remaining clusters by the desired shape: an ellipse for rice and a rectangle for glass rods, and both had the same area and aspect ratio as the cluster. This is illustrated on Fig. 4.5.

Fig. 4.5: Comparison of a particle and the expected particle shape. Green pixels are part of the particle and they located inside the fitted shape. The blue points are outside the fit. The red pixels are not filled by the particle, but they are inside the desired shape. Rice grains were fitted with an ellipse (left) while glass rods were fitted by a rectangle (right).

The pixels of the object which are outside the drawn shape are marked with blue on the figure and counted. If the ratio of these pixels and the total area is too large, then the object is rejected as it has a shadow connecting to it, or it is slightly hidden by a white grain. The accepted ratio was adjusted for each material, but it was approximately 5% for each sample.

To demonstrate the above described particle detection method a sequence of images is shown on Fig. 4.6, where two particles can be followed during flow. The gray level of the clusters shows in which step the actual object was filtered out. The surrounding white particles (see Fig. 4.2), clusters with a wrong size (shown on Fig. 4.3), a wrong aspect ratio (see Fig. 4.4) or not the desired shape (described on Fig. 4.5) are all sorted out. The detected grains are indicated with black and have a white mark at their center of mass. These particles are used in our statistical analysis.
Fig. 4.6: The result of the Particle Tracking code illustrated on an image sequence. The white areas on the image are pixels having an intensity above the threshold value. Light gray areas are clusters with wrong size, middle gray items have a wrong aspect ratio and dark gray areas have a shape different from the desired. The detected particles are indicated with black and a $2 \times 2$ pixel white box is put to their center of mass for making them distinguishable from the gray elements on the picture. Not all the frames are presented here because the local displacement at the actual radius is relatively small.

Pairing the detected particles on subsequent images in general is only possible if the particle movements are relatively small (less than half of the grain diameter). However, in our measurements we have an additional information about the flow field generated by the PIV algorithm, which allows us to increase the maximal displacement of the grains to one particle diameter.

Pairing of the particles on subsequent images is the easier part of the particle tracking problem for our optical measurements. The detected grains are relatively far from each other as only the colored particles can be recognized and many of them are not clearly visible on the images. However, sometimes one grain on the first frame can have more than one potential pair on the second frame. In this case the change in the visible area, the radial coordinate of the center of mass and the aspect ratio were used to choose the correct pair for the actual particle. The accuracy of this method was monitored by counting how many times do we pair two separate particles on the first image with the same grain on the second image. For our measurements this was about one occasion out of 5000 pairs. If we did not find the pair of one particle from the first frame on the second one, we tried to find the pair of the grain on the third image to improve the statistics of the data.

My particle tracking algorithm provided a set of tracer particle positions, orientations and angular velocities. This large amount of particle data was analyzed in Sec. 5.2.
5. Results and discussion

5.1 Wide shear zones

In this section I discuss our experimental results focusing on the features of wide shear zones in homogeneous materials. First in Sec. 5.1.1 the method of determining the shear zone position and width is described in details. Then the results obtained at the surface (optical measurements) are presented in Sec. 5.1.2, which describe the evolution of the shear zone with a high accuracy due to averaging a large number of recordings. Later in Sec. 5.1.3 I will describe the results of the bulk measurements (MRI and X-ray CT), where the structure of the shear zone has been determined inside the material providing additional information to the surface measurements.

5.1.1 Determination of the shear zone

The position and width of the shear zones were determined in the linear and the cylindrical shear cell in a slightly different way. In the linear split-bottom shear cell the two sliders are moved with respect to each other to generate a shear flow. The movement was realized slowly in small steps, so that the flow remains quasistatic. Optical images were taken with a high resolution camera after each small displacement of the slider. The displacement of the material can be determined by the PIV method discussed earlier. Here the movement of the material was determined only parallel to the displacement of the slider, since flow fluctuations in the perpendicular direction average out to zero, thus they were neglected. Three examples of the displacement profiles $D(y)$ are shown on Fig. 5.1, where $y$ is the coordinate perpendicular to the flow direction and $w$ is the width of the sheared region. In this experiment spherical glass beads of $d = 0.25$ mm were used, the system was started from an initially random configuration and the filling height was $H = 68d$. 
Fig. 5.1: Three normalized displacement profiles observed at the surface of the sample in the linear shear cell at different stages of the process, at total slider displacements of 0.8d (black), 5.4d (red) and 50.2d (green). Each profile is obtained by a small displacement of one L shaped slider. The black and red curves correspond to the transient, while the green one describes the stationary state. The experimental data were fitted with an error function (see Eq. 5.1), and the resulting curves are indicated with thin solid lines. The values of the fitting parameter $+w$ and $-w$ are indicated with dashed lines for all three cases.

Each displacement profile calculated from subsequent images was fitted with an error function:

$$D(y) = \frac{1}{2} \left( 1 + \text{erf} \left( \frac{y - y_0}{\sqrt{2} w} \right) \right)$$

(5.1)

Using this formula has the advantage of providing the same zone width $w$ as if we calculate the derivative of the displacement profile $D'(y)$ and fit it by a Gaussian function. We should note here that instead of normalizing the amplitude of this function we used an additional fitting parameter to actually capture the amplitude, and used this information to determine the local strain $\gamma$.

In the cylindrical geometry a somewhat different method is necessary due to the different displacement field. Here the tangential displacement profiles $D(r)$ are detected, one example is shown on Fig. 5.2 (upper panel). For small radii the displacement corresponds to a rigid body motion, meaning that the material is not sheared in that region. For high radii the grains are not moving. For intermediate values of the radius we can observe a smooth transition between this two cases. The increase of the local strain between the two frames $\Delta \gamma(r)$ is given by the following expression:

$$\Delta \gamma(r) = \frac{dD(r)}{dr} - \frac{D(r)}{r}$$

(5.2)
Fig. 5.2: The displacement profile (top panel) and the local strain (bottom panel) in the cylindrical cell, measured on the surface. The displacement for small radii corresponds to a rigid body rotation indicated by red dashed line. The strain is fitted by a Gaussian function (see Eq. 5.3) illustrated with green dashed line on the figure. Only the strain curve can be fitted due to the rigid body rotation of the inner disk.

The resulting $\Delta \gamma(r)$ curve is shown on Fig. 5.2 (lower panel). In this geometry the determination of the zone width is possible only by fitting the strain profile by a Gaussian function:

$$
\Delta \gamma(r) = c \exp \left( -\frac{(r - r_0)^2}{2w^2} \right),
$$

where the amplitude of the curve is $c$ and $r_0$ denotes the radius corresponding to the highest shear.

We used the above described method to calculate the zone width in the optical measurements both in the straight and the cylindrical cell. For the three dimensional images obtained by MRI we used the same method for determining the displacement field in each horizontal slice. For the case of the X-ray tomographic images however we used a different method. Here we were interested not only in the displacement field but also aimed to determine the orientation of the elongated grains. Therefore
we identified the particles, determined their position and orientation (for the case of elongated grains) and followed their motion on the subsequent frames. We calculated the displacement field for 1cm thick layers, but for studying the time evolution - where only about 500 particles are located in the sheared region - averaging two layers was necessary to reduce the noise.

The evolution of the system is best characterized by describing it as a function of the applied strain. However, the strain is not homogeneous through the sample, as it is smaller at the sides of the deformation zone than in the middle of it. To define a characteristic strain $\gamma$ we take an average value in the central part of the zone (measured in the stationary state), which is the region in between the green dotted lines on Fig. 5.1.

5.1.2 Development of the shear zone at the surface

Straight shear cell

In the linear shear cell two kind of materials were used during the experiments, which are shown on Fig. 3.6. One of them is glass beads consisting of spheres with a polidispersity of 10%, which is sufficient to suppress shear induced crystallization. The other sample consists of sand particles sieved to the same size (equal particle volume) as the glass beads, but the grains have a very different shape. Figure 5.3 shows the zone width $w(\gamma)$ as a function of the local strain, on the upper panel $w$ is in particle diameter units while on the lower one it is normalized by its stationary value. In these experiments the system was started from an initially random configuration, and the filling height was $H = 68d$ for both materials. Three measurements were carried out and averaged to reduce statistical noise.

The experimental curves for the two materials show a similar tendency, as in both cases the zone is wider at the beginning, and then the width decreases to its final value. This is connected to the evolution of the contact network as initially the force chains have no preferred direction, but later they are more likely to enclose an angle of about $45^\circ$ with the flow direction, as discussed in Sec. 2.1.2. This process starts around $\gamma \approx 0.1$ and finishes at about $\gamma \approx 1$.

The most important difference between the two data sets is that for irregular grains the transient is significantly longer. We should also note that the ratio of the initial and
the final zone width is much larger for sand than for glass beads. The first observation can be attributed to the difference in the shape of the grains. For the case of sand particles the actual extension of the grains in the flow direction slightly changes due to their continuous rotation, while for spheres it remains constant, which can explain the longer transient for irregular sand particles.

The final value of $w$ is considerably lower for the shape-anisotropic particles than for beads. A similar observation was mentioned in an earlier paper by Fenistein et al. [31], although in a contradictory manner. In the text of that article the authors stated that "irregular particles display narrower zones than spherical ones of similar diameter", but on Fig. 5(c) contradictory information was presented. Most probably two symbols were mixed on the figure.

We should note here that we repeated these measurements with a different initial condition, where after filling the experimental cell tapping was applied to generate a higher initial packing fraction and a different inner structure. The resulting zone

---

**Fig. 5.3:** Evolution of zone width $w$ as a function of the strain $\gamma$, starting from random packing, measured in the linear cell. On the upper graph $w$ is in actual particle diameter units while on the lower one it is normalized by its stationary value.
width showed huge fluctuations making the experiments ill suited for further analysis, therefore we will not include these results in this work. Nevertheless the stationary values of \( w \) were the same within error as for the experiments without tapping.

**Cylindrical shear cell**

We investigated the evolution of the surface width of the zone not only in the straight, but also in the cylindrical shear cell. In the following I will present these results. We built a motorized shear cell and recorded the images by a high speed camera. The filling height was set to \( H = 15.6d \) for silica gel beads and corundum while it was \( H = 14.7d \) for glass rods. A compromise was necessary to choose a large enough recording area, a high enough frame rate and a long enough image sequence, while the total number of recorded pixels are limited by the memory of the camera. We recorded a \( 300 \times 1024 \) pixel area with 60 frames per second. Since repeating the experiments was easier in this setup, we recorded 100 image sequences for all materials and averaged the extracted numbers to keep the level of the statistical noise low.

Two initial configurations were studied here. When the system is started from a random configuration, the resulting curves are comparable to the measurements in the straight cell. The evolution of the zone width is shown on Fig. 5.4. All the earlier statements are true for this experimental data. The strong decreasing tendency of the curves are similar. As we increase the grain shape anisotropy, the initial transient will be longer, and the ratio of the initial and the final zone width is larger. The spherical silica gel beads form a wider zone in the stationary state than the irregular corundum grains.

However, these measurements cover a much wider range in \( \gamma \), and contain data for elongated particles as well. Due to the wider \( \gamma \) range it was revealed that below \( \gamma \approx 0.1 \) the zone width is approximately constant, except for the case of rods. For this sample a slight increase in \( w \) was observed in the beginning of the process, which can be attributed to the actual lateral extent of the elongated grains. All the \( w(\gamma) \) curves have a minimum between \( \gamma = 1 \) and 5, meaning that the zone width slightly increases before it reaches the stationary value.

The other initial condition was prepared by shearing the sample to one direction until the stationary state was reached, and then the direction of shear was reversed
Fig. 5.4: The zone width $w$ as a function of the strain $\gamma$, starting from random configuration, measured in the cylindrical cell. Three materials were investigated. The two panels are similar to those presented in Fig. 5.1.

and the measurement was started. As the force chain network has a preferred direction which is neither parallel nor perpendicular to the flow, reversing the shear leads to a change in this preferred direction, as it was discussed in Sec. 2.1.2. We repeated all the measurements by using this type of initial configuration, and the corresponding curves are shown on Fig.5.5.

The most striking difference compared to the initially random system was that at the very beginning of the process the zone was wider than any time during the previous experiment. This can be explained by the fact that during shearing small voids are created behind the particles. When the opposite shear is applied, the grains can easily move backwards and fall into these small empty places. As these voids are gradually filled with particles, the zone becomes narrower. Therefore below $\gamma \approx 0.1$ the zone
width $w$ is not constant as earlier. At around $\gamma \approx 0.2$ the zone width stops decreasing, and it starts to increase slightly, but later it has an other decreasing part. After about $\gamma \approx 1$ the system mostly forgets its original state and we get a good agreement between the measurements started from the two different initial configurations. For elongated grains the increasing part is more pronounced and the agreement is found only above $\gamma \approx 20$. The final small increasing interval before the stationary state is less significant than in the earlier measurements.

### 5.1.3 Evolution in the cross section

Two different three dimensional imaging devices were used to reveal the position and width of the shear zone inside the material. The MRI machine was invoked to map the
bulk properties in a linear shear cell, and an X-ray CT device was applied to study a
cylindrical shear flow. The MRI scanner is tuned to detect the hydrogen concentration,
therefore the nearly spherical rape-seed was chosen as a sample material due to its
high oil content. The displacement of the material was determined for each pair of
subsequent images by applying the PIV method. The resulting zone width data for
the whole cross section is plotted on Fig. 5.6. This measurement was started with a
random initial condition, the first displacement field of the system is indicated by light
green and the last one is with black.

![Graph showing zone width data](image)

**Fig. 5.6:** The width of the zone in the cross section for different time steps, measured
with MRI. The color is light green for the first scans while it is black for the
stationary state. Measurements done with nearly spherical rape-seeds taking
benefit of their high oil content.

The MRI data revealed that the narrowing of the shear zone is present not only at the
surface of the sample, but also everywhere inside the material. Comparing horizontal
layers at different heights the length of the transient appears to be nearly independent
of the vertical position, but unfortunately the sampling frequency is not enough to
provide a clear evidence for that statement. For calculating the stationary shape of the
shear zone the last 10 steps are averaged in order to reduce the numerical error, and
the result is shown on Fig. 5.7 (left panel).

On this figure the zone width is normalized by the surface width and the local height
is divided by the total height of the material. It is clearly shown that close to the bottom
Fig. 5.7: The normalized shear zone profile in the stationary state in the linear geometry (left) and the shear zone profile for three types of grains in the cylindrical shear cell (right). The dashed line on the left panel corresponds to the quarter of a circle as suggested by Ries et al. [42]. On the right side three stationary zone positions (indicated with solid lines and points) and widths (marked with dashed lines on both sides) are shown for wooden pegs with varied aspect ratio.

slit the width increases rapidly, and close to the surface it becomes nearly perpendicular to the upper boundary. A previous numerical work by Ries et al. [42] stated that the width of the zone can be well fitted by a quarter of a circle. Our experimental data is in coincidence with their observation.

On the right panel of the Fig. 5.7 the shapes of the sheared region are indicated in the cylindrical shear cell for three elongated wooden pegs with different aspect ratio \( L/d \). The width of the shear zone shows a similar height dependence as in the straight cell but the resolution is worse. Due to the cylindrical geometry the middle of the shear zone is not vertical as for the linear cell (this information is not plotted on the graph), but with increasing \( z \) it drifts towards the center of rotation.

### 5.1.4 Summary

In this section, we presented our results focusing on the position and width of the shear zones formed in homogeneous granular materials. The evolution of the zone width on the surface was found to be practically the same in our straight and cylindrical setup, meaning that the actual curvature of the zone in the cylindrical cell does not have
a significant influence. Shearing an initially random system, the zone width is nearly constant in the early stage of the process (local strain $\gamma \approx 0.1$), but then it strongly decreases. For the case of spheres this happens when the applied strain is between $0.1 < \gamma < 1$. Increasing the shape-anisotropy of the particles the narrowing process occurs later. Bulk measurements confirmed that the narrowing process occurs with a comparable deformation scale inside the material. A further analysis of the three dimensional scans is located in Sec. 5.2.5.

### 5.2 Orientation of elongated particles under shear

In the following section I describe our experimental results about the behavior of granular materials consisting of elongated grains subjected to shear flow. In section 5.1 we determined the geometrical features (position and shape) of the sheared region, now we will focus on the motion of grains in this region. Due to shear the elongated grains are continuously rotating, but the actual rotation speed strongly depends on the actual orientation and the local configuration. Most of the experiments are carried out in the cylindrical cell. In section 5.2.1 the distributions of particle orientations are presented for stationary shear for particles with different shape and elongation. Then the average alignment angle and the order parameter are defined and measured in section 5.2.2. Later the evolution of the shear induced ordering is discussed in section 5.2.3, when the system is started from different initial conditions. In section 5.2.5 the connection between the packing density, the zone width and the ordering is analyzed. Finally, in section 5.2.4 we analyze how the rotation velocity of the particles depends on their orientation.

#### 5.2.1 Distribution of particle orientations in stationary shear flow

We recorded optical and X-ray tomographic images and detected the orientation of particles by using a Particle Tracking algorithm. The actual orientation of a grain is measured by the signed angle between the symmetry axis of the particle and the local streamline, as it is illustrated on Fig. 5.8. I will refer to this angle as the alignment
angle \( \theta \). This angle is invariant for a 180 degree of rotation, therefore the values of \( \theta \) can be defined for example as \(-90^\circ < \theta \leq 90^\circ\). Positive \( \theta \) values correspond to the inclination towards the velocity gradient.

\[
\begin{align*}
\theta & \quad \text{inclination towards the velocity gradient.}
\end{align*}
\]

**Fig. 5.8:** The orientation of a grain is denoted by the angle \( \theta \) enclosed by the major axis of the particle and the local flow direction.

In the cylindrical geometry used in these measurements the streamlines are tangential. The circular rotating plate had a diameter of \( D_p \approx 80 \, d \) and an angular velocity of \( \Omega \). The container diameter was large enough \( (D_c \approx 140 \, d) \) to minimize boundary induced orientation in the shear band, while typical filling height was \( H/d \approx 11 \) (unless stated otherwise). In this geometry the local shear rate in the stationary state strongly depends on the radius as it is shown on Fig. 5.2, which allowed us to map the shear rate dependence of the alignment in a single experiment. In addition, in the optical measurements three different angular velocities \( \Omega \) were applied to expand the range of the applied shear rate. The purpose of the X-ray CT experiments was to compare the properties of shear induced orientation at the surface and in the bulk. As we will see these data are very similar, thus the data extracted from surface particles by the other (optical) method are representative for the global order in the shear zone, with only slight variations.

The orientation was studied for prolate particles with different length to diameter ratios \( L/d \). Moreover, two particle shapes were used to explore the shape dependence of the orientational properties: nearly ellipsoidal particles (rice grains), and cylindrical particles (glass rods and wooden pegs). On Fig. 5.9 the orientational distributions of the particles in stationary shear are shown for 6 types of grains. The rods with the lowest and highest aspect ratio \( L/d \) were examined in the X-ray CT device while the other 4 distributions were extracted from optical images.
Fig. 5.9: Distribution of the particle orientation $\theta$ in stationary shear for grains with different shape and elongation. For rods with $L/d = 5.0$ and 2.0 data was obtained by X-ray CT, other data obtained by optical particle detection. The vertical blue line corresponds to the average orientation $\Theta_{av}$, which is determined by fitting a Gaussian function. The background level is significantly higher for smaller $L/d$ and for rods.

The angular distributions were calculated using about 200,000 data points for the optical measurements, while the X-ray data collected from the surface region of the shear zone for pegs with $L/d = 5.0$ and 2.0 are consisting of about 10,000 data points. We included data from only the central part of the sheared region, where spatial variation of the shear rate is relatively small. Due to the smaller number of data points, the scatter of the distributions is larger for the X-ray data. The graphs show that for more elongated grains the orientation is stronger since the distributions are narrower. For rods the higher background level can be attributed to the edges of the particles, where the forces can change their direction more frequently leading to stronger statistical noise. Here the average particle orientations $\Theta_{av}$ were determined by fitting the data with a Gaussian function while we took care of the offset and the periodic boundary conditions. The resulting $\Theta_{av}$ values are illustrated with blue vertical lines. An other
important difference between the two particle shapes is that for rods the distribution is broader than for rice with similar aspect ratio. An important factor leading to this can be demonstrated taking the case of the rods with \( L/d = 1.4 \), for which the angular distribution is shown on Fig. 5.10 (upper panel).

![Fig. 5.10: Distribution of the particle orientation \( \theta \) for rods with \( L/d = 1.4 \), determined from optical experiments (upper panel). The vertical blue line corresponds to the average orientation \( \Theta_{av} \) as on Fig. 5.9. The two peaks correspond to the two possible orientations illustrated on the lower panel.](image)

For this material we can observe two peaks in the distribution function, and the offset is large compared to the peaks. This indicates that these grains align with their longest extension (diagonal) in the preferential direction \( \Theta_{av} \), as illustrated on Fig. 5.10 (lower panel). Since we determine the particle orientation \( \theta \) by measuring the orientation of the symmetry axis, the resulting distribution of the detected angles has two peaks. The distance between the two maxima is close to \( 2 \arctan(1/1.4) = 71^\circ \), which correspond to the two particle configurations. The effective average alignment angle in this case is between the angles corresponding to the two peaks, i.e. around \( \Theta_{av} = 29^\circ \). The peak at around 1\(^\circ\) has a higher probability because of the particles have a lower extension perpendicular to the flow.
This argument is valid for all samples with cylindrical grains. For large $L/d$ ratios the two peaks can not be separated, but this effect leads to a widening in the angular distribution. Note that wooden pegs with $L/d = 5.0$ the edges of the cylinders are rounded, as it is seen in Fig. 3.13, therefore this widening can not be observed.

5.2.2 The average alignment angle and the order parameter

For particles detected in three dimensions the shear-induced orientational order is monitored by diagonalizing the symmetric traceless order tensor $T$:

$$T_{ij} = \frac{3}{2N} \sum_{n=1}^{N} \left[ \vec{e}_i^{(n)} \vec{e}_j^{(n)} - \frac{1}{3} \delta_{ij} \right], \quad (5.4)$$

where $\vec{e}_i^{(n)}$ is a unit vector along the long axis of particle $n$, and the sum is over all $N$ detected particles. The eigenvalues of $T$ are $e_1 < e_2 < e_3$. The order parameter is equal to the largest eigenvalue $S = e_3$. The average alignment angle $\Theta_{av}$ is given by the angle between the eigenvector of $e_3$ and the streamlines, and positive values correspond to tilting towards the velocity gradient, in agreement with the earlier definition in Sec. 5.2.1. Note that the two calculation methods of $\Theta_{av}$ gave a similar result. A second, biaxial order parameter $D$ is defined as the difference of the two smaller eigenvalues $D = e_2 - e_3$. The ordering was studied in two regions of the samples. The nearly 10 000 detected particles with a center of mass in the approximately 1 cm deep top layer are directly comparable to the optical data. In the next about 2 cm thick layer below the surface roughly 20 000 particle positions were captured, and the difference between the surface and bulk orientation was tested.

In the optical measurements we tested the shear rate dependence of the average alignment angle $\Theta_{av}$ and the order parameter $S$. We divided the shear zone into 15 bands with a radial expansion of $0.19 \text{ cm} \approx d$ and determined the orientational distribution of particles with their center of mass falling in each band separately. The average alignment angle was determined for each band by fitting the angular distributions with Gaussian functions. The time averaged shear rate $\dot{\gamma}$ was a relatively well defined constant for all bands. On Fig. 5.11 the resulting values of $\Theta_{av}$ are shown as a function of the shear rate. Three different angular rotation speeds were applied with
\( \Omega = 0.26, 0.52 \) and 4.2 \text{ rad/s}, and the corresponding shear rate ranges are indicated on the figure with horizontal black lines.

\[ \gamma = \frac{\dot{\gamma}}{d} \sqrt{\frac{\rho}{P}} \]

\( \rho \) is the density of particles and \( P \) is the pressure. In our experiments no additional forces were applied and therefore \( P \) can be estimated by taking the hydrostatic pressure. Thus the inertial number spans approximately a range between 0.0002 \( \leq I \leq 0.4 \), meaning that we explored both the quasistatic and dense flow regimes.

For all the samples studied \( \Theta_{\text{av}} \) is well defined, and it is essentially independent of the shear rate for the the whole range of \( \dot{\gamma} \) investigated here. In order to determine the flow regime of the system introduced in Sec. 1.4 in terms of the importance of inertial effects, we calculated the corresponding inertial numbers \( I \) [26]. The inertial number is defined as \( I = \gamma d \sqrt{\rho/P} \), where \( \rho \) is the density of particles and \( P \) is the pressure. In our experiments no additional forces were applied and therefore \( P \) can be estimated by taking the hydrostatic pressure. Thus the inertial number spans approximately a range between 0.0002 \( \leq I \leq 0.4 \), meaning that we explored both the quasistatic and dense flow regimes.
The order parameter $S$ was calculated simultaneously with the average alignment angle. The X-ray measurements showed that the average orientation is essentially horizontal, the out of plane tilt is below $3^\circ$. Thus, for the optical measurements we can calculate $S$ by measuring the in plane variations of the particle orientations and estimating the out of plane tilt (vertical component) of the grains from the apparent shortening of the particles. This calculation method is not fully accurate as the particles are not identical, and we cannot measure the actual length of a grain. However, the average deviation in the vertical and the horizontal direction was similar. The result is shown on Fig. 5.12, where the same symbols are used as for Fig. 5.11. The shear rate ranges corresponding to the three different values of the angular velocity $\Omega$ are again marked.

![Graph showing $S$ as a function of shear rate $\dot{\gamma}$ for two kinds of particle shapes, with symbols that are the same as on Fig. 5.11. For a given material $S$ is nearly constant, although for rice it slightly decreases with increasing the shear rate.](image)

**Fig. 5.12:** The order parameter $S$ as a function of the shear rate $\dot{\gamma}$ for two kinds of particle shapes, with symbols that are the same as on Fig. 5.11. For a given material $S$ is nearly constant, although for rice it slightly decreases with increasing the shear rate.

For the case of nearly ellipsoidal particles (rice) the order parameter slightly decreases with increasing shear rate, which means that ordering becomes somewhat
weaker as interparticle collisions become stronger. Interestingly this tendency is not seen for glass rods.

The next important step in characterizing the nature of shear induced alignment is to determine how this phenomenon depends on the aspect ratio of the particles. For this purpose we averaged all data for $\Theta_{av}$ taken at different shear rates for a given material, which results a single value for $\Theta_{av}$ for each sample. These data are shown on Fig. 5.13. The glass rods with an aspect ratio of $L/d = 1.4$ are excluded from the further analysis due to the different nature of orientation with multiple preferred directions. However, the resulting data points are in agreement with the tendencies discussed hereafter.

![Fig. 5.13: The average orientational angle $\Theta_{av}$ as a function of the aspect ratio $L/d$ of the particles. Both optical and X-ray CT data are shown. Data for rice and rods are displayed with different symbols, and the results of the bulk measurements are indicated with empty symbols. The blue point was taken for a particularly large filling height, $H/d = 34$. The cylinders with $L/d = 1.4$ are not included because the angular distribution is qualitatively different from all other samples.](image)

The data clearly show that for more elongated grains $\Theta_{av}$ becomes smaller, meaning that the average particle orientation is closer to the flow direction. The data obtained by X-ray tomography revealed that the alignment angle is somewhat smaller in the bulk than in the surface layer. Focusing on the effect of grain shape, we find that for rod like particles $\Theta_{av}$ is somewhat smaller than for nearly ellipsoidal rice grains with similar aspect ratio. In order to test whether the proximity of the lower boundary has an effect
on the orientation due to the relatively small filling height, a benchmark measurement has been carried out using a filling height \( H/d = 34 \), which was approximately 3 times larger than in the other measurements. As it is seen on Fig. 5.13 the alignment angle was found to have the same value as for the measurement with \( H/d = 11 \) within error.

We can replot the same data for \( \Theta_{av} \) as a function of the order parameter \( S \), as it is shown on Fig. 5.14.

![Graph](image.png)

**Fig. 5.14:** The average orientational angle \( \Theta_{av} \) as a function of the order parameter \( S \) of the system. The symbols used here are the same as on Fig. 5.13. The cylinders with \( L/d = 1.4 \) are not included.

For particles with smaller \( \Theta_{av} \) (larger aspect ratio) the ordering is stronger. Inside the material a larger ordering can be observed than at the surface layer. The measurement with the larger filling height \( H/d = 34 \) shows that \( S \) slightly increases for shallow samples.

### 5.2.3 Evolution of the alignment

Until now we focused on the properties of the average alignment in stationary shear. In the following we wish to quantify the evolution of the average grain alignment and the order parameter for two initial configurations: when the system is started from an initially random orientation, and when the flow direction is reversed after the system reached the stationary state. In Sec. 5.1.2 we have already characterized how the zone width changes during such transients. In order to capture the change in the grain...
orientations with good statistics during these transients, a significantly larger amount of data had to be taken compared to the stationary measurements presented in Sec. 5.2.2. Therefore these measurements were performed with one selected sample, glass rods with $L/d = 3.5$. For the case of rice we had a further technical difficulty, namely the colored tracer particles lost their color during extensive measurements, thus their identification became gradually more difficult. The advantage of the glass rods was that this erosion was not present for these particles.

The evolution of the system starting from an initially random configuration is shown on Fig. 5.15, where the number of detected particles is indicated by the gray level of the plot. The orientations were collected from about 1300 measurements. Only the center of the sheared region was used as in Sec. 5.2.1.

![Fig. 5.15](image)

**Fig. 5.15:** Evolution of the orientational distribution of the particles as a function of the local deformation $\gamma$. Darker areas correspond to higher probability densities. The system was started with random orientation, the plot corresponds to the average of 1300 recordings. Measurement done with glass rods with an aspect ratio of $L/d = 3.5$, data collected optically from the surface of the sample.

As Fig. 5.15 shows, the initially random orientation is relatively quickly destroyed. The particles on average start to rotate with a decreasing $\theta$, in other words their rotational direction is dictated by the vorticity of the shear flow. The average alignment
angle $\Theta_{av}$ is strongly decreasing due to the mostly negative rotational direction of the particles. The stationary state is reached at about $\gamma \approx 5$.

The evolution of the alignment in the oppositely sheared system is shown on Fig. 5.16. For this case slightly more measurements were carried out, about 1700 recordings are processed.

![Graph showing the evolution of the orientational distribution of the particles for the oppositely sheared initial condition.](image)

**Fig. 5.16:** Evolution of the orientational distribution of the particles for the oppositely sheared initial condition. The same materials and evaluation method was applied as for the random initial condition on Fig. 5.15. The graph shows the average of 1700 measurements.

In the measurements with reversed shear direction the majority of the grains rotate with a positive angular velocity, but some particles leave this main branch and rotate in the direction determined by the flow field. We can calculate both $\Theta_{av}$ and $S$ as a function of the local strain to extract the characteristic deformation scales for both initial conditions. The resulting curves are shown on Fig. 5.17.

When the system is started from a random configuration, the average alignment decreases exponentially to its stationary value with a characteristic strain scale $\gamma = 2.57$. Here we calculated $\Theta_{av}$ introduced in Sec. 5.2.1, again taking care of the offset and the periodic nature of the alignment angle. The evolution of the order parameter from zero to its final value is 3 to 4 times faster (with $\gamma = 0.73$) than the evolution of
The average alignment angle $\Theta_{av}$ is found to be determined dominantly by the local neighborhood of the particles, but the shear flow also has an influence.

For the transient corresponding to the case of reversed shear we find more complex dynamics. The average alignment angle mostly increases during the process, only a small overshoot can be observed. The order parameter strongly decreases in the first part of the process, when a significant fraction of the grains rotate backwards in the shear flow. In the second part of the process the order parameter increases back to the initial value. It is not straightforward to determine the characteristic strain scales, as exponential decay functions do not fit the experimental curves well. However, the evolution of $\Theta_{av}$ is slightly faster and the change in $S$ is significantly slower compared to...
to the random initial configuration. This is connected to the fact that first the original ordered state gets destroyed, and then a new ordered state develops.

### 5.2.4 Rotation of individual particles

The rotation of individual grains in the horizontal plane was studied in optical measurements for elongated glass rods with an aspect ratio of $L/d = 3.5$. The stationary state as well as the two transients discussed in Sec. 5.2.3 were analyzed. Here only the center of the sheared region is averaged as in Secs. 5.2.1 and 5.2.3, where spatial variations of the shear rate are relatively small. Under stationary shear most particles are close to the average alignment angle, as it is shown on Figs. 5.8 and 5.9. To get a better insight to the dynamical processes, we show the evolution of $\theta$ for several grains for the case of stationary shear on Fig. 5.18.

![Figure 5.18: Particle orientations $\theta$ as a function of strain $\gamma$ in the stationary state, where $\gamma = 0$ corresponds to the beginning of the measurement. The dashed lines show the quickly rotating particles far from the average alignment angle $\Theta_{av} \approx 10^\circ$. These measurements were done with glass cylinders of an aspect ratio of $L/d = 3.5$, data were collected optically from the surface of the sample.](image)

We see that the angle of the particles near the preferred alignment is fluctuating, and positive and negative angular velocities have essentially the same probability. But the grains, which are strongly misaligned, turn very quickly in the direction dictated by the flow. The dashed lines highlight three rapid particle rotations far from $\Theta_{av} \approx 10^\circ$. 
Data in this plot were selected strictly from the center of the shear zone (1d interval in radius) where the shear rate is the highest, and 10 measurements are plotted with different colors.

In order to accurately characterize the angular velocity $\omega = d\theta/dt$ of the particles we matched the detected grains for all subsequent image pairs. The value of $\omega$ should be normalized by the local shear rate $\dot{\gamma}$ if we want to compare data originating from different regions of the sample. This normalized rotation speed naturally depends on the actual orientation of the grain, therefore we collected the angular velocity distributions for different $\theta$ values separately. The resulting distribution is shown on Fig. 5.19.

![Fig. 5.19: Distribution of the normalized rotation velocity $\omega/\dot{\gamma}$ of the particles as a function of their orientation $\theta$ in stationary shear. The colors indicate the probability of a rotation velocity at a certain $\theta$. The optical measurements were done with glass rods with $L/d = 3.5$.](image)

Close to $\Theta_{av}$ the maximum of the angular velocity distribution is close to 0, but for the particles which perpendicular to it, this maximum is located at around 60 deg/unit strain. Note that the rotation speed of $\omega/\dot{\gamma} \approx -1$ rad/unit strain corresponds to the local vorticity of the shear flow. The number of detected particles between $-20^\circ < \theta < 40^\circ$ are much larger than outside this region, which is reflected in the noise level of the probability density.

We calculated the averages of the normalized rotation velocity $\omega_{av}/\dot{\gamma}$, not only for glass rods with $L/d = 3.5$ but for more materials, as it is seen on Fig. 5.20. Here
4 data sets represent the optical measurements, while an other 3 data sets originate from the X-ray CT measurements. The nearly ellipsoidal rice grains are indicated with dashed lines, and the cylindrical particles are shown with solid lines on the figure. Note that the wooden pegs with $L/d = 5.0$ and 3.3 are slightly rounded. On the right graph a numerical calculation is included for comparison by Jeffery [57], which gives the rotational velocity of a lone hard ellipsoid in a sheared Newtonian liquid. The main feature of all the curves are the same: the particles rotate slower when they are parallel to the shear flow, than when they are perpendicular to it.

The difference between the two sets of experimental curves can be attributed to the different particle detection algorithm. On the optical images only the fully visible grains can be captured, and the grains tilted higher than 30° out of the horizontal surface can not be captured accurately. But on the tomographic images almost all the grains are detected, and therefore nearly all grain rotations are registered. As a consequence, despite its higher noise level the bulk data is more reliable for studying the particle rotation. On the X-ray CT data we can find that for more elongated particles the difference between the lowest and highest $\omega_{av}/\dot{\gamma}$ is increasing, which is in agreement with the calculation of Jeffery.
In Sec. 5.2.3 we found that the evolution of the angular distribution strongly depends on the initial condition. The change in the angular distribution is essentially driven by the rotation of the grains, so we calculated the average rotation velocities for both transient measurements, similarly to the stationary state. The result can be seen on Fig. 5.21.

![Graph showing normalized average rotation velocity of particles](image)

**Fig. 5.21:** Normalized average rotation velocity of the particles $\omega_{av}/\dot{\gamma}$ as a function of their orientation $\theta$ during the two types of transients from optical data. Here glass rods with $L/d = 3.5$ were used. The red crosses correspond to the initial transient while the black circles were calculated in the stationary state, where both $\Theta_{av}$ and $S$ are close to their final value (see Fig. 5.17).

To see the difference between the transient and the stationary state we determined the stationary curve for both cases by averaging the rotational data after a certain $\gamma$ value. The change in $\omega_{av}/\dot{\gamma}$ is clearly seen: for both initial conditions the average rotational speed is positive between about $-15^\circ$ and $15^\circ$, but later it becomes negative for all $\theta$ values. It means that at the beginning of the processes more particles rotate backwards than in the stationary state.

### 5.2.5 Connection with the zone width and the density

In this section I will provide bulk data obtained by MRI and CT and characterize the evolution of various quantities during the initial transient. In Sec. 5.1.3 I have already presented MRI as well as CT data about the geometry of the shear zone in the stationary state. For both methods we have the possibility to calculate other relevant parameters of the system and monitor their evolution. The most important quantity
is the packing density, which is the ratio between the occupied volume of the particles and the total volume of the sample. In the X-ray CT measurements elongated particles were also investigated, where the average alignment angle $\Theta_{av}$ and the order parameter $S$ are accessible by detecting the particle positions and orientations.

On the MRI scans of the straight shear cell, the packing density and the zone width was monitored, and the result is shown on Fig. 5.22. With this method, the detected intensity of the particles was not homogeneous through the sample. However, distinguishing voxels filled with particle and air a binarization using a constant threshold was sufficient. Another problem was that within each grain not all the voxels are visible because some parts of the grains contain much less oil than the typical value. Therefore, the detected initial packing density was normalized to 1 and all the scans were nor-

Fig. 5.22: (upper panel) The relative packing density profile of rape-seeds recorded with the MRI in four stages of the process. The material is dilated due to the Reynolds dilation. The shear zone is located in between the dashed lines for the stationary state, which was shown on Fig. 5.6. (lower panel) The relative packing density (black) and the zone width (red) as a function of the strain, both are averaged for $z > 8d$. The lines are exponential decay fits. The characteristic strain scales describing the decrease of density and zone width are close to each other.
malized by the same factor as the first one. For calculating the relative packing density and the zone width of the sample we averaged these numbers for the upper 60% of the cross section for heights $z > 8d$, as within this part vertical spatial variations are not significant. Examples of the relative density profiles are shown on the upper panel of Fig. 5.22.

Data for the relative density was averaged for the region where the stationary shear is located, which is marked with vertical dashed lines on the upper panel. The change in the density and the zone width can be found on the lower panel of Fig. 5.22. Both data sets were fitted by an exponential decay function, and the characteristic strain scale for the relative density decrease is $\gamma = 0.22$ while for the narrowing of the shear zone it is $\gamma = 0.15$.

The CT tomograms were analyzed more in details, more types of materials were tested and a larger number of scans were produced. Here the cylindrical shear cell is applied during the measurements, and packing density is calculated from the binarized scans and data are presented without any normalization. We are focusing on relative density changes during the transient. The absolute value of the density depends on the applied local threshold value for the binarization, thus accurate comparison with other data in the literature will not be conclusive. On Fig. 5.23 one can see the evolution of the packing density, the zone width, the average alignment angle and the order parameter for four different materials.

The nearly spherical peas have no elongation, thus for this material only the first two quantities are presented. The packing density strongly decreases for all the materials, but for rods this is more pronounced. Interestingly for the rods with the highest $L/d$ the density decrease has an overshoot, and later we can see a slightly increasing part before the stationary state is reached. For the zone width the data are noisier than those obtained by the optical method (see Fig. 5.4), but at least the significant narrowing of the zone is captured. The evolution of the average alignment angle and the order parameter are similar to the previously shown results obtained optically (see Sec. 5.1.2), but below about $\gamma \approx 0.2$ the $\Theta_{av}$ values are affected by the slightly ordered initial state, therefore those data points are excluded from the further analysis. To compare the characteristic strain scales for all of these physical quantities, we fit the curves by an
Fig. 5.23: The packing density, the zone width $w$, the average alignment angle $\Theta_{av}$ and the orientational order parameter $S$ as a function of the strain, for wooden pegs with three different aspect ratio and for nearly spherical peas. All quantities are measured simultaneously in the X-ray CT device. The characteristic $\gamma$ scales are exported by fitting exponential decay functions on all the experimental curves, which is shown on the next figure.

The blue data points correspond to the density decrease, which does not depend on the particle shape and has a typical scale between $0.15 < \gamma < 0.2$. Similarly, the strain scale for $\Theta_{av}$ (red) appears to be independent of the aspect ratio, and its value is about $\gamma \approx 1.5$, suggesting that there is no strong connection with the density as there is one order of magnitude difference between the two $\gamma$ scales. For the strain scales describing the evolution of the zone width $w$ and order parameter $S$ a systematic increase with $L/d$ can be observed, and these values are close to each other indicating that there
Fig. 5.24: The characteristic strain scales obtained from Fig. 5.23 (connected by lines) as a function of the aspect ratio $L/d$ of the particles. Additional points are taken from earlier figures for comparison. From the MRI scans of rape-seed both the density decrease and the shear zone narrowing is calculated (□, Fig. 5.22). From the optical measurements presented on Fig. 5.4 the strain scales for silica gel beads (◇) and glass rods with $L/d = 3.5$ (×) are exported. The Reynolds dilation is very robust and its $\gamma$ scale is independent of the particle shape. The strain scale of the $\Theta_{av}$ is also independent of the elongation. The $\gamma$ scales of $w$ and $S$ are systematically increasing with $L/d$.

is a connection between the two quantities. If we compare the strain scales calculated from other measurements, we find that they fit to the tendencies observed for the X-ray measurements. Note that the characteristic strain scales obtained from Fig. 5.17 (surface optical measurements) are significantly larger, but there $\Theta_{av}$ had a 3.5 times larger strain scale than $S$ had, which also fits to the tendency on Fig. 5.24.

At this point, the increase of the packing density at large $\gamma$ values for the most elongated grains shown on Fig. 5.23 can be explained. For elongated particles the orientational ordering allows us to reach a larger packing fraction. For larger $L/d$ the characteristic $\gamma$ scale is larger leading to a better temporal separation from the Reynolds
dilation. At the same time the final order parameter is larger, which has a consequence of a higher packing fraction in the stationary state.

5.2.6 Conclusions

In this section we studied the alignment and rotation of macroscopic elongated particles in shear flow. We recorded optical images about the surface and collected X-ray data from inside the sample to reveal the orientation of individual particles. We measured the orientation in the steady state of the system with nearly ellipsoidal and rod-like particles. The evolution of the orientation was monitored with two initial conditions: with random orientational distribution and when the system was sheared in the opposite direction earlier. The rotation of the elongated grains was also determined in the stationary state as well as during the two initial transients. In the last part we explored the characteristic strain scales of different physical quantities when the system is started from a random configuration.

We found that in the stationary state of the system the average orientation of the elongated grains is close to parallel to the flow, this orientation is slightly tilted towards the velocity gradient. The tilting angle or average alignment angle $\Theta_{av}$ is independent of the shear rate across three decades, and its value is determined by the shape and aspect ratio of the particles. For a given particle shape $\Theta_{av}$ decreases with increasing aspect ratio. For nearly ellipsoidal rice particles we measured a somewhat larger $\Theta_{av}$ than for cylindrical grains with nearly the same aspect ratio. The ordering of the sample is captured by the order parameter $S$, which is almost independent of the shear rate. For larger aspect ratio and smaller alignment angle the order parameter increases. The X-ray CT measurements confirmed that the shear induced alignment is very similar in the bulk and at the surface. Below the surface we found a somewhat larger order parameter $S$ and a slightly smaller average alignment angle $\Theta_{av}$. We tested the orientational effect of the bottom wall by recording measurements with a different filling height by using the optical method. For a filling height three times larger than the original we detected almost the same $\Theta_{av}$ and a 10% lower $S$.

The shear rate independence of the average orientation of the particles for granular materials is similar to the flow alignment of nematic liquid crystals. For nematic
liquid crystals the average orientation of the molecules is described by the director. This director encloses a small angle with the local flow direction, and it is shear rate independent as well. The aspect ratio of the most commonly used liquid crystalline molecules (MBBA and 5CB) is estimated to be $L/d = 2.5$ [86], which is comparable to our granular samples. However, the molecules experience attractive forces (dipole-dipole, van der Waals) as well, while the macroscopic grains investigated here interact only via hard core repulsion and friction. For nematic liquid crystals the order parameter is set by the temperature, namely for higher temperature the increasing thermal fluctuations lead to weaker order. For our granular samples it is the shear flow which generates both the ordering and the average alignment of the particles relative to the flow direction.

Despite these differences, the liquid crystalline and granular systems show remarkable similarities, for example $\Theta_{av}$ is a decreasing function of $S$ (shown on Fig. 5.14 for granular systems). Our observations suggest that the shear induced alignment is a fundamental feature of these systems, and it originates from the elongated shape of the building blocks.

When we start to shear a granular sample consisting of elongated particles from an initially random configuration, the particles in average start to rotate in the direction dictated by the flow. This results a smoothly decreasing average alignment angle $\Theta_{av}$ and an increasing order parameter $S$. If our sample was originally sheared in the opposite direction, then first we have to partially destroy the original ordering and then build up the new one. Interestingly the order parameter will reach its stationary value slower compared to an initially random system, but the initial and the stationary values of $\Theta_{av}$ ($-10^\circ$ and $10^\circ$) are close to each other and therefore its evolution was found to be faster.

The evolution of the grain alignment is strongly connected to the rotation of the particles. The average angular velocity of individual particles depends on the actual orientation. When a grain is perpendicular to $\Theta_{av}$, its average rotation speed is larger compared to the rotation speed of the aligned particles. For materials consisting of more elongated particles the ratio of these fast and slow angular velocities becomes larger. A similar behavior was found earlier for hard ellipsoids in a Newtonian fluid under shear by Jeffery [57]. Under stationary shear our granular particles on average
rotate in the direction dictated by the flow, it is only the magnitude of the angular velocity which depends on the grain orientation. In contrast to this, during the initial transients, we found that the average rotation speed changes sign for particles close to the average orientation, meaning that those particles prefer to reach $\Theta_{av}$ on a shorter path by rotating opposite to the vorticity of the shear flow.

We compared the evolution of the ordering with the width of the sheared region and the packing density for particles with different aspect ratio. The characteristic deformation scale corresponding to the Reynolds dilation is independent of the particle shape. The evolution of the average alignment angle also does not change systematically with the elongation of the grains. The zone width and the order parameter have a strain scale close to each other and these scales are increasing with the aspect ratio.

5.3 Shear localization in layered granular materials

In this section I will present our experimental results about layered (heterogeneous) granular samples under quasistatic shear. In section 5.1 we quantified the position and the width of the sheared region for homogeneous samples, and concluded that the shear zone width changes with the particle shape. In this section we will only study the position of the shear zone. In Sec. 2.3.1 I summarized the previous results for layered systems and described the analogy between the position of the shear zone in granular materials and the optical path in geometric optics. These former works focused on a limited number of experimental configurations.

In the following we will explore further configurations where the shear zone takes a complex shape in granular samples in straight shear cells. We will use spherical and irregular shaped particles within one sample and determine the position of the shear zone by optical (excavation technique) and MRI measurements. The most important parameter of the materials used in our experiments is the effective friction coefficient, which is larger for irregular particles. The materials, their main properties, as well as the measurement methods are described in Sec. 3.2.2 and 3.2.3. In section 5.3.1 a series of experimental configurations and the expected results are described. Then we will
study two new experimental geometries. In the first one described in Sec. 5.3.2 the interface of the materials is a vertical plane, which leads to a configuration similar to the limiting case of total reflection in geometric optics. For the second geometry in Sec. 5.3.3 the interface between the different materials is horizontal, and both ends of the shear zone is located in the material with the higher effective friction coefficient.

5.3.1 Theoretical expectations

The position of the shear zone can be calculated by determining the path corresponding to the smallest shear force, as discussed earlier in Sec. 2.3.1. This variational problem can be reduced to the integral (2.2), or if the pressure $p$ is constant in the actual sample, the integral has an even simpler form (see Eq. 2.3). To reveal all the cases with qualitatively different zone positions, we can study a series of experiments as shown on Fig. 5.25.

![Fig. 5.25: A series of hypothetical configurations, where the layer boundary is a straight line, and this interface as well as the right end point of the shear zone is fixed. Only the left end of the zone is moved downwards as we go from panel a to d. The shear is applied by moving one of the container walls as indicated by red arrows. We can find 4 qualitatively different zone positions. The first case is similar to the refraction in geometric optics, and this phenomenon was studied earlier in the literature [84, 85]. The second case is the limiting case of total reflection. In the third case the shear zone is expelled from the higher friction part of the sample, but later the zone returns to it. The last case shows that if the two end points are too far from the lower friction part, the shear zone will be straight just like for a homogeneous sample. In the second and third case we expect the same angle of incidence $\beta_c$ independent of the exact geometry.](image)

The end point of the shear zone on the right side is held fixed at a height falling within the higher friction part of the sample, and the left one is moved downwards as
we go over the four different configurations presented in Fig. 5.25a-d. When the other end point of the zone is located in the lower friction part, the shear zone is forced to cross the layer boundary between the two materials (see Fig. 5.25a). This corresponds to the case of shear zone refraction, which was studied previously in two papers [84, 85].

When one end point is located at the layer boundary (see Fig. 5.25b), we expect that the zone goes parallel to the interface of the two materials until a certain point. This point is determined by the angles of incidence, namely the Snell’s law for granulates in Eq. 2.4 should be applied with one angle of 90°:

$$\frac{1}{\sin \beta_{\text{crit}}} = \frac{\mu_h}{\mu_l}$$

(5.5)

where $\mu_h$ and $\mu_l$ are the effective friction coefficients of the materials with higher and lower friction, and $\beta_{\text{crit}}$ is the critical angle of incidence. If the line connecting the two end points of the shear zone encloses a larger angle with the layer boundary than this critical angle $\beta_{\text{crit}}$, a straight shear zone gives a smaller integral. The actual experimental setup and the results are described in Sec. 5.3.2.

If both end points of the shear zone are located in the higher friction part, but close to them a lower friction part of the sample is present, we expect that the zone is expelled from the higher friction part, lies parallel to the layer boundary in the lower friction part of the sample, and later it returns to the higher friction part (shown on Fig. 5.25c). In classical optics no such a phenomena is observed. For this case, in both boundary crossing points we estimate the angles of incidence similar to the limiting case of total reflection. We studied this phenomenon in a different setup, which is detailed in Sec. 5.3.3. Finally, when the two end points of the zone are far from the lower friction part of our sample, a straight path gives the smallest integral, and the zone is lying entirely in the higher friction part (see Fig. 5.25d).

### 5.3.2 The limiting case of total reflection

An illustration of the experimental setup is shown on Fig. 5.26. The shear cell consisting of two L shaped sliders had a width of $W = 4.0$ cm and a height of $H = 3.4$ cm. The location of the lower end point of the shear zone is given by the setup, but the other end can be located anywhere on the free surface. The pressure is not constant through
the sample, which influences the shape of the shear zone. The ratio of the effective friction coefficients can be estimated by the angles of repose $\theta_c$ of the two materials:

$$\frac{\mu_h}{\mu_l} = \frac{\tan \theta_c^{\text{corundum}}}{\tan \theta_c^{\text{glass}}} = \frac{\tan 33.2^\circ}{\tan 21.9^\circ} \approx 1.63$$  \hspace{1cm} (5.6)

\[ \text{Fig. 5.26:} \] Sketch of the experimental configuration, and the expected shear zone position (indicated with a red line) in the limiting case of total reflection.

Both excavation and MRI techniques were applied to study this geometry. The location of the vertical interface of the two materials was varied from $y \approx 0.3$ cm to $y \approx 1.0$ cm. The results for an intermediate distance $y = 0.65$ cm is shown on Fig. 5.27.

As it is seen, we get a very similar result for both measurement methods. The shear zone leaves the high friction part, changes direction at the layer boundary and goes to the top in the low friction part close to the interface of the two materials.

The position of the shear zone and the interface for all 4 measurements detected by excavation can be found on Fig. 5.28. The solid lines show the position of the shear zone, while the dashed lines indicate the layer boundary. The different colors correspond to different measurements.

The pressure gradient leads to a curved shear zone close to the bottom. When the interface is at a larger distance from the bottom slit, the lower end of the zone becomes more vertical. Even for the highest $y$ value we found qualitatively the same behavior.

We can calculate the critical distance $y_c$ between the layer boundary and the bottom slit, for which the integral corresponding to the above observed path and a purely
Fig. 5.27: Total displacement $D(y,z)$ of the material in the limiting case of total reflection for the position of the interface $y = 0.65$ cm, measured by the excavation method (left) and MRI technique (right). The shear zone is not vertical as for a homogeneous sample, but it goes to the right where the lower friction part is located. Note that the initial transient is subtracted for the MRI measurements.

When the pressure $p$ is constant, a simple calculation gives the following expression:

Fig. 5.28: The position of the shear zone (solid lines) and the interface of the two materials (dashed lines) for different measurements with excavation. The zone is curved close to the bottom indicating that the pressure plays an important role there.
\[ y_c = H \sqrt{\frac{\mu^h - \mu^l}{\mu^h + \mu^l}} \approx 1.66 \text{ cm} \quad (5.7) \]

If we take into account that \( p \) is zero at the top surface and it increases linearly with depth, then the calculation becomes more complex. Assuming straight shear zones in both layers we reach to a quadratic equation where the smaller solution gives us a better estimation of the critical distance:

\[ y_c = H \left( \sqrt{\left(\frac{\mu^h}{\mu^l}\right)^2 - 1} - \sqrt{\left(\frac{\mu^h}{\mu^l}\right)^2 - \frac{\mu^h}{\mu^l}} \right) \approx 0.93 \text{ cm} \quad (5.8) \]

In our measurements the critical distance \( y_c \) turned to be somewhat larger than about 1 cm.

### 5.3.3 Expulsion of the shear zone

The second type of experiments was realized in a different measurement cell as shown on Fig. 5.29. Here the bottom wall of the container is moved according to the red arrow to generate a shear flow. The higher friction layer is located at the bottom, while the material with lower friction was placed above it. On the top of the sample a weight is put to decrease the relative pressure contrast inside the granular material. The weight was about 3 times heavier than the granulate below. The physical dimensions of the

\[ \text{Fig. 5.29: The experimental setup for studying the expulsion of the shear zone. We estimate the position of the zone indicated by white line.} \]
setup were slightly different, its length was about 70 cm and its cross section had a width of $W = 4.5$ cm and a height of $H = 4.0$ cm.

We repeated the measurement with different thickness of the higher friction layer between $z = 0.3$ cm and $z = 1.4$ cm. The shear zone position was detected only by the excavation method because it provides results with a similar accuracy compared to the MRI method, but we had the possibility of repeating the measurements many times.

Two detected displacement fields $D(y, z)$ are shown on Fig. 5.30.

![Fig. 5.30: Total displacement $D(y, z)$ of the material for the zone expulsion experiments, with a vertical position of the interface of $z = 0.85$ cm (left) and $z = 1.4$ cm (right). Data were collected by the excavation method. If the lower layer is not too thick, the shear zone has the expected shape, but for larger thicknesses the zone stays entirely in the higher friction part of the sample.](image)

When the thickness of the higher friction layer is below a certain critical value $z_c$, the shear zone first escapes from this layer, goes parallel with the layer boundary and then it returns to the higher friction material. Above $z_c$ the zone does not enter to the lower friction layer.

To capture the critical layer thickness $z_c$ similar calculations are needed as for the limiting case of total reflection in Sec. 5.3.2. Assuming constant pressure $p$ in the system, the integral leads to a similar formula as for the other geometry:

$$z_c = \frac{W}{2} \sqrt{\frac{\mu^h - \mu^l}{\mu^h + \mu^l}} \approx 1.1 \text{ cm} \quad (5.9)$$

For hydrostatic pressure - which is naturally present in real experiments without the top weight - the integrals for the paths give a quadratic equation for $z_c$ including both
the width $W$ and height $H$ of the cross section. Substituting these parameters to the equation we get $z_c = 2.5$ cm, which is more than double of what we calculated for constant pressure. This large difference indicates that the pressure plays an important role in this geometry. Due to the pressure gradient, the shape of the shear zone should be different from the path expected for constant pressure (see Fig. 5.29) consisting of 3 straight lines. For that reason we have put a weight on the top of the sample to minimize the effect of $p$. Using a weight having a mass of 3 times as large as the granular sample itself, the critical layer thickness becomes $z_c = 1.26$ cm. As the Janssen-effect leads to a saturation in the pressure (as described in Sec. 1.3), the $z_c$ should be somewhere between the values calculated with constant and hydrostatic pressure.

One of our measurements with a bottom layer thickness of $z = 1.1$ cm shows the transition between the expulsion of the shear zone and a straight zone in purely the higher friction part. This critical layer thickness is close to the calculated values. On Fig. 5.31 both the total displacement $D(y, z)$ of the material and its derivative, the applied strain $\gamma$ are presented to clearly show the positions of the sheared regions.

![Fig. 5.31: Splitting of the shear zone for the case of zone expulsion, observed with a bottom layer thickness of $z = 1.1$ cm by the excavation technique. Both the total displacement $D(y, z)$ (upper) and the strain $\gamma$ (below) are presented for the whole cross section of the experimental cell.](image-url)
For the measurements with $z < z_{c}$, we also detected the angles of incidence $\beta_{\text{crit}}$ at the layer boundary at both layer crossing points. We measured if this critical angle satisfies Eq. 5.5. On Fig. 5.32 all the detected angles are presented.

![Graph](image)

**Fig. 5.32:** The reciprocal of the sine of the angle of incidence $1/\sin \beta_{c}$ as a function of the vertical position of the layer boundary $z$. The ratio of the effective friction coefficients of the two materials $\mu_h/\mu_l \approx 1.63$ is shown with a dashed horizontal line on the graph.

These measurements confirmed that the Snell’s law for granular samples describes correctly the angles of incidence for the case of zone expulsion. All the points of $1/\sin \beta_{c}$ scatter around the expected value 1.63. Note that the transition between the shear zone expulsion and the straight zone is affected by the local configuration, as for $z = 1.2 \text{ cm}$ we detected only the expulsion of the shear zone without any sign of a purely horizontal zone.

### 5.3.4 Summary

In this section we studied shear zones in granular systems where two different materials are placed in the measurement cell. We introduced a series of experiments where we can easily distinguish the qualitatively different configurations. One of the four setups is already studied in the literature, now we focused on the unexplored cases and explored the limits of the analogy between granular materials and geometric optics.
We found that in the limiting case of the total reflection the shear zone will propagate parallel to the interface of the two materials at the layer boundary, which is not the case for light beams in classical optics as the intensity of the beam weakens to zero in the limiting case. A critical distance between the starting point of the shear zone (the bottom slit) and the layer boundary should exist, where the integrals of \( p\mu \) for this path and for a vertical shear path are equal. This critical distance \( y_c \) was calculated with two different assumptions for the pressure in the system. In our experiments we found that \( y_c > 1.0 \text{ cm} \).

When both ends of the shear zone are located in the higher friction part of the sample, and in the surrounding area a lower friction part is present, the zone can be formed along a path which partially leaves the higher friction material and lies in the lower friction part as close as possible to the layer boundary. In geometric optics no such a behavior is observed. The critical thickness of the bottom layer is estimated and the measurements are consistent with the calculations. For both geometries we expect the same critical angle of incidence on the side with the higher friction material, which is determined by the ratio of the effective friction coefficients of the two materials. This critical angle is detected at two positions in this geometry, and it scatters around the estimated value.

Two experimental methods were applied for this study: excavation and MRI technique. With excavation we can only measure the total displacement of the material. The achievable horizontal resolution is limited by the optical camera, but the vertical resolution is only about 0.1 cm. Using the MRI device has the advantage of a temporal resolution, which allows us to separate the initial transient effects and the steady state of the system. However, the spatial resolution in the horizontal plate is not as accurate as for the optical method. Both methods gave a very similar result meaning that the initial transient effects do not have a strong effect on the displacement field.
6. Summary and outlook

In my doctoral work I have investigated various aspects of shear zone formation in granular media experimentally. In the first part of this work, I focused on the formation of wide shear zones in homogeneous granular materials and the time evolution of the zone width, before it reaches the stationary shape. Then I introduced our results on the position and orientation of elongated grains during shear. At last the shape of the shear zone in layered granular materials were investigated. The main findings of our studies are summarized here.

The wide shear zones were generated in two types of split-bottom shear cells. In both cases two parts of the setup are moved with respect to each other to generate a shear flow. In the straight shear cell the side walls were two L shaped sliders except for one case discussed later. In the cylindrical geometry the bottom of the container consisted of a circular plate and a ring around it, and shear was generated by rotating the bottom circular plate. There is no fundamental difference between the two geometries, but for a specific question there are practical arguments for choosing one or the other. In the linear cell understanding the flow profile is easier due to the simple translation of one of the sliders, but the total applied shear is limited by the size of the setup. In the cylindrical geometry the flow profile is somewhat more complex as the streamlines are curved, but practically an infinitely large shear deformation could be achieved.

Both two and three dimensional measurement techniques were applied in order to take benefit of the complementary information provided by the different experimental methods. X-ray Computed Tomography (X-ray CT), nuclear Magnetic Resonance Imaging (MRI) as well as the excavation method helped to reveal the internal structure of the material, while optical imaging was used to collect large amounts of data from the surface layer. The latter technique allowed to reduce the statistical noise of the data by averaging numerous measurements. The measurement device often limits the size of the experimental cell, thus we used the straight shear cell for the MRI measurements.
while the cylindrical shear cell was more suitable for studying shear flows with the X-ray CT device. The excavation method is an invasive technique for optically detecting the displacement of the sample below the surface as it is realized by removing the upper surface of the material layer by layer. This technique can only reveal the total displacement of the sample in contrast to the three dimensional imaging techniques, where monitoring the movement of the grains was possible step by step.

I have characterized the evolution of the shear zone width in both the straight and the cylindrical shear cell with optical cameras, X-ray CT and MRI techniques. Spherical, irregular and elongated particles were involved in these experiments. I have determined the width of the sheared region by fitting an error function to the surface displacement profile for the straight cell, or by fitting a Gaussian function to the shear rate profile for the cylindrical cell. We prepared two initial configurations: a random distribution of the particles or when the direction of shear was reversed after the stationary state was reached. The straight and the cylindrical shear cell resulted a very similar zone width curve for the case of the random initial configuration indicating that the curvature of the cylindrical cell had no significant effect on the zone width. The three dimensional measurements revealed that the evolution of the zone width was similar at the surface layer and in the bulk of the sample. In the stationary state of the system the zone widths in the two transient measurements were equal, and interestingly for angular sand particles the stationary width was smaller than for spherical glass beads.

For all the materials the main tendency in the evolution of the shear zone width was similar. Starting from a random configuration, the zone width was wider at the beginning of the process, later it strongly decreased, and finally it slightly increased back to reach its stationary value. The strong decrease occurred roughly between a local strain of 0.1 and 1, and for particles with higher shape anisotropy it occurred at larger strain. When the direction of shear was reversed the shear zone was even wider than in the random configuration. This can be understood if we take into account that shearing leads to an anisotropic force network involving small empty volumes behind the grains. Reversing the shear results an easier grain motion and a wide shear zone until these empty spaces are filled. Thus, in this case the zone width strongly decreased until about a local strain of 0.1 was reached. After this the zone width slightly increased. Above a local strain of 1 the system forgot its initial state and the evolution became
very similar to the case of random initial configuration. The non-monotonic zone width change provides a new challenge to the existing theoretical models. More and more numerical works are able to reproduce the formation of wide shear zones in split bottom geometries, but up to now these were tested mostly only for the stationary state of the system.

A considerable part of this doctoral work was to develop a specific particle tracking algorithm for detecting the position and angle of elongated particles in shear flows to determine their movement and rotation on the optical images. This software has routines of identifying the cylindrical or nearly elliptical (rice) grains. The particles were detected also on the three dimensional CT scans using an other software. In addition to the particle tracking, Particle Image Velocimetry (PIV) was used to determine the flow field of the granular materials on the top surface. The PIV software detected the average displacement of a dozen of grains with a sub-pixel resolution, which information was used to determine the applied local strain of the sample.

The orientation of elongated particles was tracked in the cylindrical shear cell by analyzing optical and X-ray CT data. Nearly ellipsoidal rice grains as well as cylindrical glass rods with different aspect ratio were used in the optical experiments, while slightly rounded cylindrical wooden pegs with three different aspect ratio were examined in the CT measurements. The average orientation of the particles was nearly parallel to the flow direction, and inclined towards the velocity gradient at a small angle $\Theta_{av}$. Grains with larger elongation had a narrower orientational distribution with a smaller background level. For the case of cylindrical glass rods with small aspect ratio the orientational distribution was unusual with two peaks, which corresponded to the two possible orientations of the largest extensions (diagonals) of the grains.

On the optical images I determined the orientation of elongated grains using my particle tracking code. The average orientation angle of the particles was measured in the stationary state as a function of shear rate $\dot{\gamma}$. A wide range of shear rates was covered by applying three different rotation speeds of the bottom plate and taking benefit of spatial variations of $\dot{\gamma}$. The value of $\Theta_{av}$ was measured to be practically independent of the shear rate for the whole range (three decades) of $\dot{\gamma}$. The order parameter describing the strength of the orientational ordering slightly decreased for high shear rates as the inertial effects increased the fluctuations in the grain orientation.
The average alignment angle decreased with increasing aspect ratio of the grains, and rod-like grains had a smaller $\Theta_{av}$ than rice particles with the same elongation. The value of $\Theta_{av}$ decreased with increasing order parameter meaning that particles with larger aspect ratio are more likely to orient close to the flow direction, and the width of the orientational distribution (which is closely connected to the order parameter) becomes smaller. The above described observations indicate that ordering of the particles can influence the behavior of the flowing material. Further investigations would be necessary to determine how the ordering affects the flowing properties in other configurations, for example in silos, which are important for agricultural and industrial applications.

With the help of my particle tracking code, the evolution of the shear alignment for two types of initial conditions was determined, similarly to the zone width measurements: from an initially random orientational distribution and when the direction of shear was reversed. For the random initial condition the grains rotated mostly in the direction dictated by the flow, therefore the average alignment angle $\Theta_{av}$ continuously decreased while the orientational order parameter increased. For the case of the reversed shear, first the original ordering got destroyed, and many of the grains rotated backwards to reach the new alignment angle. The net rotation of the grains was calculated to study the orientation process more in details. In the stationary state of the system the particles in average rotated in the direction determined by the flow field, but the rotation speed depended on the orientation $\theta$. Grains almost parallel to the flow were rotating slower than the ones perpendicular to it. During both initial transients the average rotation speed had an opposite sign within a certain range of $\theta$. This was expected for reversed shear but a very similar rotation speed was measured for the initially random configuration. The X-ray CT data turned to be more accurate for calculating the rotation of the grains as essentially all the particles were detected and their two Euler angles were captured directly. Since X-ray CT time was limited, we could only determine the distribution of grain rotation speeds only for the stationary case with this method. For the initial transients these data were collected only by the optical method.

The evolution of the zone width, packing density and ordering was measured simultaneously for spherical and elongated particles with various aspect ratio for an initially random orientation of the grains. The experiments were recorded with the X-
ray CT device by using the cylindrical shear cell. Additional MRI measurements were performed with the straight shear cell for comparison, but only with nearly spherical particles. The characteristic strain scales corresponding to the Reynolds dilation, the narrowing of the shear zone, the decreasing of the average alignment angle and the increasing of the order parameter were compared by fitting exponential decay functions to all of the measured curves. The Reynolds dilation had a typical strain scale independent of the particle shape. The strain scale extracted from the MRI data was in accordance with this observation. The evolution of $\Theta_{\text{av}}$ was also independent of the elongation of the grains, but the corresponding strain scale was much larger than that of the Reynolds dilation. For both the narrowing of the shear zone and for the increase of the orientational order parameter the characteristic strain scale increased with the aspect ratio, which indicated that these two quantities can be connected. A slight increase in the packing density for large shear strain values was observed for elongated particles, which is connected to the orientational ordering. It would be interesting to relate the formation of the shear zone with the development of the force network in the sample. Unfortunately, our measurement techniques did not allow the determination of the force network.

We believe that the physics behind this zone width change could be better explained by relating it to the formation of the force network during shear for various grain aspect ratios. The shear deformation needed to reach the critical state can be estimated by the largest deformation scale among the above described processes. Thus the history of the granular sample will only be erased when reaching this deformation. As a consequence, if elongated particles are used, we need to shear the sample longer to reach the steady state compared to spherical or irregular grains. This observation can be important not only for physicists, but also for engineers working in agriculture or in industry.

The complex shape of the shear zone were studied for layered granular samples consisting of materials with two different effective friction coefficient. I performed the experiments in the straight shear cell by applying the excavation technique, and additional MRI scans were recorded. One configuration was analyzed in previous studies, now we focused on two further configurations. Since the earlier studies showed that many aspect of shear zone formation in layered granular systems and the trajectory of
a light beam propagating through a layered material are analogous, we also focused on the similarities between these two systems.

In the first type of layered granular configurations the container consisted of two L shaped sliders and the interface between the low and high friction part was a vertical plane. In this geometry we varied the distance between the bottom slit and the vertical layer boundary, but the shear zone started always in the high friction part of the sample. The results of the experiments matched our expectations, as the zone left the high friction part, changed direction at the layer boundary and reached the top surface vertically next to the layer boundary. This configuration was confirmed by two detection methods: MRI and excavation of the material. In geometric optics this case is analogous to the limiting case of total reflection, but in optics the intensity of the light beam goes to zero and total reflection occurs at the critical angle of incidence. A similar critical angle was expected for granulates, which was studied in the other experimental geometry.

For the second type of layered granular configurations both ends of the shear zone was forced to be in the high friction part of the sample. Shear was induced by moving the bottom wall of the container and the low friction material was placed to the top of the high friction one. In this geometry the shear zone was expelled from the layer with larger internal friction. The angles of incidence were expected to have the same critical value for both layer crossing points as in the first type of layered configurations. The angles determined from the measurements scattered around the expected value. In both types of layered experiments the above described shear zones occurred only if the low friction layer was close to the estimated position of the shear zone for a homogeneous sample. We estimated the critical distance of the low friction layer, and the results of the measurements were consistent with the calculations.

As the layered granular systems are typical in geology, studying them during shear can help us to better understand some natural processes occurring in the upper part of the Earth’s crust. For example, after earthquakes, there are certain regions where the relative movement of the neighboring tectonic plates is almost parallel to the interface. In a general case, the relative movement can be separated to a normal and a tangential component, and the tangential one describes the amount of sustained shear. Our
experiments, combined with the fluctuating narrow band model, provide a new tool for geologists to analyze a large set of collected data from a new point of view.


List of publications

Publications in peer reviewed scientific journals:


Publication in a peer reviewed conference proceedings:


Publication in Hungarian:


The publications below are not included in this dissertation:


Theses

1. For sheared granular samples I determined experimentally the evolution of the width of the shear zones formed in split-bottom shear cells by using optical imaging. The width of the sheared region as a function of the local strain was found to have a non-monotonic shape including a strongly decreasing part for two different initial conditions (random and oppositely sheared). The local strain, at which the sharp zone width decrease happens, was found to be larger for elongated grains than for spherical particles, while the ratio between the initial and the stationary zone width increased with the shape anisotropy of the grains [P1].

2. Using X-ray Computed Tomography I characterized the packing density of the grains, the orientational ordering as well as the width of the shear zone simultaneously as a function of the local strain for an initially random configuration using both elongated and spherical granular materials. The characteristic strain rates describing Reynolds dilation and the evolution of the average alignment did not depend on the shape of the particles, but strain scales corresponding to the narrowing of the shear zone and the evolution of the orientational order parameter systematically increased with the particle elongation [P1].

3. I have developed a particle tracking software which detected the orientation and position of granular particles consisting of elongated grains in shear flows recorded by optical imaging. Under stationary shear the average grain alignment was found to be not parallel to the streamlines, but it was slightly tilted towards the velocity gradient. The alignment angle was measured to be independent of the shear rate across three decades. The shear induced orientational order increased and the alignment angle systematically decreased with increasing grain aspect ratio. Comparing nearly ellipsoidal and cylindrical particles with the same aspect ratio, the cylindrical grains had a smaller average alignment angle. The alignment of elongated particles was found to be very similar at the surface and inside the material determined by X-ray Computed Tomography measurements [P2, P3, P5].

4. I determined the evolution of the alignment of granular particles in shear flows started from a random orientation distribution or from an oppositely sheared configuration by analyzing optical measurements. In the case of the initially random
distribution, the average alignment angle was found to decrease to its stationary value, and the order parameter was measured to increase monotonically until it reached its final value. For the case of reversed shear, the average alignment angle increased, i.e. the grains reached the new preferred direction by rotating backwards, and the order parameter decreased at the beginning of the transient before increasing back to the stationary state, as the original ordering got destroyed first. The angular velocity of individual grains was also determined. This depends on the local configuration and the interactions with the neighbors, thus involves large statistical noise. The average rotation speed of many grains was found to depend systematically on the grain orientation. Namely, particles parallel to the flow are in average rotating slower than those perpendicular to the flow direction. Grains under stationary shear rotate typically in the direction dictated by the flow, however, during the initial transients, particles which are close to parallel to the streamlines in average rotate backwards [P2, P3, P5].

5. I determined experimentally the complex shape of the shear zones for layered granular systems consisting of two kinds of particles with different effective friction coefficients by applying the excavation method and analyzing Magnetic Resonance Imaging measurements. When one end of the shear zone was forced to be in the high friction part of the sample, the shear zone was found to leave the high friction part, it changed its direction at the vertical layer boundary and it propagated parallel to the layer boundary lying entirely in the low friction part of the sample. In a different geometry, where both ends of the shear zone were forced to be in the high friction part of the sample, the shear zone left the high friction part, it went parallel to the layer boundary between the two materials, and then it returned to the high friction layer. Both observations were found to appear only if the low friction part is close enough to the fixed ends of the shear zone. The position of the layer boundary was calculated, which corresponds to the transition between the cases when the zone crosses the boundary and when it does not cross [P4].
Résumé

This dissertation is dedicated to reveal the unexplored properties of shear zones - regions where the shear is localized - in dry granular materials experimentally by applying optical and additional three dimensional imaging techniques.

The evolution of the width of the shear zone was measured for spherical, irregular and elongated granular particles. Starting from a random configuration, the width of the sheared region was found to be large at the beginning of the process, and later it decreased to its stationary value for all kinds of particles studied. This decrease occurs at larger strain for particles with larger shape-anisotropy. For a different initial condition, when the direction of the shear was reversed, the zone width was measured to be larger than any time during the random transient. After the local strain reaches unity (the grains in the shear zone passed their neighbors), the system forgot its original state, and the behavior becomes identical to the other initial condition.

I determined the orientation of elongated granular particles under shear for nearly ellipsoidal rice grains and cylindrical glass rods by developing a particle tracking code. The particles had a preferred direction almost parallel to the flow direction, and they are tilted towards the velocity gradient by an angle called average alignment angle. This alignment angle was found to be independent of the shear rate across three decades. For grains with larger aspect ratio the alignment angle was smaller and the order parameter - which monitors the strength of the ordering - was larger. The evolution of the orientational order was measured for two initial conditions. The particles in average rotated in the direction dictated by the shear flow in the stationary state, but many of them rotated backwards during both types of transients. The characteristic strain scales describing the evolution of the packing density, the average alignment angle, the order parameter and the narrowing of the zone width were calculated simultaneously.

I have investigated experimentally the complex shape of the shear zones in layered granular samples consisting of two materials with different frictional coefficients. Two new configurations were examined: one which is similar to the limiting case of total reflection in geometric optics, and the other one where the shear zone is expelled from the high friction layer. The criteria for the occurrence of these phenomena were estimated.
Összefoglaló

A disszertáció fő célkitűzése a nyírás zónák – olyan térrészek, ahol a nyírás lokalizálódik – még feltérképezetlen tulajdonságainak kísérleti feltárása száraz szemcsés anyagokban optikai, továbbá három dimenziós képalkotási technikák segítségével.

Megmértem a nyírás zóna szélességének fejlődését gömb szerű, szabálytalan alakú, valamint elnyújtott alakú részecsékre. Véletlenszerű kezdeti konfigurációnból indítva a rendszer a nyírt tartomány szélessége kezdetben nagyobbán adódott, majd később jelentősen lecsökkent az összes vizsgált anyag esetében. A csökkenési stádium későbbre toldódott nagyobb alak-anizotrópiájú részecsék vizsgálatakor. Egy másik kezdeti állapot tranziens sztanúlnyomozva, ahol a nyírás irányát a mérés megkezdése előtt megfordítottuk, a zóna szélességét nagyobbán mértek, mint bármikor az előzőr enlített tranziens alatt. Miután az elszenvedett lokális nyírás elérte az egységnyi értéket (tehát a szomszédos szemcsék egy részecskenyit mozdultak egymáshoz képest), a rendszer elfelejtette az eredeti állapotát, ugyanis a kölönbszoró kezdeti állapotokra azonos viselkedést tapasztaltunk.

Egy részecske nyomon követő programot fejlesztettem, amellyel meghatároztam az elnyújtott részecskekből álló anyagok rendeződését nyírás hatására ellipszoidhoz hasonló alakú rizsszemek, valamint henger alakú üvegrudak esetére. A részecsék tipikusan közel párhuzamosan álltak a folyás irányával, attól kissé a sebességgradiens irányába dölték ki, melynek mértékét átlagos orientációs szöggel jellemzettük. Ez a szög függetlennek adódott a nyírás ráta-tól három nagyságrenden keresztül. Elnyújtottabb részecsékre az átlagos orientációs szöget kisebbnek mértek, míg a rendparaméter – ami az orientáció erősségét jellemzi – nagyobbán adódott. A rendeződés időbeli fejlődését két kezdeti állapotra vizsgáltuk. A részecsék átlagosan a folyás által meghatározott irányba forogtak a stacionárius állapotban, de sokuk forgott vissza felé a kétfele kezdeti tranziens alatt. A térkitöltés, az átlagos orientációs szög, a rendparaméter, valamint a zóna szélességének fejlődését jellemző nyírású skálákat egyazon mérés során sikerült meghatározunk.

Kísérletileg vizsgáltam a rétegzett szemcsés anyagokban megfigyelhető nyírású zóna alakját, ahol a különbször rétegek eltérő effektív súrlódási együttthatójú anyagokból álltak. Két új elrendezést tanulmányoztunk: az egyikben az optikából ismert teljes visszaverődéséhez hasonló eredményt kaptunk, a másikban pedig a nyírású zóna kíszorult a nagyobb súrlódási együttthatójú anyagból. Mindkét jelenség fellépéséhez szükséges feltételekre becsült adtunk.
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