Grand Canonical Potential for a Static Quark–Anti-quark Pair at $\mu \neq 0$

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We present numerical results on the static quark–anti-quark grand canonical potential in full QCD at non-vanishing temperature (T) and quark chemical potential (μ). Non-zero μ-s are reached by means of multi-parameter reweighting. The dynamical staggered simulations were carried out for $n_f = 2+1$ flavors with physical quark masses on 4×12^3 lattices.

1. INTRODUCTION

Forces among infinitely heavy quarks separated by distance r are of great significance both at $T = 0$ and when they are surrounded by an interacting medium with temperature T. The lattice formulation of their free energy was successfully carried out already 20 years ago [1]. Numerous lattice results are available in quenched [2] and also in full QCD [3,4] at $T > 0$.

There are recent calculations to determine the free energy in channels with definite color transformation properties. The gauge invariance and the interpretation of these singlet/octet channels in view of [5,6] raised some new, interesting questions.

All the previous results were obtained at vanishing baryonic chemical potential. Lattice simulations at $\mu \neq 0$ are difficult, because they are hindered by the sign problem of the fermion determinant. Recently, new techniques have been proposed to study lattice QCD at non-vanishing μ [7,8,9,10]. In this paper we use the multi-parameter reweighting to reach non-zero μ values. We determine the grand canonical potential of the heavy quark–anti-quark system up to $\mu = 140$ MeV in quark chemical potential on $N_t = 4$ lattices for physical quark masses. We use the same configurations as have been used in [11].

2. FORMALISM

The free energy of a QCD system at $\mu = 0$ with a static quark and anti-quark separated by distance r can be expressed as follows [1]:

$$\exp(-F_{q\bar{q}}(T,r)/T + C) = \langle L(r)L^\dagger(0)\rangle_T \qquad (1)$$

where $L(r) = \frac{1}{3}\mathrm{tr}\prod_{\tau=0}^{N_\tau-1} U_4(\tau,r)$ is the trace of the temporal Wilson line, i.e. the Polyakov loop and the $U_4(\tau,r)$-s are the link matrices in the time direction. C is needed for renormalization, and depends only on the lattice spacing (see below). $F_{q\bar{q}}$ is the additional free energy due to the presence of the heavy quark–anti-quark pair.

We can ask for the grand canonical potential of the heavy quark system, if we place it into a medium with T temperature and μ chemical potential. One can modify the above formula in an obvious way

$$\exp(-\Phi_{q\bar{q}}(T,\mu,r)/T + C) = \langle L(r)L^\dagger(0)\rangle_{T,\mu}, \qquad (2)$$

where $\Phi_{q\bar{q}}$ is the additional grand canonical potential due to the presence of the heavy quark–anti-quark.

Both $F_{q\bar{q}}$ and $\Phi_{q\bar{q}}$ contain the self-energy of the static quarks. At $T = 0$, $\mu = 0$ one usually normalizes the potential at a given distance: e.g. by demanding that $V_{q\bar{q}}(r_0) = 0$. Here we used the Sommer scale ($r_0 = 0.49$ fm). The r independent shift, which is applied to the potential to satisfy the renormalization condition, is just the self-energy at a given lattice spacing. This $T = 0$ shift has to be removed from the finite T, μ potentials as well. Using these renormalized grand canonical potentials the renormalization of the Polyakov-loop can also be done (e.g. [12]).

In eq. (2) we need the expectation value of the Polyakov-loop correlator for non-vanishing μ val-

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ues. For moderate chemical potentials one can use the multi-parameter reweighting: one generates configurations at $\mu = 0$ and reweights the generated ensemble in the β and μ parameters, simultaneously. The new parameters are usually chosen to improve the overlap between the simulated and the target configurations, or saying it in another way to reduce the systematic errors of reweighting to as small as possible. Starting from the transition point at $\mu = 0$ the overlap is maximized along the transition line. However one is not forced to follow the transition line during the reweighting. The transition ensemble contains enough information about both the confined and the deconfined phases so a constant T reweighting is also possible. (Though this reweighting cannot reach as far in μ as the one along the transition line.) The reweighting formula for the Polyakov-loop correlator is:

$$\langle L(r)L^\dagger(0)\rangle_{T,\mu} = \frac{\langle L(r)L^\dagger(0)w(T,\mu)\rangle_{T_c,0}}{\langle w(T,\mu)\rangle_{T_c,0}} \qquad (3)$$

After diagonalizing the transformed fermion matrix the w weights can be calculated for arbitrary μ values using the explicit formula of [13].

3. RESULTS AT FINITE μ

We studied QCD with 2+1 flavors of dynamical staggered quarks at physical quark masses, that is at $m_{u,d} = 0.0096$ and $m_s = 0.25$ on 4×12^3 lattices. 150 000 configurations were simulated at the critical gauge coupling: $\beta_c = 5.1893$. The scale was set using the Sommer prescription. The Wilson-loops were measured on 24×12^3 lattices for relatively high quark masses. The scale for the physical quark mass is obtained from a chiral extrapolation: $r_0/a = 1.77(2)$. Note that the Sommer-scale depends weakly on the quark mass, therefore the extrapolation is quite safe.

Since β changes along the critical line, it is necessary to deal with the problem of renormalization. In order to follow our renormalization prescription described in sec. 2 we calculated the $T = 0$ potentials not only at β_c but at further two β values. From the values of the potentials at a reference distance (in our case r_0) one gets a β dependent shift, which is used to renormalize the potentials along the critical line.

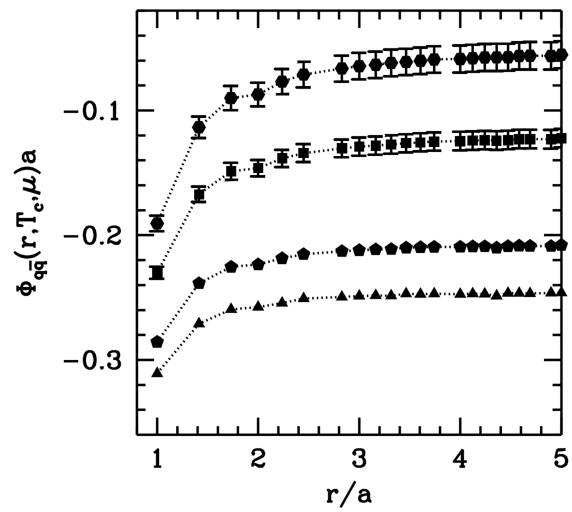

Figure 1. $\Phi_{q\bar{q}}$ along the $T = T_c(\mu = 0)$ line for $\mu a = 0, 0.05, 0.10, 0.15$. Higher μ corresponds to lower curve.

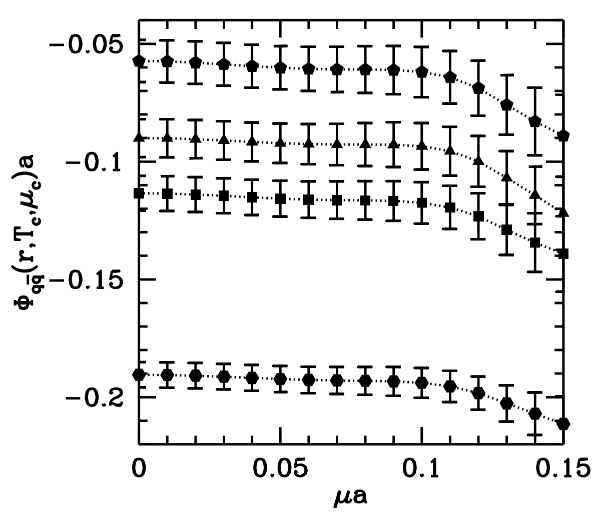

Figure 2. The $\Phi_{q\bar{q}}$ as the function of μ for distances $(r/a)^2 = 1, 2, 3, 20$ along the transition line. Larger r corresponds to higher curve.

On Fig. 1 the potential is plotted as the function of the distance for various chemical potentials. The reweighting was done along a constant temperature line. For higher μ values the potential flattens out at short distances, whereas for smaller μ values it reaches its asymptotic value only at larger distances.

Fig. 2 shows the potentials as the function of μ. The reweighting is done along the transition line. Until the critical point (which is around $\mu a \approx 0.18$) only slight changes can be observed. After this point the errors get enlarged. Since there is only a small change in β, the renormalization has only a small effect on the result.

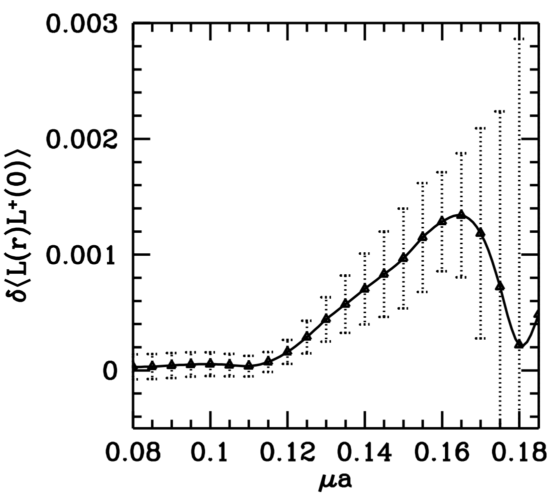

Figure 3. $\delta\langle L(r)L^\dagger(0)\rangle = \langle L(r)L^\dagger(0)\rangle_\mu - \langle L(r)L^\dagger(0)\rangle_{\mu=0}$, i.e. the difference in the correlator to the $\mu=0$ case as the function of μ. The distance is $(r/a)^2 = 13$.

It can be instructive to examine the change in the Polyakov-loop correlator along the transition line. Fig. 3 shows the difference in the correlator to the $\mu=0$ case for a given distance. The correlator starts to grow intensively before the second order endpoint. Unfortunately, at the same time the statistical errors get enlarged.

4. SUMMARY

We have determined the heavy quark–antiquark grand canonical potentials for non-vanishing μ values on $N_t = 4$ lattices. We have presented results at fixed T and various μ values as a function of r. We have found no significant change in the potential along the transition line. Around the critical endpoint we have observed a rise in the correlations. The renormalization at $T{=}0/T \neq 0$ and $\mu{=}0/\mu \neq 0$ was also discussed.

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