Heavy quark systems at finite temperature from lattice QCD

PhD Theses

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Introduction

QCD is the theory of the Strong interactions. The most important properties of QCD are probably quark confinement and asymptotic freedom. On the one hand, the elementary particles of QCD - the quarks and the gluons - are not observable in nature. What we see as final and initial states in scattering experiments are bound states called hadrons. On the other hand, as the momentum transfer in QCD processes becomes higher, the effective coupling decreases, asymptotically approaching zero. This makes perturbative predictions possible at large energies, where QCD gives a consistent explanation of a variety of experimental data. At low energies however, the coupling constant becomes larger, and perturbation theory breaks down. At low energies, we need non-perturbative calculation methods, to get the predictions of QCD. The *ab initio* method of solving QCD in this case is lattice field theory. Here we define the theory on a finite space-time lattice, and with gradual decrease in the lattice spacing, and continuum extrapolation, we can get a prediction of continuum QCD.

An important consequence of asymptotic freedom is the fact that at high temperatures, the coupling becomes smaller. We expect the bound states to dissolve, and a new kind of matter, the so called quark-gluon plasma, to manifest. Both the transition and the quark-gluon plasma itself can be studied with lattice methods. For example, we know from lattice simulations, that the transition is an analytic cross-over with a pseudo-critical temperature of $T_c \approx 150 MeV$.

There is also considerable experimental work being done on the nature of hot QCD. Experiments at the CERN SPS, at RHIC and at the LHC have all given considerable contributions to the field. The theoretical work presented in my thesis provides input to the phenomenological models trying to understand these experiments.

I investigate bound states of heavy quark-antiquark pairs (heavy quarkonia) in a finite temperature medium. These have been under heavy investigation, since J/Ψ suppression[1] is a possible signature for quark-gluon plasma formation in heavy ion experiments. Such a suppression pattern has indeed been found at the experiments of the CERN SPS, Brookhaven RHIC and CERN LHC colliders. However, the actual theoretical prediction for the finite temperature behavior of the J/Ψ in QCD is still poorly understood. Thus, we can't be sure about the actual interpretation of the observed suppression.

Analysis of charmonium spectral functions

The main problem is that lattice QCD calculations are done in Euclidean space-time, i.e. in imaginary time. Calculating real-time properties, i.e. performing an analytic continuation in time however, is a difficult problem. The most important quantity is the so-called spectral function (SF). If we knew the SF, we would know the dissociation temperature of J/Ψ, as well as the temperature dependence of it's width, and the diffusion coefficient of the charm quarks. The SF is related to the Euclidean correlators by the following integral transformation:

$$G(t,T) = \int_0^\infty d\omega \frac{\cosh\left(\omega\left(t-\frac{1}{2T}\right)\right)}{\sinh\left(\frac{\omega}{2T}\right)} A(\omega,T) \equiv \int_0^\infty d\omega K(\omega,t,T) A(\omega,T). \quad (1)$$

This transformation has to be inverted to obtain the SFs. This is a well-known hard (ill-posed) problem, since the Euclidean correlator is not very sensitive to the details of the SF. The usual approach is the so-called Maximum Entropy Method (MEM), that was first proposed in lattice QCD by Ref. [2]. Here one has to maximize:

$$Q = -\frac{1}{2}\chi^2 + \alpha S, \quad (2)$$

where χ^2 is the usual, it measures how close the reconstruction is to the data, and the S Shannon-Jaynes entropy measures how close the reconstruction is to the so-called prior function $m(\omega)$:

$$S = \int d\omega \left(A(\omega) - m(\omega) - A(\omega) \ln \left(A(\omega)/m(\omega)\right)\right). \tag{3}$$

After obtaining the optimal $A(\omega)$ at a given α, the parameter α has to be averaged over. In the first publication related to my PhD thesis [3] I used MEM to study the spectral function of psuedoscalar and vector $\bar{c}c$ correlators at finite temperature:

1. I created a C++ code for the Maximum Entropy analyis of correlators in Euclidean field theories.
2. I tested the reliability in the code and the Maximum Entropy Method in general by mock data analysis. This is done in the following way:
   (a) I write down a SF $A_{\text{mock}}(\omega)$, and calculate the Euclidean correlator from equation (1). I add Gaussian errors and generate mock configurations.
   (b) I reconstruct these using MEM the same way I would reconstruct the lattice data.
   (c) I compare the reconstructed spectral function with $A_{\text{mock}}(\omega)$.

   The most important result of this analysis is that the reconstruction can give the position of the ground state peak relatively accurately, but not the width.
3. I calculated the J/Ψ (vector) and η_c (pseudoscalar) correlators at finite temperature using the lattice QCD code of the Budapest-Wuppertal collaboration. I reconstructed the spectral functions using MEM. This is the first lattice study of the charmonium spectral functions that uses $2+1$ flavours of dynamical quarks. Earlier work mostly used the quenched approximation. We used dynamical Wilson quarks, a pion mass of $m_\pi = 545$MeV, and a lattice spacing of $a = 0.057$fm. I made a full error analysis of the position of the ground state peak, and found that up to the temperature $1.3T_c$, the position is consistent with a constant in both the vector and pseudoscalar channels, with an errorbar of 30% at the highest temperature.
4. I calculated the following quantity [4]:

$$G/G_{\text{rec}} = \frac{G\,(t,T)}{G_{\text{rec}}\,(t,T)} = \frac{G(t,T)}{\int A(\omega, T_{\text{ref}}) K(\omega,t,T) \mathrm{d}\omega}. \tag{4}$$

   To calculate this quantity the spectral function only has to be reconstructed at one temperature T_{ref}, which can be chosen to be the lowest available temperature, where the reconstruction is the most reliable, since we have the highest number of data points in this case. An other advantage is that this ratio can be calculated at higher temperatures, where the MEM reconstruction is not that reliable. I calculated this ratio up to a temperature of $1.4T_c$. An important property of G/G_{rec} is that if the temperature dependence in G only comes from the integral kernel, and not the spectral function, this ratio will be one. This is exactly what happens in the pseudoscalar channel, but not in the vector channel(Figure 2). Here, we can go one step further and define the mid-point subtracted version [5]:

$$\frac{G^-}{G^-_{\text{rec}}} = \frac{G\,(t,T) - G\,(N_t/2,T)}{G_{\text{rec}}\,(t,T) - G_{\text{rec}}\,(N_t/2,T)}, \tag{5}$$

which has the property that if the temperature dependence of the spectral function only comes from a narrow transport peak at $\omega = 0$, or in other words, if the temperature dependence of the Euclidean correlator only comes from the integral kernel and from a temperature dependent, but Euclidean time independent constant, then this ratio will be 1. This is what happens in the vector channel (Figure 3).

The conclusion of this analysis is therefore the following: Up to a temperature of $1.4T_c$ the spectral function of the pseudoscalar channel is approximately temperature independent. The vector channel spectral function is consistent with a temperature independent part at $\omega > 0$ and a temperature dependent transport peak at $\omega \approx 0$.

The correlator of Polyakov loops

In the second part of my PhD thesis I consider the static (or infinite mass) limit of $\bar{Q}Q$ systems. Here there is a non-perturbatively well defined quantity, that can be calculated with Euclidean methods directly (without analytic continuation). This static $\bar{Q}Q$ pair free energy, or more precisely, the excess free energy we get by inserting two static charge in the medium. This free energy is related to the correlator of Polyakov loops:

$$F_{\bar{Q}Q}(r) = -T \ln C(r,T) = -T \ln \left\langle \sum_{\mathbf{x}} \mathrm{Tr} L(\mathbf{x}) \mathrm{Tr} L^{+}(\mathbf{x}+\mathbf{r}) \right\rangle , \tag{6}$$

and is an important input for phenomenological models. For the simulations, we used $2+1$ flavours of staggered quarks, with physical quark masses. My contributions are:

5. Here, similarly to Ref. [6] the renormalization prescription of the free energy only uses finite temperature lattices. However, [6] used Wilson fermions, where the implementation of such a prescription is straightforward. Therefore, I introduced a non-perturbative renormalization procedure for the static quark and static quark-antiquark pair free energies, that only uses finite temperature lattices, and is applicable to simulations with staggered fermions.

6. I calculated the continuum limit extrapolations of the renormalized static single quark free energy in the temperature range of $130 - 390$ MeV.

7. I calculated the continuum limit extrapolations of the renormalized quark-antiquark pair free energy in the temperature rang of $150 - 350$MeV (Figure 4).

8. I calculated the electric and magnetic screening masses in the correlator of Polyakov loops in the temperature range of $160 - 450$MeV (Figure 5).

   (a) Magnetic gluons are even under Euclidean time reflection $\mathcal{R}$, while electric gluons are odd. This makes it possible to study the two electric and magnetic sectors separately [7]. We can introduce two correlators, whose sum is equal to the Polyakov loop correlator. These are the magnetic correlator in the $\mathcal{R}(\mathcal{C}) = +(+)$ sector:

$$C_{M+}(r,T) \equiv \left\langle \sum_{\mathbf{x}} \mathrm{Tr} L_{M+}(\mathbf{x}) \mathrm{Tr} L_{M+}(\mathbf{x}+\mathbf{r}) \right\rangle , \tag{7}$$

   and the electric correlator in the $\mathcal{R}(\mathcal{C}) = -(-)$ sector:

$$C_{E-}(r,T) \equiv - \left\langle \sum_{\mathbf{x}} \mathrm{Tr} L_{E-}(\mathbf{x}) \mathrm{Tr} L_{E-}(\mathbf{x}+\mathbf{r}) \right\rangle , \tag{8}$$

where $\mathrm{Tr} L_{M+} = \mathrm{Re}\,\mathrm{Tr} L$ and $\mathrm{Tr} L_{E-} = i\,\mathrm{Im}\,\mathrm{Tr} L$.

(b) As the Polyakov loop has a multiplicative UV divergence, to calculate the screening masses one does not need renormalization.

(c) I used a Yukawa potential type ansatz to fit the screening masses. For the correlated fitting I used a similar procedure as in Ref. [8]. A careful choice of the fit interval in r is needed. I used hypothesis testing (Kolmogorov-Smirnov test) to choose the fit interval. After systematic and statistical error estimation I also calculated the continuum limit extrapolation.

(d) If we compare our results of the results in the literature, we find that our results are closest to the estimates from dimensionally reduced effective field theory [9].

This work was published in Ref. [10].

Hivatkozások

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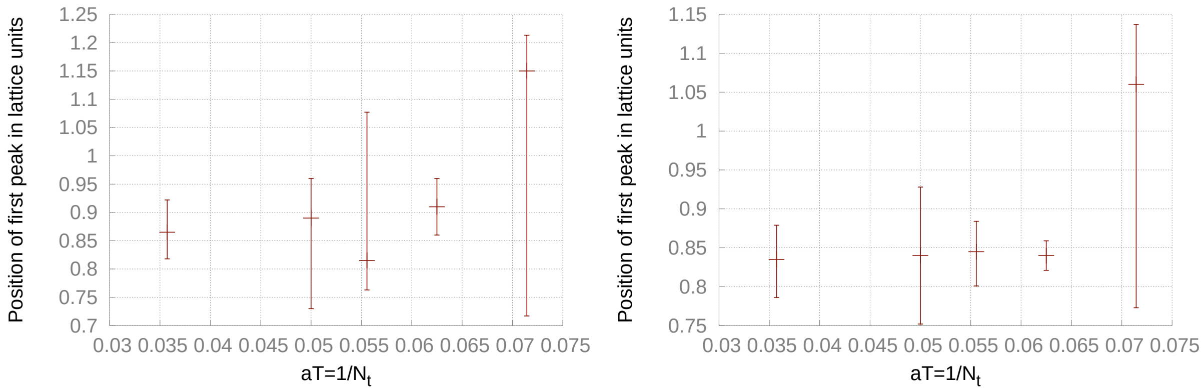
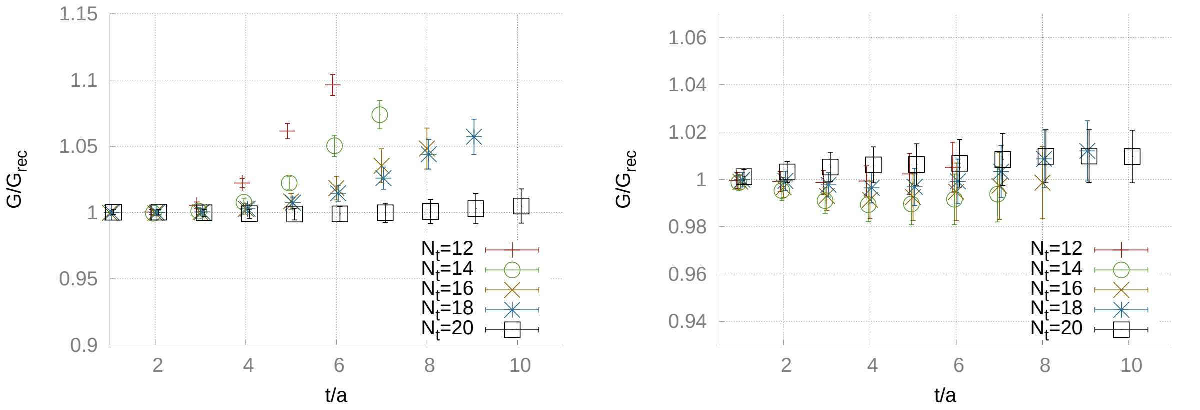
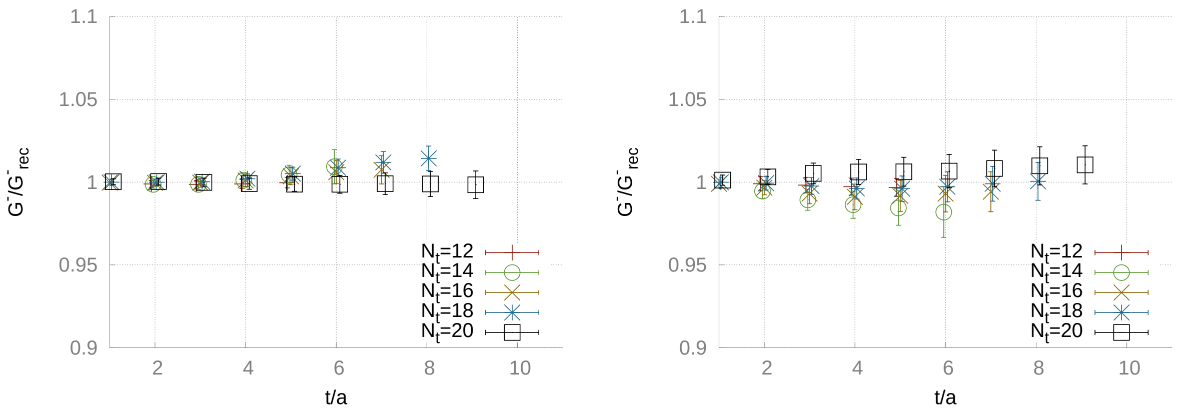


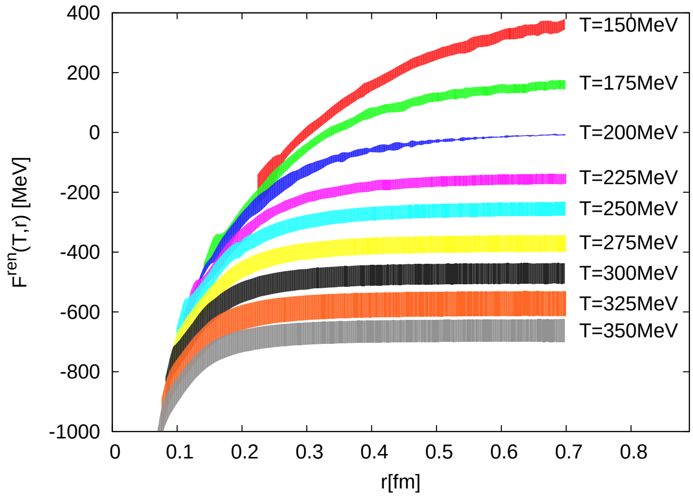
1. ábra. The position of the first peak of the SF in the vector (left) and pseudoscalar (right) channels as a function of the temperature. $N_t = 14$ corresponds to $T = 1.3T_c$.



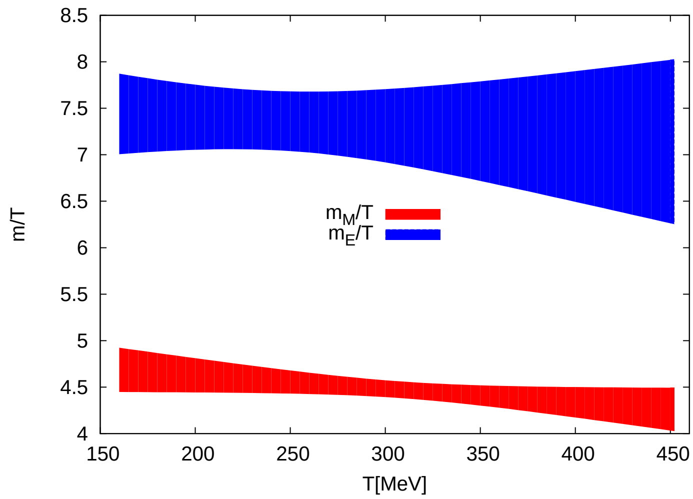
2. ábra. The ratio $G/G_{\rm rec}$ in the vector (left) and pseudoscalar (right) channels at different temperatures. $N_t = 12$ corresponds to $T = 1.4T_c$.



3. ábra. The ratio $G^-/G^-_{\rm rec}$ in the vector (left) and pseudoscalar (right) channels at different temperatures. $N_t = 12$ corresponds to $T = 1.4T_c$.



4. ábra. The continuum extrapolated static $\bar{Q}Q$ free energy at different temperatures.



5. ábra. The continuum extrapolated electric and magnetic screening masses as a function of temperature.